Problem 1.1 [Difficulty: 3]

1.1 A number of common substances are

Tar	Sand
"Silly Putty"	Jello
Modeling clay	Toothpaste
Wax	Shaving cream

Some of these materials exhibit characteristics of both solid and fluid behavior under different conditions. Explain and give examples.

Given: Common Substances

Tar	Sand
"Silly Putty"	Jello
Modeling clay	Toothpaste
Wax	Shaving cream

Some of these substances exhibit characteristics of solids and fluids under different conditions.

Find: Explain and give examples.

Solution: Tar, Wax, and Jello behave as solids at room temperature or below at ordinary pressures. At high pressures or over long periods, they exhibit fluid characteristics. At higher temperatures, all three liquefy and become viscous fluids.

Modeling clay and silly putty show fluid behavior when sheared slowly. However, they fracture under suddenly applied stress, which is a characteristic of solids.

Toothpaste behaves as a solid when at rest in the tube. When the tube is squeezed hard, toothpaste "flows" out the spout, showing fluid behavior. Shaving cream behaves similarly.

Sand acts solid when in repose (a sand "pile"). However, it "flows" from a spout or down a steep incline.

Problem 1.2 [Difficulty: 2]

1.2 Give a word statement of each of the five basic conservation laws stated in Section 1.2 as they apply to a system.

Given: Five basic conservation laws stated in Section 1.2

Write: A word statement of each, as they apply to a system.

Solution: Assume that laws are to be written for a *system*.

a. Conservation of mass — The mass of a system is constant by definition.

b. Newton's second law of motion — The net force acting on a system is directly proportional to the product of the system mass times its acceleration.

c. First law of thermodynamics — The change in stored energy of a system equals the net energy added to the system as heat and work.

d. Second law of thermodynamics — The entropy of any isolated system cannot decrease during any process between equilibrium states.

e. Principle of angular momentum — The net torque acting on a system is equal to the rate of change of angular momentum of the system.

Problem 1.3

[Difficulty: 3]

1.3 The barrel of a bicycle tire pump becomes quite warm during use. Explain the mechanisms responsible for the temperature increase.

Open-Ended Problem Statement: The barrel of a bicycle tire pump becomes quite warm during use. Explain the mechanisms responsible for the temperature increase.

Discussion: Two phenomena are responsible for the temperature increase: (1) friction between the pump piston and barrel and (2) temperature rise of the air as it is compressed in the pump barrel.

Friction between the pump piston and barrel converts mechanical energy (force on the piston moving through a distance) into thermal energy as a result of friction. Lubricating the piston helps to provide a good seal with the pump barrel and reduces friction (and therefore force) between the piston and barrel.

Temperature of the trapped air rises as it is compressed. The compression is not adiabatic because it occurs during a finite time interval. Heat is transferred from the warm compressed air in the pump barrel to the cooler surroundings. This raises the temperature of the barrel, making its outside surface warm (or even hot!) to the touch.

Problem 1.4

(Difficulty: 1)

1.4 Very small particles moving in fluids are known to experience a drag force proportional to speed. Consider a particle of net weight W dropped in a fluid. The particle experiences a drag force, $F_D = kV$, where V is the particle speed. Determine the time required for the particle to accelerate from rest to 95 percent of its terminal speed, V_t, in terms of k, W, and g.

Given: Data on oxygen tank.

Find: Mass of oxygen.

Solution: Compute tank volume, and then use oxygen density to find the mass.

The given or available data is: $D = 16 \cdot ft$ $\quad p = 1000 \cdot psi$ $\quad T = (77 + 460) \cdot R$ $\quad T = 537 \cdot R$

$$R_{O2} = 48.29 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$$

For oxygen the critical temperature and pressure are: $T_c = 279 \cdot R$ $\quad p_c = 725.2 \cdot psi$ (data from NIST WebBook)

so the reduced temperature and pressure are:

$$T_R = \frac{T}{T_c} = 1.925 \qquad p_R = \frac{p}{p_c} = 1.379$$

Using a compressiblity factor chart: $Z = 0.948$ Since this number is close to 1, we can assume ideal gas behavior.

Therefore, the governing equation is the ideal gas equation $p = \rho \cdot R_{O2} \cdot T$ and $\rho = \frac{M}{V}$

where V is the tank volume $V = \frac{\pi \cdot D^3}{6}$ $\quad V = \frac{\pi}{6} \times (16 \cdot ft)^3$ $\quad V = 2144.7 \cdot ft^3$

Hence:

$$M = V \cdot \rho = \frac{p \cdot V}{R_{O2} \cdot T} \qquad M = 1000 \cdot \frac{lbf}{in^2} \times 2144.7 \cdot ft^3 \times \frac{1}{48.29} \cdot \frac{lbm \cdot R}{ft \cdot lbf} \times \frac{1}{537} \cdot \frac{1}{R} \times \left(\frac{12 \cdot in}{ft}\right)^2$$

$$M = 11910 \cdot lbm$$

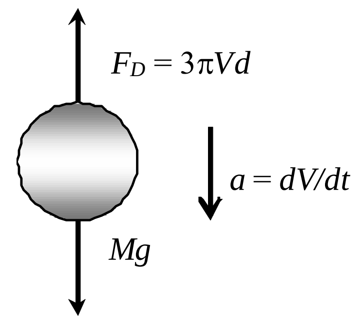
Problem 1.5

[Difficulty: 4]

1.5 In a combustion process, gasoline particles are to be dropped in air at 200°F. The particles must drop at least 10 in. in 1 s. Find the diameter d of droplets required for this. (The drag on these particles is given by $F_D = \pi\mu V d$, where V is the particle speed and μ is the air viscosity. To solve this problem, use *Excel's Goal Seek*.)

NOTE: Drag formula is in error: It should be:

$$F_D = 3\cdot\pi\cdot V\cdot d$$



Given: Data on sphere and formula for drag.

Find: Diameter of gasoline droplets that take 1 second to fall 10 in.

Solution: Use given data and data in Appendices; integrate equation of motion by separating variables.

The data provided, or available in the Appendices, are:

$$\mu = 4.48\times 10^{-7}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \qquad \rho_w = 1.94\cdot\frac{\text{slug}}{\text{ft}^3} \qquad SG_{gas} = 0.72 \qquad \rho_{gas} = SG_{gas}\cdot\rho_w \qquad \rho_{gas} = 1.40\cdot\frac{\text{slug}}{\text{ft}^3}$$

Newton's 2nd law for the sphere (mass M) is (ignoring buoyancy effects)

$$M\cdot\frac{dV}{dt} = M\cdot g - 3\cdot\pi\cdot\mu\cdot V\cdot d$$

so

$$\frac{dV}{g - \frac{3\cdot\pi\cdot\mu\cdot d}{M}\cdot V} = dt$$

Integrating twice and using limits

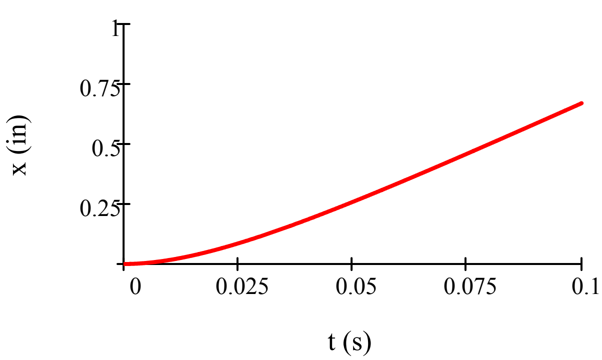
$$V(t) = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(1 - e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t}\right) \qquad x(t) = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left[t + \frac{M}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t} - 1\right)\right]$$

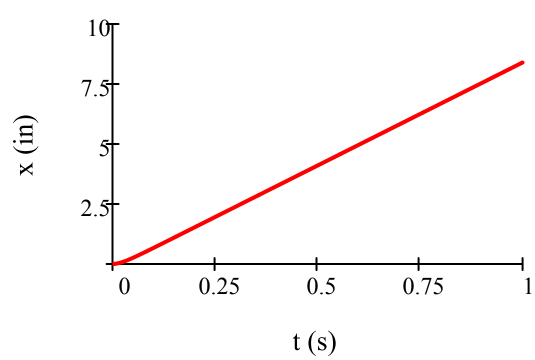
Replacing M with an expression involving diameter d

$$M = \rho_{gas}\cdot\frac{\pi\cdot d^3}{6} \qquad x(t) = \frac{\rho_{gas}\cdot d^2\cdot g}{18\cdot\mu}\cdot\left[t + \frac{\rho_{gas}\cdot d^2}{18\cdot\mu}\cdot\left(e^{\frac{-18\cdot\mu}{\rho_{gas}\cdot d^2}\cdot t} - 1\right)\right]$$

This equation must be solved for d so that $x(1\cdot s) = 10\cdot in$. The answer can be obtained from manual iteration, or by using *Excel's Goal Seek*.

$$d = 4.30\times 10^{-3}\cdot in$$





Note That the particle quickly reaches terminal speed, so that a simpler approximate solution would be to solve $Mg = 3\pi\mu Vd$ for d, with $V = 0.25$ m/s (allowing for the fact that M is a function of d)!

Problem 1.6

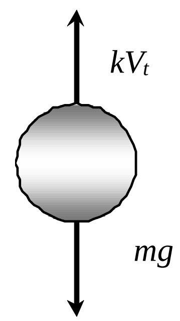
[Difficulty: 3]

1.6 In a pollution control experiment, minute solid particles (typical mass 1×10^{-13} slug) are dropped in air. The terminal speed of the particles is measured to be 0.2 ft/s. The drag of these particles is given by $F_D = kV$, where V is the instantaneous particle speed. Find the value of the constant k. Find the time required to reach 99 percent of terminal speed.

Given: Data on sphere and terminal speed.

Find: Drag constant k, and time to reach 99% of terminal speed.

Solution: Use given data; integrate equation of motion by separating variables.



