Chapter One: Management Science

PROBLEM SUMMARY

1. Total cost, revenue, profit, and break-even
2. Total cost, revenue, profit, and break-even
3. Total cost, revenue, profit, and break-even
4. Break-even volume
5. Graphical analysis (1–2)
6. Graphical analysis (1–4)
7. Break-even sales volume
8. Break-even volume as a percentage of capacity (1–2)
9. Break-even volume as a percentage of capacity (1–3)
10. Break-even volume as a percentage of capacity (1–4)
11. Effect of price change (1–2)
12. Effect of price change (1–4)
13. Effect of variable cost change (1–12)
14. Effect of fixed cost change (1–13)
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16. Effect of fixed cost change (1–7)
17. Effect of variable cost change (1–7)
18. Break-even analysis
19. Break-even analysis
20. Break-even analysis; profit analysis
21. Break-even analysis; indifference (1–20)
22. Break-even analysis
23. Break-even analysis; volume and price analysis
24. Break-even analysis
25. Break-even analysis
26. Break-even analysis; profit analysis
27. Break-even analysis; price and volume analysis
28. Break-even analysis; profit analysis
29. Break-even analysis; profit analysis
30. Break-even analysis; profit analysis
31. Break-even analysis
32. Multiproduct break-even analysis
33. Decision analysis
34. Expected value
35. Linear programming
36. Linear programming
37. Linear programming
38. Linear programming
39. Forecasting/statistics
40. Linear programming
41. Waiting lines
42. Shortest route

PROBLEM SOLUTIONS

1. a) $v = 300,\ c_f = \$8{,}000,$

$c_v = \$65$ per table, $p = \$180;$

$\text{TC} = c_f + vc_v = \$8{,}000 + (300)(65) = \$27{,}500;$

$\text{TR} = vp = (300)(180) = \$54{,}000;$

$Z = \$54{,}000 - 27{,}500 = \$26{,}500$ per month

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{8{,}000}{180 - 65} = 69.56$ tables per month

2. a) $v = 12{,}000,\ c_f = \$18{,}000,\ c_v = \$0.90,$

$p = \$3.20;$

$\text{TC} = c_f + vc_v$

$= 18{,}000 + (12{,}000)(0.90)$

$= \$28{,}800;$

$\text{TR} = vp = (12{,}000)(\$3.20) = \$38{,}400;$

$Z = \$38{,}400 - 28{,}800 = \$9{,}600$ per year

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{18{,}000}{3.20 - 0.90} = 7{,}826$

3. a) $v = 18{,}000,\ c_f = \$21{,}000,\ c_v = \$.45,$

$p = \$1.30;$

$\text{TC} = c_f + vc_v = \$21{,}000 + (18{,}000)(.45) = \$29{,}100;$

$\text{TR} = vp = (18{,}000)(1.30) = \$23{,}400;$

$Z = \$23{,}400 - 29{,}100 = -\$5{,}700$ (loss)

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{21{,}000}{1.30 - .45} = 24{,}705.88$ yd per month

4. $c_f = \$25{,}000,\ p = \$.40,\ c_v = \$.15,$

$v = \dfrac{c_f}{p - c_v} = \dfrac{25{,}000}{.40 - .15} = 100{,}000$ lb per month

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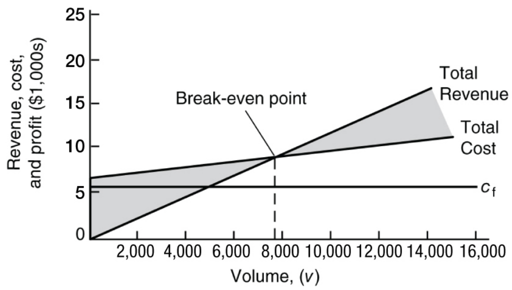

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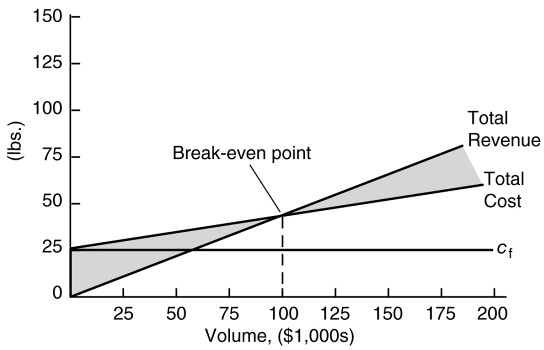


7. $v = \frac{c_f}{p - c_v} = \frac{\$25{,}000}{30 - 10} = 1{,}250$ dolls

8. Break-even volume as percentage of capacity

$= \frac{v}{k} = \frac{7{,}826}{12{,}000} = .652 = 65.2\%$

9. Break-even volume as percentage of capacity

$= \frac{v}{k} = \frac{24{,}750.88}{25{,}000} = .988 = 98.8\%$

10. Break-even volume as percentage of

capacity $= \frac{v}{k} = \frac{100{,}000}{120{,}000} = .833 = 83.3\%$

11. $v = \frac{c_f}{p - c_v} = \frac{18{,}000}{2.75 - 0.90} = 9{,}729.7$ cupcakes

It increases the break-even volume from 7,826 to 9,729.7 per year.

12. $v = \frac{c_f}{p - c_v} = \frac{25{,}000}{.60 - .15} = 55{,}555.55$ lb

per month; it reduces the break-even volume from 100,000 lb per month to 55,555.55 lb.

13. $v = \frac{c_f}{p - c_v} = \frac{25{,}000}{.60 - .22} = 65{,}789.47$ lb

per month; it increases the break-even volume from 55,555.55 lb per month to 65,789.47 lb per month.

14. $v = \frac{c_f}{p - c_v} = \frac{39{,}000}{.60 - .22} = 102{,}613.57$ lb

per month; it increases the break-even volume from 65,789.47 lb per month to 102,631.57 lb per month.

15. Initial profit: $Z = vp - c_f - vc_v = (9{,}000)(.75) - 4{,}000 - (9{,}000)(.21) = 6{,}750 - 4{,}000 - 1{,}890 =$ \$860 per month; increase in price: $Z = vp - c_f - vc_v = (5{,}700)(.95) - 4{,}000 - (5{,}700)(.21) = 5{,}415 - 4{,}000 - 1{,}197 =$ \$218 per month; the dairy should not raise its price.

16. $v = \frac{c_f}{p - c_v} = \frac{35{,}000}{30\text{–}10} = 1{,}750$

The increase in fixed cost from \$25,000 to \$35,000 will increase the break-even point from 1,250 to 1,750 or 500 dolls; thus, he should not spend the extra \$10,000 for advertising.

17. Original break-even point (from problem 7) = 1,250

New break-even point:

$v = \frac{c_f}{p - c_v} = \frac{17{,}000}{30 - 14} = 1{,}062.5$

Reduces BE point by 187.5 dolls.

18. a) $v = \frac{c_f}{p - c_v} = \frac{\$27{,}000}{8.95 - 3.75} = 5{,}192.30$ pizzas

b) $\frac{5{,}192.3}{20} = 259.6$ days

c) Revenue for the first 30 days $= 30(pv - vc_v)$

$= 30[(8.95)(20) - (20)(3.75)]$

$= \$3{,}120$

\$27,000 − 3,120 = \$23,880, portion of fixed cost not recouped after 30 days.

New $v = \frac{c_f}{p - c_v} = \frac{\$23{,}880}{7.95 - 3.75} = 5{,}685.7$ pizzas

Total break-even volume = 600 + 5,685.7 = 6,285.7 pizzas

$$\text{Total time to break-even} = 30 + \frac{5{,}685.7}{20}$$
$$= 314.3 \text{ days}$$

19. a) Cost of Regular plan = $55 + (.33)(260 minutes)
= $140.80

Cost of Executive plan = $100 + (.25)(60 minutes)
= $115

Select Executive plan.

b) 55 + (x − 1,000)(.33) = 100 + (x − 1,200)(.25)

− 275 + .33x = .25x − 200

x = 937.50 minutes per month or 15.63 hrs.

20. c_f = $26,000

c_v = $0.67 ($5.36/8 = 0.67)

p = $3.75

$$v = \frac{26{,}000}{3.75 - 0.67}$$

= 8,442 slices

Forecasted annual demand = (540)(52) = 28,080

Z = $91,260 – 44,813.6 = 46,446.4

21.