The data provided are: $M = 1 \times 10^{-13}\cdot\text{slug} \quad V_t = 0.2\cdot\frac{\text{ft}}{\text{s}}$

Newton's 2nd law for the general motion is (ignoring buoyancy effects) $M\cdot\frac{dV}{dt} = M\cdot g - k\cdot V \quad (1)$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $M\cdot g = k\cdot V_t$ so $k = \frac{M\cdot g}{V_t}$

$$k = 1 \times 10^{-13}\cdot\text{slug} \times 32.2\cdot\frac{\text{ft}}{\text{s}^2} \times \frac{\text{s}}{0.2\cdot\text{ft}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad k = 1.61 \times 10^{-11}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}}$$

To find the time to reach 99% of V_t, we need $V(t)$. From 1, separating variables $\frac{dV}{g - \frac{k}{M}\cdot V} = dt$

Integrating and using limits $t = -\frac{M}{k}\cdot\ln\left(1 - \frac{k}{M\cdot g}\cdot V\right)$

We must evaluate this when $V = 0.99\cdot V_t \qquad V = 0.198\cdot\frac{\text{ft}}{\text{s}}$

$$t = -1 \times 10^{-13}\cdot\text{slug} \times \frac{\text{ft}}{1.61 \times 10^{-11}\cdot\text{lbf}\cdot\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\cdot\ln\left(1 - 1.61 \times 10^{-11}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}} \times \frac{1}{1 \times 10^{-13}\cdot\text{slug}} \times \frac{\text{s}^2}{32.2\cdot\text{ft}} \times \frac{0.198\cdot\text{ft}}{\text{s}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)$$

$t = 0.0286\,\text{s}$

Problem 1.7

(Difficulty: 2)

1.7 A rocket payload with a weight on earth of $2000\ lbf$ is landed on the moon where the acceleration due to the moon's gravity $g_m \approx \frac{g_e}{6}$. Find the mass of the payload on the earth and the moon and the payload's moon weight.

Given: Rocket payload weight on earth $W_e = 2000\ lbf$. The acceleration due to the moon's gravity $g_m \approx \frac{g_e}{6}$.

Find: The mass of payload on earth M_e and on moon M_m in SI and EE units. The payload's moon weight W_m.

Solution:

Basic equation: Newton's law applied to mass and weight

$$M = \frac{W}{g}$$

Gravity on the moon relative to that on Earth:

$$g_m \approx \frac{g_e}{6}$$

The value of gravity is:

$$g_e = 32.2\ \frac{ft}{s^2}$$

The mass on earth is:

$$M_e = \frac{W_e}{g_e} = \frac{2000\ lbf}{32.2\ \frac{ft}{s^2}.} = 62.1\ slug$$

The mass on moon is the same as it on earth:

$$M_m = 62.1\ slug$$

The weight on the moon is then

$$W_m = M_m g_m = M_m\left(\frac{g_e}{6}\right) = M_e\left(\frac{g_e}{6}\right) = \frac{W_e}{6} = 333\ lbf$$

Problem 1.8

(Difficulty: 1)

1.8 A cubic meter of air at 101 kPa and 15 °C weighs 12.0 N. What is its specific volume? What is the specific volume if it is cooled to -10 °C at constant pressure?

Given: Specific weight $\gamma = 12.0\ \frac{N}{m^3}$ at 101 kPa and 15 °C.

Find: The specific volume v at 101 kPa and 15 °C. Also the specific volume v at 101 kPa and -10 °C.

Assume: Air can be treated as an ideal gas

Solution:

Basic equation: ideal gas law:

$$pv = RT$$

The specific volume is equal to the reciprocal of the specific weight divided by gravity

$$v_1 = \frac{g}{\gamma}$$

Using the value of gravity in the SI units, the specific volume is

$$v_1 = \frac{g}{\gamma} = \frac{9.81\ \frac{m}{s^2}}{12.0\ N} = 0.818\ \frac{m^3}{kg}$$

The temperature conditions are

$$T_1 = 15\ °C = 288\ K, \qquad T_2 = -10\ °C = 263K$$

For v_2 at the same pressure of 101 kPa and cooled to -10 °C we have, because the gas constant is the same at both pressures:

$$\frac{v_1}{v_2} = \frac{\frac{RT_1}{p}}{\frac{RT_2}{p}} = \frac{T_1}{T_2}$$

So the specific volume is

$$v_2 = v_1 \frac{T_2}{T_1} = 0.818 \frac{m^3}{kg} \times \frac{263\ K}{288\ K} = 0.747\ \frac{m^3}{kg}$$

Problem 1.9

(Difficulty: 2)

1.9 Calculate the specific weight, specific volume and density of air at 40°F and 50 $psia$. What are the values if the air is then compressed isentropically to 100 psia?

Given: Air temperature: 40°F, Air pressure 50 psia.

Find: The specific weight, specific volume and density at 40°F and 50 psia and the values at 100 psia after isentropic compression.

Assume: Air can be treated as an ideal gas

Solution:

Basic equation: $pv = RT$

The absolute temperature is

$$T_1 = 40\text{°F} = 500°R$$

The gas constant is

$$R = 1715\ \frac{ft \cdot lbf}{slug \cdot °R}$$

The specific volume is:

$$v_1 = \frac{RT_1}{p} = \frac{1715\ \dfrac{ft \cdot lbf}{slug \cdot °R}}{50psia \times \dfrac{144in^2}{ft^2}} \times 500°R = 119.1\ \frac{ft^3}{slug}$$

The density is the reciprocal of the specific volume

$$\rho_1 = \frac{1}{v_1} = 0.0084\ \frac{slug}{ft^3}$$

Using Newton's second law, the specific weight is the density times gravity:

$$\gamma_1 = \rho g = 0.271\ \frac{lbf}{ft^3}$$

For the isentropic compression of air to 100 psia, we have the relation for entropy change of an ideal gas:

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

The definition of an isentropic process is

$$s_2 = s_1$$

Solving for the temperature ratio

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{R/c_p}$$

The values of R and specific heat are

$$R = 1715\ \frac{ft \cdot lbf}{slug \cdot {}^\circ R} = 53.3\ \frac{ft \cdot lbf}{lb \cdot {}^\circ R} = 0.0686\ \frac{Btu}{lb \cdot {}^\circ R}$$

$$c_p = 0.24 \frac{Btu}{lbm\ R}$$

The temperature after compression to 100 psia is

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{R/c_p} = 500\ R \left(\frac{100\ psia}{50\ psia}\right)^{0.0686/0.24} = 610\ {}^\circ R$$

$$p_2 = 100\ psia = 14400 \frac{lbf}{ft^2}$$

The specific volume is computed using the ideal gas law:

$$v_2 = \frac{RT_2}{p_2} = \frac{1715\ \frac{ft \cdot lbf}{slug \cdot {}^\circ R}}{100 psia \times \frac{144 in^2}{ft^2}} \times 610.00{}^\circ R = 72.6\ \frac{ft^3}{slug}$$

The density is the reciprocal of the specific volume

$$\rho_2 = \frac{1}{v_2} = 0.0138\ \frac{slug}{ft^3}$$

The specific weight is:

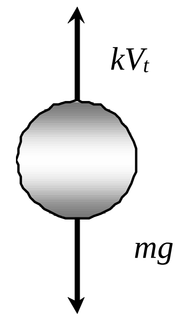
$$\gamma_2 = \rho_2 g = 0.444\ \ \frac{lbf}{ft^3}$$

Problem 1.10

[Difficulty: 5]

1.10 For Problem 1.6 , find the distance the particles travel before reaching 99 percent of terminal speed. Plot the distance traveled as a function of time.

Given: Data on sphere and terminal speed from Problem 1.6 .



Find: Distance traveled to reach 99% of terminal speed; plot of distance versus time.

Solution: Use given data; integrate equation of motion by separating variables.

The data provided are: $M = 1 \times 10^{-13}\cdot\text{slug} \qquad V_t = 0.2\cdot\dfrac{\text{ft}}{\text{s}}$

Newton's 2nd law for the general motion is (ignoring buoyancy effects) $\quad M\cdot\dfrac{dV}{dt} = M\cdot g - k\cdot V \quad (1)$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $\quad M\cdot g = k\cdot V_t \quad$ so $\quad k = \dfrac{M\cdot g}{V_t}$

$$k = 1 \times 10^{-13}\cdot\text{slug} \times 32.2\cdot\frac{\text{ft}}{\text{s}^2} \times \frac{\text{s}}{0.2\cdot\text{ft}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad k = 1.61 \times 10^{-11}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}}$$

To find the distance to reach 99% of V_t, we need $V(y)$. From 1: $\quad M\cdot\dfrac{dV}{dt} = M\cdot\dfrac{dy}{dt}\cdot\dfrac{dV}{dy} = M\cdot V\cdot\dfrac{dV}{dy} = M\cdot g - k\cdot V$

Separating variables $\quad \dfrac{V\cdot dV}{g - \dfrac{k}{M}\cdot V} = dy$

Integrating and using limits $\quad y = -\dfrac{M^2\cdot g}{k^2}\cdot\ln\left(1 - \dfrac{k}{M\cdot g}\cdot V\right) - \dfrac{M}{k}\cdot V$

We must evaluate this when $\quad V = 0.99\cdot V_t \qquad V = 0.198\cdot\dfrac{\text{ft}}{\text{s}}$

$$y = \left(1\cdot 10^{-13}\cdot\text{slug}\right)^2\cdot\frac{32.2\cdot\text{ft}}{\text{s}^2}\cdot\left(\frac{\text{ft}}{1.61\cdot 10^{-11}\cdot\text{lbf}\cdot\text{s}}\right)^2\cdot\left(\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)^2\cdot\ln\left(1 - 1.61\cdot 10^{-11}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}}\cdot\frac{1}{1\cdot 10^{-13}\cdot\text{slug}}\cdot\frac{\text{s}^2}{32.2\cdot\text{ft}}\cdot\frac{0.198\cdot\text{ft}}{\text{s}}\cdot\frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)\ldots$$
$$+ 1\cdot 10^{-13}\cdot\text{slug} \times \frac{\text{ft}}{1.61\cdot 10^{-11}\cdot\text{lbf}\cdot\text{s}} \times \frac{0.198\cdot\text{ft}}{\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$y = 4.49 \times 10^{-3}\cdot\text{ft}$$

Alternatively we could use the approach of Problem 1.12 and first find the time to reach terminal speed, and use this time in y(t) to find the above value of y:

From 1, separating variables $\quad \dfrac{dV}{g - \dfrac{k}{M}\cdot V} = dt$

Integrating and using limits $\quad t = -\dfrac{M}{k}\cdot\ln\left(1 - \dfrac{k}{M\cdot g}\cdot V\right) \quad (2)$

We must evaluate this when $V = 0.99 \cdot V_t$ $\quad V = 0.198 \cdot \frac{ft}{s}$

$$t = 1 \times 10^{-13} \cdot slug \times \frac{ft}{1.61 \times 10^{-11} \cdot lbf \cdot s} \cdot \frac{lbf \cdot s^2}{slug \cdot ft} \cdot \ln\left(1 - 1.61 \times 10^{-11} \cdot \frac{lbf \cdot s}{ft} \times \frac{1}{1 \times 10^{-13} \cdot slug} \times \frac{s^2}{32.2 \cdot ft} \times \frac{0.198 \cdot ft}{s} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)$$

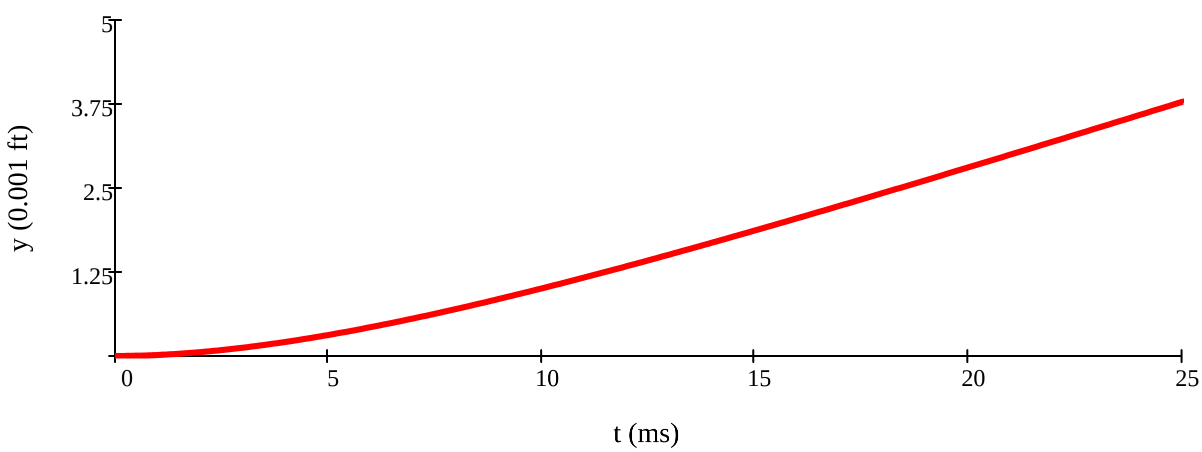
$$t = 0.0286\,s$$

From 2, after rearranging $\quad V = \frac{dy}{dt} = \frac{M \cdot g}{k} \cdot \left(1 - e^{-\frac{k}{M} \cdot t}\right)$

Integrating and using limits $\quad y = \frac{M \cdot g}{k} \cdot \left[t + \frac{M}{k} \cdot \left(e^{-\frac{k}{M} \cdot t} - 1\right)\right]$

$$y = 1 \times 10^{-13} \cdot slug \times \frac{32.2 \cdot ft}{s^2} \times \frac{ft}{1.61 \times 10^{-11} \cdot lbf \cdot s} \cdot \frac{lbf \cdot s^2}{slug \cdot ft} \cdot \left[0.0291 \cdot s \ldots + 10^{-13} \cdot slug \cdot \frac{ft}{1.61 \times 10^{-11} \cdot lbf \cdot s} \cdot \frac{lbf \cdot s^2}{slug \cdot ft} \cdot \left(e^{-\frac{1.61 \times 10^{-11}}{1 \times 10^{-13}} \cdot .0291} - 1\right)\right]$$

$$y = 4.49 \times 10^{-3} \cdot ft$$



This plot can also be presented in Excel.

Problem 1.11

[Difficulty: 4]

1.11 A sky diver with a mass of 70 kg jumps from an aircraft. The aerodynamic drag force acting on the sky diver is known to be $F_D = kV^2$, where $k = 0.25\ \text{N}\cdot\text{s}^2/\text{m}^2$. Determine the maximum speed of free fall for the sky diver and the speed reached after 100 m of fall. Plot the speed of the sky diver as a function of time and as a function of distance fallen.

Given: Data on sky diver: $M = 70\cdot\text{kg}$ $\quad k = 0.25\cdot\dfrac{\text{N}\cdot\text{s}^2}{\text{m}^2}$

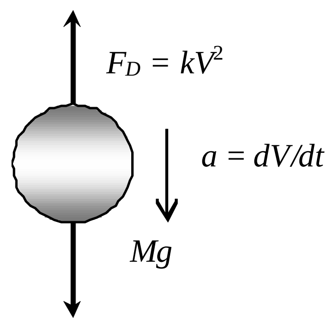
Find: Maximum speed; speed after 100 m; plot speed as function of time and distance.

Solution: Use given data; integrate equation of motion by separating variables.

Treat the sky diver as a system; apply Newton's 2nd law:

Newton's 2nd law for the sky diver (mass M) is (ignoring buoyancy effects): $M\cdot\dfrac{dV}{dt} = M\cdot g - k\cdot V^2 \quad (1)$

(a) For terminal speed V_t, acceleration is zero, so $M\cdot g - k\cdot V^2 = 0$ so $V_t = \sqrt{\dfrac{M\cdot g}{k}}$



$$V_t = \left(70\cdot\text{kg}\times 9.81\cdot\frac{\text{m}}{\text{s}^2}\times\frac{\text{m}^2}{0.25\cdot\text{N}\cdot\text{s}^2}\cdot\frac{\text{N}\cdot\text{s}^2}{\text{kg}\times\text{m}}\right)^{\frac{1}{2}} \qquad V_t = 52.4\frac{\text{m}}{\text{s}}$$

(b) For V at $y = 100$ m we need to find $V(y)$. From (1) $M\cdot\dfrac{dV}{dt} = M\cdot\dfrac{dV}{dy}\cdot\dfrac{dy}{dt} = M\cdot V\cdot\dfrac{dV}{dt} = M\cdot g - k\cdot V^2$

Separating variables and integrating:

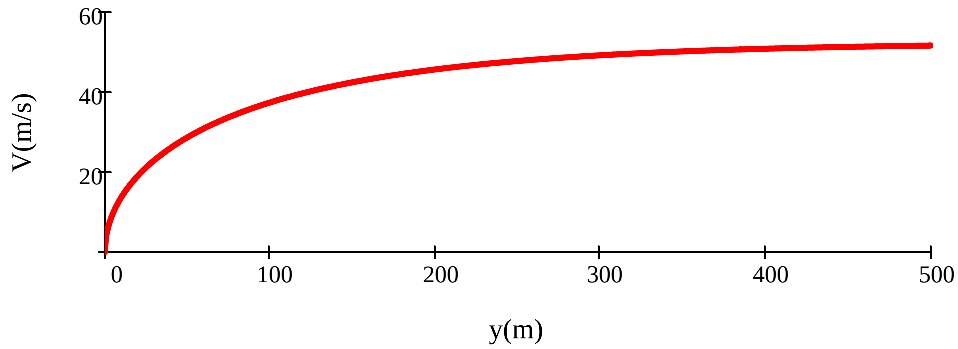
$$\int_0^V \frac{V}{1-\frac{k\cdot V^2}{M\cdot g}}\,dV = \int_0^y g\,dy$$

so $\quad \ln\left(1-\dfrac{k\cdot V^2}{M\cdot g}\right) = -\dfrac{2\cdot k}{M}y \quad$ or $\quad V^2 = \dfrac{M\cdot g}{k}\cdot\left(1-e^{-\frac{2\cdot k\cdot y}{M}}\right)$

Hence $\quad V(y) = V_t\cdot\left(1-e^{-\frac{2\cdot k\cdot y}{M}}\right)^{\frac{1}{2}}$

For $y = 100$ m:

$$V(100\cdot\text{m}) = 52.4\cdot\frac{\text{m}}{\text{s}}\cdot\left(1-e^{-2\times 0.25\cdot\frac{\text{N}\cdot\text{s}^2}{\text{m}^2}\times 100\cdot\text{m}\times\frac{1}{70\cdot\text{kg}}\times\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}}\right)^{\frac{1}{2}} \qquad V(100\cdot\text{m}) = 37.4\cdot\frac{\text{m}}{\text{s}}$$

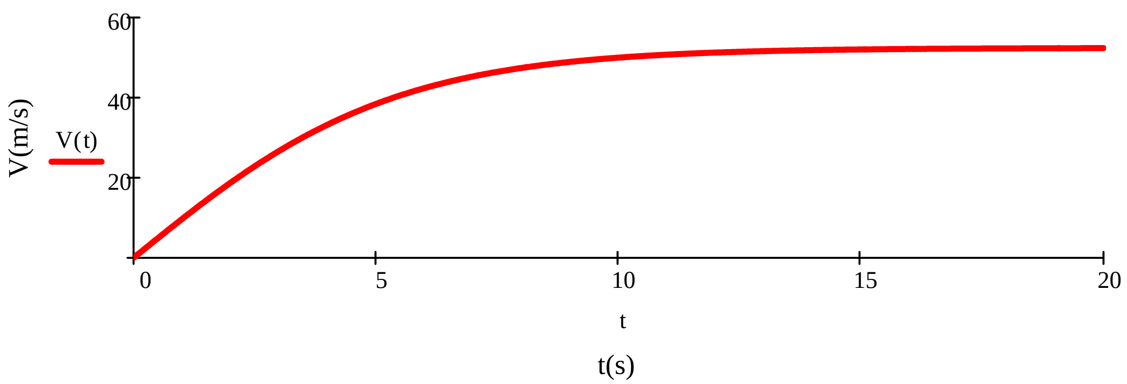


(c) For $V(t)$ we need to integrate (1) with respect to t: $M \cdot \frac{dV}{dt} = M \cdot g - k \cdot V^2$

Separating variables and integrating: $$\int_0^V \frac{V}{\frac{M \cdot g}{k} - V^2}\, dV = \int_0^t 1\, dt$$

so $$t = \frac{1}{2} \cdot \sqrt{\frac{M}{k \cdot g}} \cdot \ln\left(\left|\frac{\sqrt{\frac{M \cdot g}{k}} + V}{\sqrt{\frac{M \cdot g}{k}} - V}\right|\right) = \frac{1}{2} \cdot \sqrt{\frac{M}{k \cdot g}} \cdot \ln\left(\frac{|V_t + V|}{|V_t - V|}\right)$$

Rearranging $$V(t) = V_t \cdot \frac{\left(e^{2 \cdot \sqrt{\frac{k \cdot g}{M}} \cdot t} - 1\right)}{\left(e^{2 \cdot \sqrt{\frac{k \cdot g}{M}} \cdot t} + 1\right)}$$ or $$V(t) = V_t \cdot \tanh\left(V_t \cdot \frac{k}{M} \cdot t\right)$$



The two graphs can also be plotted in Excel.