OLD	New

26,000 + (.67)v = 30,000 + (.48)v

.19v = 4,000

v = 21,053 slices

Z = New profit – old profit

Z = $47,781.60 – 46,446.40

= $1,335.20

Purchase equipment

22. a) $$14{,}000 = \frac{7{,}500}{p - .35}$$

p = $0.89 to break even

b) If the team did not perform as well as expected the crowds could be smaller; bad weather could reduce crowds and/or affect what fans eat at the game; the price she charges could affect demand.

c) This will be a subjective answer, but $1.25 seems to be a reasonable price.

$Z = vp - c_f - vc_v$

Z = (14,000)(1.25) − 7,500 − (14,000)(0.35)

= 17,500 − 12,400

= $5,100

23. a) c_f = $1,700

c_v = $12 per pupil

p = $75

$$v = \frac{1{,}700}{75 - 12}$$

= 26.98 or 27 pupils

b) $Z = vp - c_f - vc_v$

$5,000 = v(75) − $1,700 − v(12)

63v = 6,700

v = 106.3 pupils

c) $Z = vp - c_f - vc_v$

$5,000 = 60$p$ − $1,700 − 60(12)

60p = 7,420

p = $123.67

24. a) c_f = $350,000

c_v = $12,000

p = $18,000

$$v = \frac{c_f}{p - c_v}$$

$$= \frac{350{,}000}{18{,}000 - 12{,}000}$$

= 58.33 or 59 students

b) Z = (75)(18,000) − 350,000 − (75)(12,000)
= $100,000

c) Z = (35)(25,000) − 350,000 − (35)(12,000)
= 105,000

This is approximately the same as the profit for 75 students and a lower tuition in part (b).

25. p = $400

c_f = $8,000

c_v = $75

Z = $60,000

$$v = \frac{Z + c_f}{p - c_v}$$

$$v = \frac{60{,}000 + 8{,}000}{400 - 75}$$

v = 209.23 teams

26. Fixed cost (c_f) = 875,000

Variable cost (c_v) = $200

Price (p) = (225)(12) = $2,700

$v = c_f/(p - c_v)$ = 875,000/(2,700 – 200)
= 350

With volume doubled to 700:

Profit (Z) = (2,700)(700) – 875,000 – (700)(200)
= \$875,000

27. Fixed cost (c_f) = 100,000

Variable cost (c_v) = \$(.50)(.35) + (.35)(.50) + (.15)(2.30)
= \$0.695

Price (p) = \$6

Profit (Z) = (6)(45,000) – 100,000 – (45,000)(0.695)
= \$138,725

This is not the financial profit goal of \$150,000.

The price to achieve the goal of \$150,000 is,

$p = (Z + c_f + vc_v)/v$
= (150,000 + 100,000 + (45,000)(.695))/45,000
= \$6.25

The volume to achieve the goal of \$150,000 is,

$v = (Z + c_f)/(p - c_v)$
= (150,000 + 100,000)/(6 – .695)
= 47,125

28. a) Monthly fixed cost (c_f) = cost of van/60 months + labor (driver)/month
= (21,500/60) + (30.42 days/month)(\$8/hr)(5 hr/day)
= 358.33 + 1,216.80
= \$1,575.13

Variable cost (c_v) = \$1.35 + 15.00
= \$16.35

Price (p) = \$34

$v = c_f/(p - v_c)$
= (1,575.13)/(34 – 16.35)

v = 89.24 orders/month

b) 89.24/30.42 = 2.93 orders/day – Monday through Thursday

Double for weekend = 5.86 orders/day – Friday through Sunday

Orders per month = approximately (18 days)(2.93 orders) + (12.4 days)(5.86 orders)
= 125.4 delivery orders per month

Profit = total revenue – total cost
= $vp - (c_f + vc_v)$
= (125.4)(34) – 1,575.13 – (125.4)(16.35)
= 638.18

29. a) $v = \dfrac{c_f}{p - c_v} = \dfrac{500}{30 - 14}$

v = 31.25 jobs

b) (8 weeks)(6 days/week)(3 lawns/day) = 144 lawns

Z = (144)(30) – 500 – (144)(14)

Z = \$1,804

c) (8 weeks)(6 days/week)(4 lawns/day) = 192 lawns

Z = (192)(25) – 500 – (192)(14)

Z = \$1,612

No, she would make less money than (b)

30. a) $v = \dfrac{c_f}{p - c_v} = \dfrac{700}{35 - 3}$

v = 21.88 jobs

b) (6 snows)(2 days/snow)(10 jobs/day) = 120 jobs

Z = (120)(35) – 700 – (120)(3)