Problem 1.12 [Difficulty: 3]

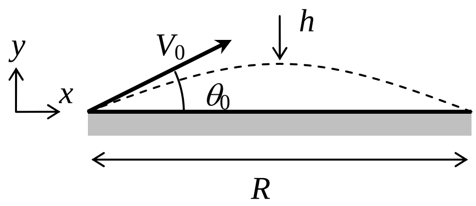
1.12 The English perfected the longbow as a weapon after the Medieval period. In the hands of a skilled archer, the longbow was reputed to be accurate at ranges to 100 m or more. If the maximum altitude of an arrow is less than $h = 10$ m while traveling to a target 100 m away from the archer, and neglecting air resistance, estimate the speed and angle at which the arrow must leave the bow. Plot the required release speed and angle as a function of height h.

Given: Long bow at range, R = 100 m. Maximum height of arrow is h = 10 m. Neglect air resistance.

Find: Estimate of (a) speed, and (b) angle, of arrow leaving the bow.

Plot: (a) release speed, and (b) angle, as a function of h

Solution: Let $\vec{V}_0 = u_0\hat{i} + v_0\hat{j} = V_0(\cos\theta_0\hat{i} + \sin\theta_0\hat{j})$



$\Sigma F_y = m\frac{dv}{dt} = -mg$, so $v = v_0 - gt$, and $t_f = 2t_{v=0} = 2v_0/g$

Also, $mv\dfrac{dv}{dy} = -mg,\ v\,dv = -g\,dy,\ 0 - \dfrac{v_0^2}{2} = -gh$

Thus $h = v_0^2/2g$ (1)

$$\Sigma F_x = m\frac{du}{dt} = 0,\ \text{so } u = u_0 = \text{const, and } R = u_0 t_f = \frac{2u_0v_0}{g} \quad (2)$$

From Eq. 1: $v_0^2 = 2gh$ (3)

From Eq. 2: $u_0 = \dfrac{gR}{2v_0} = \dfrac{gR}{2\sqrt{2gh}} \qquad \therefore\ u_0^2 = \dfrac{gR^2}{8h}$

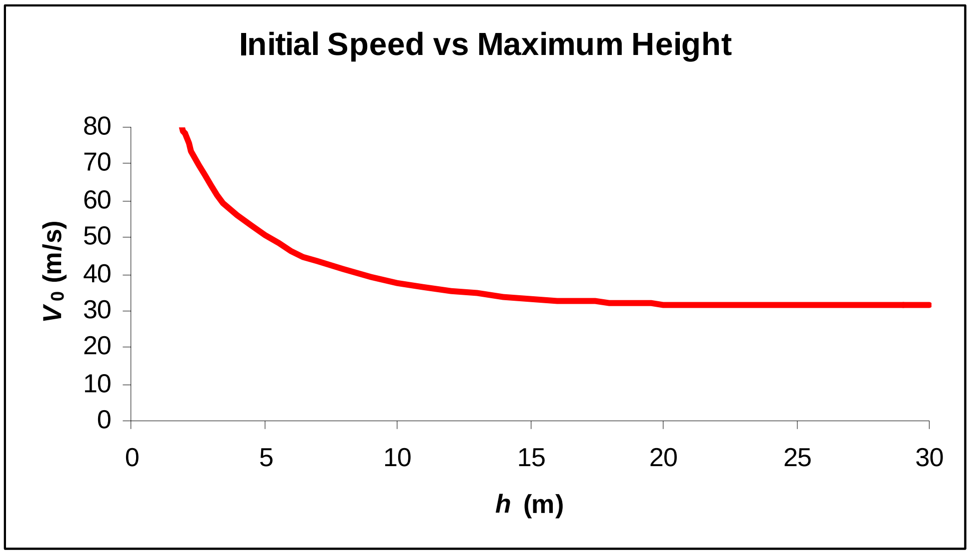
Then $V_0^2 = u_0^2 + v_0^2 = \dfrac{gR^2}{8h} + 2gh$ and $V_0 = \left(2gh + \dfrac{gR^2}{8h}\right)^{\frac{1}{2}}$ (4)

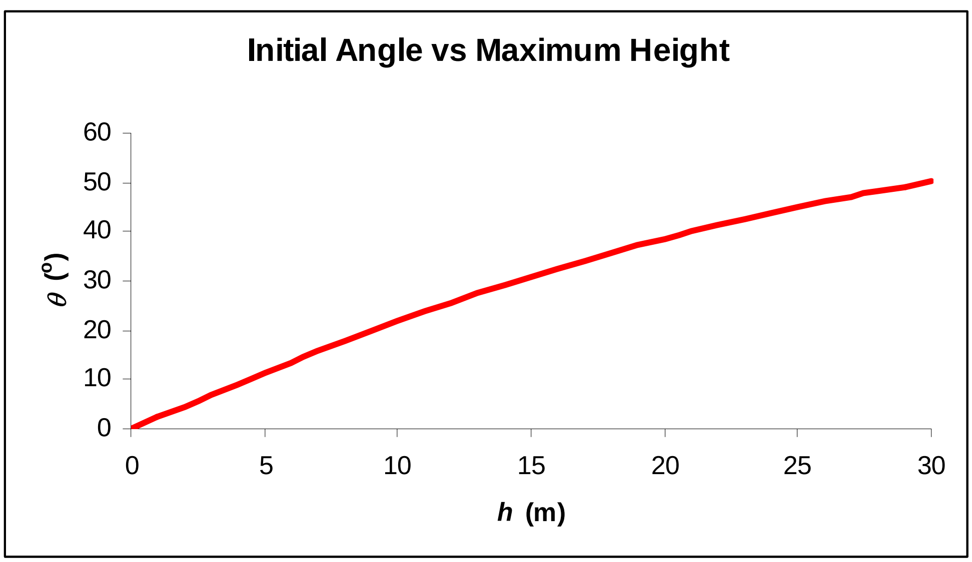
$$V_0 = \left(2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 10\ \text{m} + \frac{9.81}{8}\frac{\text{m}}{\text{s}^2} \times 100^2\ \text{m}^2 \times \frac{1}{10\ \text{m}}\right)^{\frac{1}{2}} = 37.7\frac{\text{m}}{\text{s}}$$

From Eq. 3: $v_0 = \sqrt{2gh} = V_0 \sin\theta,\ \theta = \sin^{-1}\dfrac{\sqrt{2gh}}{V_0}$ (5)

$$\theta = \sin^{-1}\left[\left(2\times 9.81\frac{\text{m}}{\text{s}^2}\times 10\text{ m}\right)^{\frac{1}{2}}\times\frac{\text{s}}{37.7\text{ m}}\right] = 21.8°$$

Plots of $V_0 = V_0(h)$ (Eq. 4) and $\theta_0 = \theta_0(h)$ (Eq. 5) are presented below:





Problem 1.13 [Difficulty: 2]

1.13 For each quantity listed, indicate dimensions using mass as a primary dimension, and give typical SI and English units:
(a) Power
(b) Pressure
(c) Modulus of elasticity
(d) Angular velocity
(e) Energy
(f) Moment of a force
(g) Momentum
(h) Shear stress
(i) Strain
(j) Angular momentum

Given: Basic dimensions M, L, t and T.

Find: Dimensional representation of quantities below, and typical units in SI and English systems.

Solution:

(a) Power

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}} = \frac{F \cdot L}{t}$$

From Newton's 2nd law $\quad \text{Force} = \text{Mass} \times \text{Acceleration} \quad$ so $\quad F = \frac{M \cdot L}{t^2}$

Hence $\quad \text{Power} = \frac{F \cdot L}{t} = \frac{M \cdot L \cdot L}{t^2 \cdot t} = \frac{M \cdot L^2}{t^3} \qquad \frac{kg \cdot m^2}{s^3} \qquad \frac{slug \cdot ft^2}{s^3}$

(b) Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2} \qquad \frac{kg}{m \cdot s^2} \qquad \frac{slug}{ft \cdot s^2}$$

(c) Modulus of elasticity

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2} \qquad \frac{kg}{m \cdot s^2} \qquad \frac{slug}{ft \cdot s^2}$$

(d) Angular velocity

$$\text{AngularVelocity} = \frac{\text{Radians}}{\text{Time}} = \frac{1}{t} \qquad \frac{1}{s} \qquad \frac{1}{s}$$

(e) Energy

$$\text{Energy} = \text{Force} \times \text{Distance} = F \cdot L = \frac{M \cdot L \cdot L}{t^2} = \frac{M \cdot L^2}{t^2} \qquad \frac{kg \cdot m^2}{s^2} \qquad \frac{slug \cdot ft^2}{s^2}$$

(f) Moment of a force

$$\text{MomentOfForce} = \text{Force} \times \text{Length} = F \cdot L = \frac{M \cdot L \cdot L}{t^2} = \frac{M \cdot L^2}{t^2} \qquad \frac{kg \cdot m^2}{s^2} \qquad \frac{slug \cdot ft^2}{s^2}$$

(g) Momentum

$$\text{Momentum} = \text{Mass} \times \text{Velocity} = M \cdot \frac{L}{t} = \frac{M \cdot L}{t} \qquad \frac{kg \cdot m}{s} \qquad \frac{slug \cdot ft}{s}$$

(h) Shear stress

$$\text{ShearStress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2} \qquad \frac{kg}{m \cdot s^2} \qquad \frac{slug}{ft \cdot s^2}$$

(i) Strain

$$\text{Strain} = \frac{\text{LengthChange}}{\text{Length}} = \frac{L}{L} \qquad \text{Dimensionless}$$

(j) Angular momentum

$$\text{AngularMomentum} = \text{Momentum} \times \text{Distance} = \frac{M \cdot L}{t} \cdot L = \frac{M \cdot L^2}{t} \qquad \frac{kg \cdot m^2}{s} \qquad \frac{slugs \cdot ft^2}{s}$$

Problem 1.14

(Difficulty: 1)

1.14 The density of a sample of sea water is 1.99 $slugs/ft^3$. What are the values in SI and EE units?

Given: The density of sea water is 1.99 $slugs/ft^3$

Find: The density of sea water in SI and EE units

Solution:

For SI unit:

The relations between the units are $1\ m = 3.28\ ft$, $1\ kg = 0.0685\ slug$

$$\rho = 1.99\frac{slug}{ft^3} = \frac{1.99 \times \frac{1}{0.0685}kg}{\frac{1}{3.28^3}m^3} = 1026\ \frac{kg}{m^3}$$

For EE units:

The relation between a lbm and a slug is $1\ lbm = 0.0311\ slug$

$$\rho = 1.99\frac{slug}{ft^3} = \frac{1.99 \times \frac{1}{0.0311}lbm}{ft^3} = 64.0\ \frac{lbm}{ft^3}$$

Problem 1.15

(Difficulty: 1)

1.15 A pump is rated at 50 hp; What is the rating in kW and Btu/hr ?

Given: The pump is rated at 50 hp.

Find: The rating in kW and Btu/hr.

Solution:

The relation between the units is

$$1\ kW = 1.341\ hp$$

$$1\ \frac{Btu}{hr} = 0.000393\ hp$$

The power is then

$$P = 50\ hp = 50\ hp \times \frac{1\ kW}{1.341\ hp} = 37.3\ kW$$

$$P = 50\ hp = 50\ hp \times \frac{1\ \frac{Btu}{hr}}{0.000393}\ hp = 127{,}200\ \frac{Btu}{hr}$$

Problem 1.16

(Difficulty: 1)

1.16 A fluid occupying 3.2 m^3 has a mass of $4Mg$. Calculate its density and specific volume in SI, EE and BG units.

Given: The fluid volume $V = 3.2\ m^3$ and mass $m = 4Mg$.

Find: Density and specific volume in SI, EE and BG units.

Solution:

For SI units:

The density is the mass divided by the volume

$$\rho = \frac{m}{V} = \frac{4000\ kg}{3.2\ m^3} = 1250\ \frac{kg}{m^3}$$

The specific volume is the reciprocal of the density:

$$v = \frac{1}{\rho} = 8 \times 10^{-4} \frac{m^3}{kg}$$

For EE units:

$$1\ \frac{lbm}{ft^3} = 16.0\ \frac{kg}{m^3}$$

The density is:

$$\rho = \frac{1250}{16.0} \frac{lbm}{ft^3} = 78.0 \frac{lbm}{ft^3}$$

And the specific volume is:

$$v = \frac{1}{\rho} = \frac{1}{78.0} \frac{ft^3}{lbm} = 0.0128\ \frac{ft^3}{lbm}$$

For BG unit, the relation between slug and lbm is:

$$1\ \frac{slug}{ft^3} = 32.2\ \frac{lbm}{ft^3}$$

The density is:

$$\rho = \frac{78.0}{32.2} \frac{slug}{ft^3} = 2.43\ \frac{slug}{ft^3}$$

And the specific volume is

$$v = \frac{1}{\rho} = \frac{1}{2.43}\ \frac{ft^3}{slug} = 0.412\ \ \frac{ft^3}{slug}$$

Problem 1.17

(Difficulty: 1)

1.17 If a power plant is rated at 2000 MW output and operates (on average) at 75% of rated power, how much energy (in J and $ft \cdot lbf$) does it put out a year.

Given: The power plant is rated at $= 2000\ MW$. Efficiency $\eta = 75\%$.

Find: Energy output per year E in SI and EE units.

Solution:

For SI units:

The energy produced is a year is:

$$E = Pt \cdot \eta = 2000 \times 10^6\ W \times \left(365 \frac{day}{yr} \times 24 \frac{hr}{day} \times 3600 \frac{s}{hr}\right) s \times 0.75 = 4.73 \times 10^{16}\ J$$

For EE units:

The relation between ft-lbf and Joules is

$$1\ ft \cdot lbf = 1.356\ J$$

The energy is:

$$E = \frac{4.73 \times 10^{16}}{1.356} ft \cdot lbf = 3.49 \times 10^{16}\ ft \cdot lbf$$

Problem 1.18 [Difficulty: 2]

1.18 For each quantity listed, indicate dimensions using force as a primary dimension, and give typical SI and English units:
(a) Power
(b) Pressure
(c) Modulus of elasticity
(d) Angular velocity
(e) Energy
(f) Momentum
(g) Shear stress
(h) Specific heat
(i) Thermal expansion coefficient
(j) Angular momentum

Given: Basic dimensions F, L, t and T.

Find: Dimensional representation of quantities below, and typical units in SI and English systems.

Solution:

(a) Power $\quad \text{Power} = \dfrac{\text{Energy}}{\text{Time}} = \dfrac{\text{Force} \times \text{Distance}}{\text{Time}} = \dfrac{F \cdot L}{t} \quad \dfrac{N \cdot m}{s} \quad \dfrac{lbf \cdot ft}{s}$

(b) Pressure $\quad \text{Pressure} = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{L^2} \quad \dfrac{N}{m^2} \quad \dfrac{lbf}{ft^2}$

(c) Modulus of elasticity $\quad \text{Pressure} = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{L^2} \quad \dfrac{N}{m^2} \quad \dfrac{lbf}{ft^2}$

(d) Angular velocity $\quad \text{AngularVelocity} = \dfrac{\text{Radians}}{\text{Time}} = \dfrac{1}{t} \quad \dfrac{1}{s} \quad \dfrac{1}{s}$

(e) Energy $\quad \text{Energy} = \text{Force} \times \text{Distance} = F \cdot L \quad N \cdot m \quad lbf \cdot ft$

(f) Momentum $\quad \text{Momentum} = \text{Mass} \times \text{Velocity} = M \cdot \dfrac{L}{t}$

From Newton's 2nd law $\quad \text{Force} = \text{Mass} \times \text{Acceleration}$ so $F = M \cdot \dfrac{L}{t^2}$ or $M = \dfrac{F \cdot t^2}{L}$

Hence $\quad \text{Momentum} = M \cdot \dfrac{L}{t} = \dfrac{F \cdot t^2 \cdot L}{L \cdot t} = F \cdot t \quad N \cdot s \quad lbf \cdot s$

(g) Shear stress $\quad \text{ShearStress} = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{L^2} \quad \dfrac{N}{m^2} \quad \dfrac{lbf}{ft^2}$

(h) Specific heat $\quad \text{SpecificHeat} = \dfrac{\text{Energy}}{\text{Mass} \times \text{Temperature}} = \dfrac{F \cdot L}{M \cdot T} = \dfrac{F \cdot L}{\left(\dfrac{F \cdot t^2}{L}\right) \cdot T} = \dfrac{L^2}{t^2 \cdot T} \quad \dfrac{m^2}{s^2 \cdot K} \quad \dfrac{ft^2}{s^2 \cdot R}$

(i) Thermal expansion coefficient $\quad \text{ThermalExpansionCoefficient} = \dfrac{\left(\dfrac{\text{LengthChange}}{\text{Length}}\right)}{\text{Temperature}} = \dfrac{1}{T} \quad \dfrac{1}{K} \quad \dfrac{1}{R}$

(j) Angular momentum $\quad \text{AngularMomentum} = \text{Momentum} \times \text{Distance} = F \cdot t \cdot L \quad N \cdot m \cdot s \quad lbf \cdot ft \cdot s$

Problem 1.19

[Difficulty: 1]

1.19 Derive the following conversion factors:
(a) Convert a pressure of 1 psi to kPa.
(b) Convert a volume of 1 liter to gallons.
(c) Convert a viscosity of 1 $\mathrm{lbf \cdot s/ft^2}$ to $\mathrm{N \cdot s/m^2}$.

Given: Pressure, volume and density data in certain units

Find: Convert to different units

Solution:

Using data from tables

(a) $$1\cdot\mathrm{psi} = 1\cdot\mathrm{psi} \times \frac{6895\,\mathrm{Pa}}{1\cdot\mathrm{psi}} \times \frac{1\cdot\mathrm{kPa}}{1000\,\mathrm{Pa}} = 6.89\,\mathrm{kPa}$$

(b) $$1\cdot\mathrm{liter} = 1\cdot\mathrm{liter} \times \frac{1\cdot\mathrm{quart}}{0.946\,\mathrm{liter}} \times \frac{1\cdot\mathrm{gal}}{4\cdot\mathrm{quart}} = 0.264\,\mathrm{gal}$$

(c) $$1\cdot\frac{\mathrm{lbf\cdot s}}{\mathrm{ft}^2} = 1\cdot\frac{\mathrm{lbf\cdot s}}{\mathrm{ft}^2} \times \frac{4.448\,\mathrm{N}}{1\cdot\mathrm{lbf}} \times \left(\frac{\frac{1}{12}\cdot\mathrm{ft}}{0.0254\,\mathrm{m}}\right)^2 = 47.9\,\frac{\mathrm{N\cdot s}}{\mathrm{m}^2}$$

Problem 1.20 [Difficulty: 1]

1.20 Express the following in SI units:

(a) 5 acre · ft
(b) 150 in^3/s
(c) 3 gpm
(d) 3 mph/s

Given: Quantities in English Engineering (or customary) units.

Find: Quantities in SI units.

Solution: Use Table and other sources (e.g., Machinery's Handbook, Mark's Standard Handbook)

(a) $$3.7\cdot acre\cdot ft = 3.7\cdot acre \times \frac{4047\cdot m^2}{1\cdot acre} \times \frac{0.3048\cdot m}{1\cdot ft} = 4.56 \times 10^3\cdot m^3$$

(b) $$150\cdot\frac{in^3}{s} = 150\cdot\frac{in^3}{s} \times \left(\frac{0.0254\cdot m}{1\cdot in}\right)^3 = 0.00246\cdot\frac{m^3}{s}$$

(c) $$3\cdot gpm = 3\cdot\frac{gal}{min} \times \frac{231\cdot in^3}{1\cdot gal} \times \left(\frac{0.0254\cdot m}{1\cdot in}\right)^3\cdot\frac{1\cdot min}{60\cdot s} = 0.000189\cdot\frac{m^3}{s}$$

(d) $$3\cdot\frac{mph}{s} = 3\cdot\frac{mile}{hr\cdot s} \times \frac{1609\cdot m}{1\cdot mile} \times \frac{1\cdot hr}{3600\cdot s} = 1.34\cdot\frac{m}{s^2}$$

Problem 1.21

[Difficulty: 1]

1.21 Express the following in SI units:

(a) 100 cfm (ft^3/min)
(b) 5 gal
(c) 65 mph
(d) 5.4 acres

Given: Quantities in English Engineering (or customary) units.

Find: Quantities in SI units.