Z = \$3,140

c) (6 snows)(2 days/snow)(4 jobs/day) = 48 jobs

Z = (48)(150) – 1800 – (48)(28)

Z = \$4,056

Yes, better than (b)

d) Z = (120)(35) – 700 – (120)(18)

Z = \$1,340

Yes, still a profit with one more person

31. c_f = \$7,500

Monthly c_f = (\$2,300)(12)
= \$27,600

Total c_f = \$35,100

$c_v = 0$

p = \$0.24

$$v = \frac{c_f}{p} = \frac{35{,}100}{.24} = 146{,}250 \text{ hits per year}$$

v = 12,188 hits per month

\$45,000 = v(.24) – (12)(3,500) – (0)v

.24v = 87,000

v = 362,500

v = 30,208 hits per month

32. This is a "multiproduct" break-even problem. The formula for the break-even volume is,

$$v = \frac{\text{Total fixed cost}}{\begin{pmatrix}\text{weighted average}\\ \text{selling price}\end{pmatrix} - \begin{pmatrix}\text{weighted average}\\ \text{variable cost}\end{pmatrix}}$$

$$v = \frac{18{,}000}{[(3.20)(.70) + (2.50)(.30)] - [(.90)(.70) + (.45)(.30)]}$$

v = 8,089.89 units

cupcakes = (8,089.89)(.70)
= 5,622.92

cookies = (8,089.89)(.30)
= 2,426.97

BE sales \$ = (5,662.92)(3.20) + (2,426.97)(2.50)
= \$24,188.76

33. This is a decision analysis problem – the subject of Chapter 12. The payoff table is:

	Weather Conditions	
Decision Alternatives	Good	Bad
\$3.25	\$12,800	\$8,450
\$4.00	\$14,400	\$5,275

The student's decision depends on the degree of risk they are willing to assume.

Chapter 12 includes decision criteria for this problem.

34. This problem uses expected value for the decision alternatives in problem 30.

Expected value (\$3.25) = (\$12,800)(0.60) + (\$8,450)(0.40) = \$11,060

Expected value (\$4.00) = (\$14,400)(0.60) + (\$5,275)(0.40) = \$10,750

Although the decision to sell hotdogs for \$3.25 results in the greatest expected value, the results are so close, Annie would likely be indifferent.

35. There are two possible answers, or solution points:

$x = 25$, $y = 0$ or $x = 0$, $y = 50$

Substituting these values in the objective function:

$Z = 15(25) + 10(0) = 375$

$Z = 15(0) + 10(50) = 500$

Thus, the solution is $x = 0$ and $y = 50$

This is a simple linear programming model, the subject of the next several chapters. The student should recognize that there are only two possible solutions, which are the corner points of the feasible solution space, only one of which is optimal.

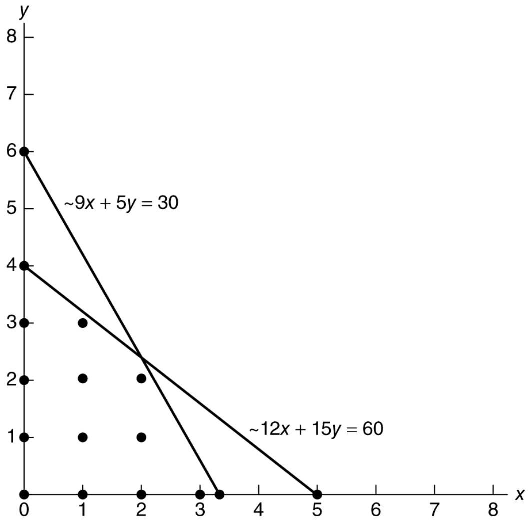
36. The solution is computed by solving simultaneous equations,

$x = 30$, $y = 10$, $Z = \$1,400$

It is the only, i.e., "optimal" solution because there is only one set of values for x and y that satisfy both constraints simultaneously.

37.