Solution: Use Table and other sources (e.g., Google)

(a) $$100 \cdot \frac{ft^3}{m} = 100 \cdot \frac{ft^3}{min} \times \left(\frac{0.0254 \cdot m}{1 \cdot in} \times \frac{12 \cdot in}{1 \cdot ft}\right)^3 \times \frac{1 \cdot min}{60 \cdot s} = 0.0472 \cdot \frac{m^3}{s}$$

(b) $$5 \cdot gal = 5 \cdot gal \times \frac{231 \cdot in^3}{1 \cdot gal} \times \left(\frac{0.0254 \cdot m}{1 \cdot in}\right)^3 = 0.0189 \cdot m^3$$

(c) $$65 \cdot mph = 65 \cdot \frac{mile}{hr} \times \frac{1852 \cdot m}{1 \cdot mile} \times \frac{1 \cdot hr}{3600 \cdot s} = 29.1 \cdot \frac{m}{s}$$

(d) $$5.4 \cdot acres = 5.4 \cdot acre \times \frac{4047 \cdot m^3}{1 \cdot acre} = 2.19 \times 10^4 \cdot m^2$$

Problem 1.22

[Difficulty: 1]

1.22 Express the following in BG units:
(a) 50 m^2
(b) 250 cc
(c) 100 kW
(d) 5 kg/m^2

Given: Quantities in SI (or other) units.

Find: Quantities in BG units.

Solution: Use appropriate Table

(a) $$50\cdot m^2 = 50\cdot m^2 \times \left(\frac{1\cdot in}{0.0254\,m} \times \frac{1\cdot ft}{12\cdot in}\right)^2 = 538\cdot ft^2$$

(b) $$250\cdot cc = 250\cdot cm^3 \times \left(\frac{1\cdot m}{100\cdot cm} \times \frac{1\cdot in}{0.0254\,m} \times \frac{1\cdot ft}{12\cdot in}\right)^3 = 8.83 \times 10^{-3}\cdot ft^3$$

(c) $$100\cdot kW = 100\cdot kW \times \frac{1000\cdot W}{1\cdot kW} \times \frac{1\cdot hp}{746\cdot W} = 134\cdot hp$$

(d) $$5\cdot\frac{kg}{m^2} = 5\cdot\frac{kg}{m^2} \times \left(\frac{0.0254\,m}{1\cdot in} \times \frac{12\cdot in}{1\cdot ft}\right)^2 \times \frac{1\cdot slug}{14.95\,kg} = 0.0318\,\frac{slug}{ft^2}$$

Problem 1.23 [Difficulty: 2]

1.23 While you're waiting for the ribs to cook, you muse about the propane tank of your barbecue. You're curious about the volume of propane versus the actual tank size. Find the liquid propane volume when full (the weight of the propane is specified on the tank). Compare this to the tank volume (take some measurements, and approximate the tank shape as a cylinder with a hemisphere on each end). Explain the discrepancy.

Given: Geometry of tank, and weight of propane.

Find: Volume of propane, and tank volume; explain the discrepancy.

Solution: Use Table and other sources (e.g., Google) as needed.

The author's tank is approximately 12 in in diameter, and the cylindrical part is about 8 in. The weight of propane specified is 17 lb.

The tank diameter is $D = 12 \cdot in$

The tank cylindrical height is $L = 8 \cdot in$

The mass of propane is $m_{prop} = 17 \cdot lbm$

The specific gravity of propane is $SG_{prop} = 0.495$

The density of water is $\rho = 998 \cdot \frac{kg}{m^3}$

The volume of propane is given by $V_{prop} = \frac{m_{prop}}{\rho_{prop}} = \frac{m_{prop}}{SG_{prop} \cdot \rho}$

$$V_{prop} = 17 \cdot lbm \times \frac{1}{0.495} \times \frac{m^3}{998 \cdot kg} \times \frac{0.454 \cdot kg}{1 \cdot lbm} \times \left(\frac{1 \cdot in}{0.0254 \cdot m}\right)^3 \qquad V_{prop} = 953 \cdot in^3$$

The volume of the tank is given by a cylinder diameter D length L, $\pi D^2 L/4$ and a sphere (two halves) given by $\pi D^3/6$

$$V_{tank} = \frac{\pi \cdot D^2}{4} \cdot L + \frac{\pi \cdot D^3}{6}$$

$$V_{tank} = \frac{\pi \cdot (12 \cdot in)^2}{4} \cdot 8 \cdot in + \pi \cdot \frac{(12 \cdot in)^3}{6} \qquad V_{tank} = 1810 \cdot in^3$$

The ratio of propane to tank volumes is $\frac{V_{prop}}{V_{tank}} = 53 \cdot \%$

This seems low, and can be explained by a) tanks are not filled completely, b) the geometry of the tank gave an overestimate of the volume (the ends are not really hemispheres, and we have not allowed for tank wall thickness).

Problem 1.24

[Difficulty: 1]

1.24 Derive the following conversion factors:

(a) Convert a volume flow rate in cubic inches per minute to cubic millimeters per minute.

(b) Convert a volume flow rate in cubic meters per second to gallons per minute (gpm).

(c) Convert a volume flow rate in liters per minute to gpm.

(d) Convert a volume flow rate of air in standard cubic feet per minute (SCFM) to cubic meters per hour. A standard cubic foot of gas occupies one cubic foot at standard temperature and pressure ($T = 15°C$ and $p = 101.3$ kPa absolute).

Given: Data in given units

Find: Convert to different units

Solution:

(a) $$1 \cdot \frac{in^3}{min} = 1 \cdot \frac{in^3}{min} \times \left(\frac{0.0254 m}{1 \cdot in} \times \frac{1000 mm}{1 \cdot m}\right)^3 \times \frac{1 \cdot min}{60 \cdot s} = 273 \cdot \frac{mm^3}{s}$$

(b) $$1 \cdot \frac{m^3}{s} = 1 \cdot \frac{m^3}{s} \times \frac{1 \cdot gal}{4 \times 0.000946 m^3} \times \frac{60 \cdot s}{1 \cdot min} = 15850\, gpm$$

(c) $$1 \cdot \frac{liter}{min} = 1 \cdot \frac{liter}{min} \times \frac{1 \cdot gal}{4 \times 0.946 \cdot liter} \times \frac{60 \cdot s}{1 \cdot min} = 0.264 \cdot gpm$$

(d) $$1 \cdot SCFM = 1 \cdot \frac{ft^3}{min} \times \left(\frac{0.0254 \cdot m}{\frac{1}{12} \cdot ft}\right)^3 \times \frac{60 \cdot min}{1 \cdot hr} = 1.70 \cdot \frac{m^3}{hr}$$

Problem 1.25

[Difficulty: 1]

1.25 The kilogram force is commonly used in Europe as a unit of force. (As in the U.S. customary system, where 1 lbf is the force exerted by a mass of 1 lbm in standard gravity, 1 kgf is the force exerted by a mass of 1 kg in standard gravity.) Moderate pressures, such as those for auto or truck tires, are conveniently expressed in units of kgf/cm^2. Convert 32 psig to these units.

Given: Definition of kgf.

Find: Conversion from psig to kgf/cm^2.

Solution: Use appropriate Table

Define kgf $\quad \text{kgf} = 1\cdot\text{kg} \times 9.81\cdot\frac{\text{m}}{\text{s}^2} \qquad \text{kgf} = 9.81\text{N}$

Then

$$32\cdot\frac{\text{lbf}}{\text{in}^2} \times \frac{4.448\,\text{N}}{1\cdot\text{lbf}} \times \frac{1\cdot\text{kgf}}{9.81\cdot\text{N}} \times \left(\frac{12\cdot\text{in}}{1\cdot\text{ft}} \times \frac{1\cdot\text{ft}}{0.3048\,\text{m}} \times \frac{1\cdot\text{m}}{100\cdot\text{cm}}\right)^2 = 2.25\frac{\text{kgf}}{\text{cm}^2}$$

Problem 1.26 [Difficulty: 2]

1.26 From thermodynamics, we know that the coefficient of performance of an ideal air conditioner (COP_{ideal}) is given by

$$COP_{ideal} = \frac{T_L}{T_H - T_L}$$

where T_L and T_H are the room and outside temperatures (absolute). If an AC is to keep a room at 20°C when it is 40°C outside, find the COP_{ideal}. Convert to an EER value, and compare this to a typical Energy Star–compliant EER value.

Given: Equation for COP_{ideal} and temperature data.

Find: COP_{ideal}, *EER*, and compare to a typical Energy Star compliant *EER* value.

Solution: Use the COP equation. Then use conversions from Table or other sources (e.g., www.energystar.gov) to find the EER.

The given data is $T_L = (20 + 273)\cdot K$ $\quad T_L = 293\cdot K$ $\quad T_H = (40 + 273)\cdot K$ $\quad T_H = 313\cdot K$

The COP_{Ideal} is $\quad COP_{Ideal} = \frac{293}{313 - 293} = 14.65$

The EER is a similar measure to COP except the cooling rate (numerator) is in BTU/hr and the electrical input (denominator) is in W:

$$EER_{Ideal} = COP_{Ideal} \times \frac{\frac{BTU}{hr}}{W} \qquad EER_{Ideal} = 14.65 \times \frac{2545\cdot\frac{BTU}{hr}}{746\cdot W} = 50.0\cdot\frac{BTU}{hr\cdot W}$$

This compares to Energy Star compliant values of about 15 BTU/hr/W! We have some way to go! We can define the isentropic efficiency as

$$\eta_{isen} = \frac{EER_{Actual}}{EER_{Ideal}}$$

Hence the isentropic efficiency of a very good AC is about 30%.

Problem 1.27 [Difficulty: 2]

1.27 The maximum theoretical flow rate (slug/s) through a supersonic nozzle is

$$\dot{m}_{max} = 2.38 \frac{A_t p_0}{\sqrt{T_0}}$$

where A_t (ft^2) is the nozzle throat area, p_0 (psia) is the tank pressure, and T_0 (°R) is the tank temperature. Is this equation dimensionally correct? If not, find the units of the 2.38 term. Write the equivalent equation in SI units.

Given: Equation for maximum flow rate.

Find: Whether it is dimensionally correct. If not, find units of 2.38 coefficient. Write a SI version of the equation

Solution: Rearrange equation to check units of 0.04 term. Then use conversions from Table or other sources (e.g., Google)

"Solving" the equation for the constant 2.38: $$2.38 = \frac{m_{max}\sqrt{T_0}}{A_t \cdot p_0}$$

Substituting the units of the terms on the right, the units of the constant are

$$\frac{slug}{s} \times R^{\frac{1}{2}} \times \frac{1}{ft^2} \times \frac{1}{psi} = \frac{slug}{s} \times R^{\frac{1}{2}} \times \frac{1}{ft^2} \times \frac{in^2}{lbf} \times \frac{lbf \cdot s^2}{slug \cdot ft} = \frac{R^{\frac{1}{2}} \cdot in^2 \cdot s}{ft^3}$$

Hence the constant is actually $$c = 2.38 \frac{R^{\frac{1}{2}} \cdot in^2 \cdot s}{ft^3}$$

For BG units we could start with the equation and convert each term (e.g., A_t), and combine the result into a new constant, or simply convert c directly:

$$c = 2.38 \frac{R^{\frac{1}{2}} \cdot in^2 \cdot s}{ft^3} = 2.38 \frac{R^{\frac{1}{2}} \cdot in^2 \cdot s}{ft^3} \times \left(\frac{K}{1.8 \cdot R}\right)^{\frac{1}{2}} \times \left(\frac{1 \cdot ft}{12 \cdot in}\right)^2 \times \frac{1 \cdot ft}{0.3048 m}$$

$$c = 0.04 \cdot \frac{K^{\frac{1}{2}} \cdot s}{m}$$ so $$m_{max} = 0.04 \frac{A_t \cdot p_0}{\sqrt{T_0}}$$ with A_t in m^2, p_0 in Pa, and T_0 in K.

Problem 1.28

[Difficulty: 1]

1.28 The mean free path λ of a molecule of gas is the average distance it travels before collision with another molecule. It is given by

$$\lambda = C\frac{m}{\rho d^2}$$

where m and d are the molecule's mass and diameter, respectively, and ρ is the gas density. What are the dimensions of constant C for a dimensionally consistent equation?

Given: Equation for mean free path of a molecule.

Find: Dimensions of C for a diemsionally consistent equation.

Solution: Use the mean free path equation. Then "solve" for C and use dimensions.

The mean free path equation is $\lambda = C \cdot \frac{m}{\rho \cdot d^2}$

"Solving" for C, and using dimensions $C = \frac{\lambda \cdot \rho \cdot d^2}{m}$

$$C = \frac{L \times \frac{M}{L^3} \times L^2}{M} = 0$$

The constant C is dimensionless.

Problem 1.29

[Difficulty: 1]

1.29 A container weighs 3.5 lbf when empty. When filled with water at 90°F, the mass of the container and its contents is 2.5 slug. Find the weight of water in the container, and its volume in cubic feet, using data from Appendix A.

Given: Data on a container and added water.

Find: Weight and volume of water added.

Solution: Use Appendix

For the empty container $W_c = 3.5 \cdot lbf$

For the filled container $M_{total} = 2.5 \cdot slug$

The weight of water is then $W_w = M_{total} \cdot g - W_c$

$$W_w = 2.5 \cdot slug \times 32.2 \cdot \frac{ft}{s^2} \times \frac{1 \cdot lbf \cdot s^2}{1 \cdot slug \cdot ft} - 3.5 \cdot lbf \qquad W_w = 77.0\,lbf$$

The temperature is $90°F = 32.2°C$ and from Table A.7 $\rho = 1.93 \cdot \frac{slug}{ft^3}$

Hence $V_w = \frac{M_w}{\rho}$ or $V_w = \frac{W_w}{g \cdot \rho}$

$$V_w = 77.0 \cdot lbf \times \frac{1}{32.2} \cdot \frac{s^2}{ft} \times \frac{1}{1.93} \cdot \frac{ft^3}{slug} \times \frac{1 \cdot slug \cdot ft}{1 \cdot lbf \cdot s^2} \qquad V_w = 1.24 ft^3$$

Problem 1.30 [Difficulty: 1]

1.30 A parameter that is often used in describing pump performance is the specific speed, $N_{S_{cu}}$, given by

$$N_{s_{cu}} = \frac{N(\text{rpm})[Q(\text{gpm})]^{1/2}}{[H(\text{ft})]^{3/4}}$$

What are the units of specific speed? A particular pump has a specific speed of 2000. What will be the specific speed in SI units (angular velocity in rad/s)?

Given: Specific speed in customary units

Find: Units; Specific speed in SI units

Solution:

The units are $\dfrac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}$ or $\dfrac{\text{ft}^{\frac{3}{4}}}{\text{s}^{\frac{3}{2}}}$

Using data from tables

$$N_{Scu} = 2000\cdot\frac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}$$

$$N_{Scu} = 2000\times\frac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}\times\frac{2\cdot\pi\cdot\text{rad}}{1\cdot\text{rev}}\times\frac{1\cdot\text{min}}{60\cdot\text{s}}\times\left(\frac{4\times0.000946\cdot\text{m}^3}{1\cdot\text{gal}}\cdot\frac{1\cdot\text{min}}{60\cdot\text{s}}\right)^{\frac{1}{2}}\times\left(\frac{\frac{1}{12}\cdot\text{ft}}{0.0254\cdot\text{m}}\right)^{\frac{3}{4}}$$

$$N_{Scu} = 4.06\cdot\frac{\frac{\text{rad}}{\text{s}}\cdot\left(\frac{\text{m}^3}{\text{s}}\right)^{\frac{1}{2}}}{\text{m}^{\frac{3}{4}}}$$

Problem 1.31

[Difficulty: 2]

1.31 Calculate the density of standard air in a laboratory from the ideal gas equation of state. Estimate the experimental uncertainty in the air density calculated for standard conditions (29.9 in. of mercury and 59°F) if the uncertainty in measuring the barometer height is ±0.1 in. of mercury and the uncertainty in measuring temperature is ±0.5°F. (Note that 29.9 in. of mercury corresponds to 14.7 psia.)

Given: Air at standard conditions – p = 29.9 in Hg, T = 59°F

Uncertainty in p is ± 0.1 in Hg, in T is ± 0.5°F

Note that 29.9 in Hg corresponds to 14.7 psia

Find: Air density using ideal gas equation of state; Estimate of uncertainty in calculated value.

Solution:

$$\rho = \frac{p}{RT} = 14.7\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lb}\cdot{}^{\text{o}}\text{R}}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{519^{\text{o}}\text{R}} \times 144\frac{\text{in}^2}{\text{ft}^2}$$

The uncertainty in density is given by

$$u_\rho = \left[\left(\frac{p}{\rho}\frac{\partial\rho}{\partial p}u_p\right)^2 + \left(\frac{T}{\rho}\frac{\partial\rho}{\partial T}u_T\right)^2\right]^{\frac{1}{2}}$$

$$\frac{p}{\rho}\frac{\partial\rho}{\partial p} = RT\frac{1}{RT} = \frac{RT}{RT} = 1; \qquad u_p = \frac{\pm 0.1}{29.9} = \pm 0.334\%$$

$$\frac{T}{\rho}\frac{\partial\rho}{\partial T} = \frac{T}{\rho}\cdot -\frac{p}{RT^2} = -\frac{p}{\rho RT} = -1; \qquad u_T = \frac{\pm 0.5}{460+59} = \pm 0.0963\%$$

Then

$$u_\rho = \left[u_p^2 + \left(-u_T\right)^2\right]^{\frac{1}{2}} = \pm\left[0.334\%^2 + \left(-0.0963\%\right)^2\right]^{\frac{1}{2}}$$

$$u_\rho = \pm 0.348\% = \pm 2.66\times 10^{-4}\,\frac{\text{lbm}}{\text{ft}^3}$$

Problem 1.32

[Difficulty: 2]

1.32 The mass of the standard American golf ball is 1.62 ± 0.01 oz and its mean diameter is 1.68 ± 0.01 in. Determine the density and specific gravity of the American golf ball. Estimate the uncertainties in the calculated values.

Given: Standard American golf ball:

$$m = 1.62 \pm 0.01 \text{oz} \quad (20 \text{ to } 1)$$
$$D = 1.68 \pm 0.01 \text{in.} \quad (20 \text{ to } 1)$$

Find: Density and specific gravity; Estimate uncertainties in calculated values.

Solution: Density is mass per unit volume, so

$$\rho = \frac{m}{V\!\!\!-} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{m}{(D/2)^3} = \frac{6}{\pi}\frac{m}{D^3}$$

$$\rho = \frac{6}{\pi} \times 1.62 \text{ oz} \times \frac{1}{(1.68)^3 \text{in.}^3} \times \frac{0.4536 \text{ kg}}{16 \text{ oz}} \times \frac{\text{in.}^3}{(0.0254)^3 \text{m}^3} = 1130 \text{ kg/m}^3$$

and

$$\text{SG} = \frac{\rho}{\rho_{H_2O}} = 1130 \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{1000 \text{ kg}} = 1.13$$

The uncertainty in density is given by

$$u_\rho = \left[\left(\frac{m}{\rho}\frac{\partial \rho}{\partial m} u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial \rho}{\partial D} u_D\right)^2\right]^{\frac{1}{2}}$$

$$\frac{m}{\rho}\frac{\partial \rho}{\partial m} = \frac{m}{\rho}\frac{1}{\forall} = \frac{\forall}{\forall} = 1; \qquad u_m = \frac{\pm 0.01}{1.62} = \pm 0.617\%$$

$$\frac{D}{\rho}\frac{\partial \rho}{\partial D} = \frac{D}{\rho}\cdot\left(-3\frac{6m}{\pi D^4}\right) = -3\frac{6}{\pi}\frac{m}{\rho D^4} = -3; \qquad u_D = \frac{\pm 0.1}{1.68} = \pm 0.595\%$$

Thus

$$u_\rho = \pm\left[u_m^2 + (-3u_D)^2\right]^{\frac{1}{2}} = \pm\left[0.617\%^2 + (-3 \times 0.595\%)^2\right]^{\frac{1}{2}} \quad u_\rho = \pm 1.89\% = \pm 21.4 \frac{\text{kg}}{\text{m}^3}$$

$$u_{SG} = u_\rho = \pm 1.89\% = \pm 0.0214$$

Finally,

$$\rho = 1130 \pm 21.4 \text{kg/m}^3 \quad (20 \text{ to } 1)$$
$$\text{SG} = 1.13 \pm 0.0214 \quad (20 \text{ to } 1)$$

Problem 1.33

[Difficulty: 2]

1.33 A can of pet food has the following internal dimensions: 102 mm height and 73 mm diameter (each ±1 mm at odds of 20 to 1). The label lists the mass of the contents as 397 g. Evaluate the magnitude and estimated uncertainty of the density of the pet food if the mass value is accurate to ±1 g at the same odds.

Given: Pet food can

$$H = 102 \pm 1\ \text{mm}\ (20\text{ to }1)$$
$$D = 73 \pm 1\ \text{mm}\quad (20\text{ to }1)$$
$$m = 397 \pm 1\ \text{g}\quad (20\text{ to }1)$$

Find: Magnitude and estimated uncertainty of pet food density.