# bowls	# mugs	Labor usage $12x + 15y < = 60$	Clay usage $9x+5y < = 30$	Profit $300x + 250y$	Possible solution?
0	1	15	5	250	yes
1	0	12	9	300	yes
1	1	27	14	550	yes
0	2	30	10	500	yes
2	0	24	18	600	yes
1	2	42	19	800	yes
2	1	39	23	850	yes
2	2	54	28	1100	yes, best solution
0	3	45	15	750	yes
3	0	36	27	900	yes
1	3	57	24	1050	yes
3	1	51	32	1150	no
2	3	69	33	1350	no
3	2	66	37	1400	no
3	3	81	42	1650	no
4	0	48	36	1200	no
0	4	60	20	1000	yes
1	4	72	29	1300	no
4	1	63	41	1450	no
2	4	84	38	1600	no
4	2	78	46	1700	no



38. Maximize $Z = \$30x_{AN} + 70x_{AJ} + 40x_{BN} + 60x_{BJ}$
subject to

$$x_{AN} + x_{AJ} = 400$$
$$x_{BN} + x_{BJ} = 400$$
$$x_{AN} + x_{BN} = 500$$
$$x_{AJ} + x_{BJ} = 300$$

The solution is $x_{AN} = 400$, $x_{BN} = 100$, $x_{BJ} = 300$, and $Z = 34{,}000$

This problem can be solved by allocating as much as possible to the lowest cost variable, $x_{AN} = 400$, then repeating this step until all the demand has been met. This is a similar logic to the minimum cell cost method.

39. This is virtually a straight linear relationship between time and site visits; thus, a simple linear graph would result in a forecast of approximately 34,500 site visits.

40. Determine logical solutions:

	Cakes	Bread	Total Sales
1.	0	2	\$12
2.	1	2	\$22
3.	3	1	\$36
4.	4	0	\$40

Each solution must be checked to see if it violates the constraints for baking time and flour. Some possible solutions can be logically discarded because they are obviously inferior. For example, 0 cakes and 1 loaf of bread is clearly inferior to 0 cakes and 2 loaves of bread. 0 cakes and 3 loaves of bread is not possible because there is not enough flour for 3 loaves of bread.

Using this logic, there are four possible solutions as shown. The best one, 4 cakes and 0 loaves of bread, results in the highest total sales of \$40.

41. This problem demonstrates the cost trade-off inherent in queuing analysis, the topic of Chapter 13. In this problem the cost of service, i.e., the cost of staffing registers, is added to the cost of customers waiting, i.e., the cost of lost sales and ill will, as shown in the following table.

Registers staffed	1	2	3	4	5	6	7	8
Waiting time (mins)	20	14	9	4	1.7	1	0.5	0.1
Cost of service (\$)	60	120	180	240	300	360	420	480
Cost of waiting (\$)	850	550	300	50	0	0	0	0
Total cost (\$)	910	670	480	290	300	360	420	480

The total minimum cost of \$290 occurs with 4 registers staffed

42. The shortest route problem is one of the topics of Chapter 7. At this point, the most logical "trial and error" way that most students will probably approach this problem is to identify all the feasible routes and compute the total distance for each, as follows:

1-2-6-9 = 228
1-2-5-9 = 213
1-3-5-9 = 211
1-3-8-9 = 276
1-4-7-8-9 = 275

Obviously inferior routes like 1-3-4-7-8-9 and 1-2-5-8-9 that include additional segments to the routes listed above can be logically eliminated from consideration. As a result, the route 1-3-5-9 is the shortest.

An additional aspect to this problem could be to have the students look at these routes on a real map and indicate which they think might "practically" be the best route. In this case, 1-2-5-9 would likely be a better route, because even though it's two miles farther it is Interstate highway the whole way, whereas 1-3-5-9 encompasses U.S. 4-lane highways and state roads.

CASE SOLUTION: CLEAN CLOTHES CORNER LAUNDRY

a) $v = \frac{c_f}{p - c_v} = \frac{1{,}700}{1.10 - .25} = 2{,}000$ items per month

b) Solution depends on number of months; 36 used here. $16,200 ÷ 36 = $450 per month, thus monthly fixed cost is $2,150

$$v = \frac{c_f}{p - c_v} = \frac{2{,}150}{1.10 - .25} = 2{,}529.4 \text{ items per month}$$

529.4 additional items per month

c) $Z = vp - c_f - vc_v$

$= 4{,}300(1.10) - 2{,}150 - 4{,}300(.25)$

$= \$1{,}505$ per month

After 3 years, $Z = \$1{,}955$ per month

d) $v = \frac{c_f}{p - c_v} = \frac{1{,}700}{.99 - .25} = 2{,}297.3$

$Z = vp - c_f - vc_v$

$= 3{,}800(.99) - 1{,}700 - 3{,}800(.25)$

$= \$1{,}112$ per month

e) With both options:

$Z = vp - c_f - vc_v$

$= 4{,}700(.99) - 2{,}150 - 4{,}700(.25)$

$= \$1{,}328$

She should purchase the new equipment but not decrease prices.

CASE SOLUTION: OCOBEE RIVER RAFTING COMPANY

Alternative 1: $c_f = \$3{,}000$

$p = \$20$

$c_v = \$12$

$$v_1 = \frac{c_f}{p - c_v} = \frac{3{,}000}{20 - 12} = 375 \text{ rafts}$$

Alternative 2: $c_f = \$10{,}000$

$p = \$20$

$c_v = \$8$

$$v_2 = \frac{c_f}{p - c_v} = \frac{10{,}000}{20 - 8} = 833.37$$

If demand is less than 375 rafts, the students should not start the business.

If demand is less than 833 rafts, alternative 2 should not be selected, and alternative 1 should be used if demand is expected to be between 375 and 833.33 rafts.

If demand is greater than 833.33 rafts, which alternative is best? To determine the answer, equate the two cost functions.

$$3{,}000 + 12v = 10{,}000 + 8v$$
$$4v = 7{,}000$$
$$v = 1{,}750$$

This is referred to as the point of indifference between the two alternatives. In general, for demand lower than this point (1,750) the alternative should be selected with the lowest fixed cost; for demand greater than this point the alternative with the lowest variable cost should be selected. (This general relationship can be observed by graphing the two cost equations and seeing where they intersect.)

Thus, for the Ocobee River Rafting Company, the following guidelines should be used:

demand < 375, do not start business; 375 < demand < 1,750, select alternative 1; demand > 1,750, select alternative 2

Since Penny estimates demand will be approximately 1,000 rafts, alternative 1 should be selected.

$Z = v_p - c_f - vc_v$

$= (1{,}000)(20) - 3{,}000 - (1{,}000)(12)$

$Z = \$5{,}000$

CASE SOLUTION: CONSTRUCTING A DOWNTOWN PARKING LOT IN DRAPER

a) The annual capital recovery payment for a capital expenditure of $4.5 million over 30 years at 8% is,

$(4{,}500{,}000)[0.08(1 + .08)^{30}] / (1 + .08)^{30} - 1$

$= \$399{,}723.45$

This is part of the annual fixed cost. The other part of the fixed cost is the employee annual salaries of $140,000. Thus, total fixed costs are,

$399,723.45 + 140,000 = $539,723.45

$v = \frac{c_f}{p - c_v}$

$= \frac{539{,}723.45}{3.20 - 0.60}$

$= 207{,}585.94$ parked cars per year

b) If 365 days per year are used, then the daily usage is,

$$\frac{207{,}585.94}{365} = 568.7 \text{ or approximately 569 cars per day}$$

This seems like a reachable goal given the size of the town and the student population.

CASE SOLUTION: A BUS SERVICE FOR DRAPER

Fixed cost (3 buses) = 1,200,000
Total Variable Cost = 591,300
Annual Revenue = 648,240

Passengers/bus/trip = 37
Passenger fare = 4
Trips/bus/day = 4
Number of buses = 3
Days/year = 365
Total annual revenue = 648,240 = (37)(4)(4)(3)(365)

Bus operating hrs/day = 18
Operating cost/hr = 90
Days/year = 365
Total annual variable cost = 591,300 = (18)(90)(365)

(a) First year loss = (1,143,060.00)

(b) Years to break even:
Loss in year 1 = –1,143,060.0
Not possible to break even

(c) 45 passengers per trip:
Annual Revenue = 788,400
First year loss = (1,002,900)
Not possible to break even
50 passengers per trip:
Annual revenue = 876,000
First year loss = (915,300)
Break even year: (3.215) years

(d) Decrease in trips:
Annual revenue = 657,000
Total variable cost = 443,475
First year loss = (986,475)
Break even year: (5.62) years
Bus operating hrs/day = 13.5
Operating cost/hr. = 90
Days/year = 365
Total annual variable cost = 443,475

(e) $1,200,000 Grant:
Fixed Cost = 0
First Year Revenue = 56,940