Solution: Density is

$$\rho = \frac{m}{\forall} = \frac{m}{\pi R^2 H} = \frac{4}{\pi}\frac{m}{D^2 H} \quad \text{or} \quad \rho = \rho(m, D, H)$$

From uncertainty analysis:
$$u_\rho = \pm\left[\left(\frac{m}{\rho}\frac{\partial\rho}{\partial m}u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2 + \left(\frac{H}{\rho}\frac{\partial\rho}{\partial H}u_H\right)^2\right]^{\frac{1}{2}}$$

Evaluating:
$$\frac{m}{\rho}\frac{\partial\rho}{\partial m} = \frac{m}{\rho}\frac{4}{\pi}\frac{1}{D^2H} = \frac{1}{\rho}\frac{4\,m}{\pi D^2 H} = 1; \qquad u_m = \frac{\pm 1}{397} = \pm 0.252\%$$
$$\frac{D}{\rho}\frac{\partial\rho}{\partial D} = \frac{D}{\rho}(-2)\frac{4m}{\pi D^3 H} = (-2)\frac{1}{\rho}\frac{4m}{\pi D^2 H} = -2; \qquad u_D = \frac{\pm 1}{73} = \pm 1.37\%$$
$$\frac{H}{\rho}\frac{\partial\rho}{\partial H} = \frac{H}{\rho}(-1)\frac{4m}{\pi D^2 H^2} = (-1)\frac{1}{\rho}\frac{4m}{\pi D^2 H} = -1; \qquad u_H = \frac{\pm 1}{102} = \pm 0.980\%$$

Substituting:
$$u_\rho = \pm\left[(1\times 0.252)^2 + (-2\times 1.37)^2 + (-1\times 0.980)^2\right]^{\frac{1}{2}}$$
$$u_\rho = \pm 2.92\%$$

$$\forall = \frac{\pi}{4}D^2H = \frac{\pi}{4}\times(73)^2\ \text{mm}^2 \times 102\ \text{mm} \times \frac{\text{m}^3}{10^9\ \text{mm}^3} = 4.27\times 10^{-4}\ \text{m}^3$$
$$\rho = \frac{m}{\forall} = \frac{397\ \text{g}}{4.27\times 10^{-4}\ \text{m}^3} \times \frac{\text{kg}}{1000\ \text{g}} = 930\ \text{kg/m}^3$$

Thus:
$$\rho = 930 \pm 27.2\ \text{kg/m}^3\ (20\text{ to }1)$$

Problem 1.34

[Difficulty: 2]

1.34 The mass flow rate in a water flow system determined by collecting the discharge over a timed interval is 0.2 kg/s. The scales used can be read to the nearest 0.05 kg and the stopwatch is accurate to 0.2 s. Estimate the precision with which the flow rate can be calculated for time intervals of (a) 10 s and (b) 1 min.

Given: Mass flow rate of water determine by collecting discharge over a timed interval is 0.2 kg/s.

Scales can be read to nearest 0.05 kg.

Stopwatch can be read to nearest 0.2 s.

Find: Estimate precision of flow rate calculation for time intervals of (a) 10 s, and (b) 1 min.

Solution: Apply methodology of uncertainty analysis,

Computing equations:

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

$$u_{\dot{m}} = \pm\left[\left(\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m}u_{\Delta m}\right)^2 + \left(\frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t}u_{\Delta t}\right)^2\right]^{\frac{1}{2}}$$

Thus

$$\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m} = \Delta t\frac{1}{\Delta t} = 1 \quad \text{and} \quad \frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t} = \frac{\Delta t^2}{\Delta m}\cdot -\frac{\Delta m}{\Delta t^2} = -1$$

The uncertainties are expected to be ± half the least counts of the measuring instruments.

Tabulating results:

Time Interval, Δt (s)	Error in Δt (s)	Uncertainty in Δt (%)	Water Collected, Δm (kg)	Error in Δm (kg)	Uncertainty in Δm (%)	Uncertainty in $\dot{m}$ (%)
10	± 0.10	± 1.0	2.0	± 0.025	± 1.25	± 1.60
60	± 0.10	± 0.167	12.0	± 0.025	± 0.208	± 0.267

A time interval of about 15 seconds should be chosen to reduce the uncertainty in results to ± 1 percent.

Problem 1.35

[Difficulty: 3]

1.35 The mass flow rate of water in a tube is measured using a beaker to catch water during a timed interval. The nominal mass flow rate is 100 g/s. Assume that mass is measured using a balance with a least count of 1 g and a maximum capacity of 1 kg, and that the timer has a least count of 0.1 s. Estimate the time intervals and uncertainties in measured mass flow rate that would result from using 100, 500, and 1000 mL beakers. Would there be any advantage in using the largest beaker? Assume the tare mass of the empty 1000 mL beaker is 500 g.

Given: Nominal mass flow rate of water determined by collecting discharge (in a beaker) over a timed interval is $\dot{m} = 100\ \mathrm{g/s}$; Scales have capacity of 1 kg, with least count of 1 g; Timer has least count of 0.1 s; Beakers with volume of 100, 500, 1000 mL are available – tare mass of 1000 mL beaker is 500 g.

Find: Estimate (a) time intervals, and (b) uncertainties, in measuring mass flow rate from using each of the three beakers.

Solution: To estimate time intervals assume beaker is filled to maximum volume in case of 100 and 500 mL beakers and to maximum allowable mass of water (500 g) in case of 1000 mL beaker.

Then $\dot{m} = \dfrac{\Delta m}{\Delta t}$ and $\Delta t = \dfrac{\Delta m}{\dot{m}} = \dfrac{\rho \Delta \forall}{\dot{m}}$

Tabulating results

$\Delta\forall = 100\ \mathrm{mL}\ 500\ \mathrm{mL}\ 1000\ \mathrm{mL}$

$\Delta t = \ 1\ \mathrm{s} \quad 5\ \mathrm{s} \quad 5\ \mathrm{s}$

Apply the methodology of uncertainty analysis, Appendix E. Computing equation:

$$u_{\dot{m}} = \pm\left[\left(\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m}u_{\Delta m}\right)^2 + \left(\frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t}u_{\Delta t}\right)^2\right]^{\frac{1}{2}}$$

The uncertainties are ± half the least counts of the measuring instruments: $\delta\Delta m = \pm 0.5\ \mathrm{g}$ $\quad \delta\Delta t = 0.05\ \mathrm{s}$

$$\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m} = \Delta t \frac{1}{\Delta t} = 1 \quad \text{and} \quad \frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t} = \frac{\Delta t^2}{\Delta m}\cdot -\frac{\Delta m}{\Delta t^2} = -1 \qquad \therefore u_{\dot{m}} = \pm\left[u_{\Delta m}{}^2 + \left(-u_{\Delta t}\right)^2\right]^{\frac{1}{2}}$$

Tabulating results:

Beaker Volume $\Delta\forall$ (mL)	Water Collected Δm(g)	Error in Δm (g)	Uncertainty in Δm (%)	Time Interval Δt (s)	Error in Δt (s)	Uncertainty in Δt (%)	Uncertainty in $\dot{m}$ (%)
100	100	± 0.50	± 0.50	1.0	± 0.05	± 5.0	± 5.03
500	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0
1000	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0

Since the scales have a capacity of 1 kg and the tare mass of the 1000 mL beaker is 500 g, there is no advantage in using the larger beaker. The uncertainty in $\dot{m}$ could be reduced to ± 0.50 percent by using the large beaker if a scale with greater capacity the same least count were available

Problem 1.36

[Difficulty: 2]

1.36 The mass of the standard British golf ball is 45.9 ± 0.3 g and its mean diameter is 41.1 ± 0.3 mm. Determine the density and specific gravity of the British golf ball. Estimate the uncertainties in the calculated values.

Given: Standard British golf ball:

$$m = 45.9 \pm 0.3 \text{ g} \quad (20 \text{ to } 1)$$
$$D = 41.1 \pm 0.3 \text{ mm} \quad (20 \text{ to } 1)$$

Find: Density and specific gravity; Estimate of uncertainties in calculated values.

Solution: Density is mass per unit volume, so

$$\rho = \frac{m}{\forall} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{m}{(D/2)^3} = \frac{6}{\pi}\frac{m}{D^3}$$

$$\rho = \frac{6}{\pi} \times 0.0459 \text{ kg} \times \frac{1}{(0.0411)^3}\text{m}^3 = 1260 \text{ kg/m}^3$$

and

$$SG = \frac{\rho}{\rho H_2O} = 1260\,\frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{1000 \text{ kg}} = 1.26$$

The uncertainty in density is given by

$$u_\rho = \pm\left[\left(\frac{m}{\rho}\frac{\partial\rho}{\partial m}u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2\right]^{\frac{1}{2}}$$

$$\frac{m}{\rho}\frac{\partial\rho}{\partial m} = \frac{m}{\rho}\frac{1}{\forall} = \frac{\forall}{\forall} = 1; \qquad u_m = \pm\frac{0.3}{45.9} = \pm 0.654\%$$

$$\frac{D}{\rho}\frac{\partial D}{\partial m} = \frac{D}{\rho}\left(-3\frac{6}{\pi}\frac{m}{D^4}\right) = -3\left(\frac{6m}{\pi D^4}\right) = -3; \qquad u_D = \pm\frac{0.3}{41.1} = \pm 0.730\%$$

Thus

$$u_\rho = \pm\left[u_m{}^2 + (-3u_D)^2\right]^{\frac{1}{2}} = \pm\left[0.654^2 + (-3\times 0.730)^2\right]^{\frac{1}{2}}$$
$$u_\rho = \pm 2.29\% = \pm 28.9 \text{ kg/m}^3$$
$$u_{SG} = u_\rho = \pm 2.29\% = \pm 0.0289$$

Summarizing

$$\rho = 1260 \pm 28.9 \text{ kg/m}^3 \; (20 \text{ to } 1)$$

$$SG = 1.26 \pm 0.0289 \; (20 \text{ to } 1)$$

Problem 1.37

[Difficulty: 3]

1.37 From Appendix A, the viscosity μ ($N \cdot s/m^2$) of water at temperature T (K) can be computed from $\mu = A10^{B/(T-C)}$, where $A = 2.414 \times 10^{-5}\ N \cdot s/m^2$, $B = 247.8$ K, and $C = 140$ K. Determine the viscosity of water at 30°C, and estimate its uncertainty if the uncertainty in temperature measurement is ± 0.5°C.

Given: Data on water

Find: Viscosity; Uncertainty in viscosity

Solution:

The data is: $A = 2.414\times 10^{-5} \cdot \frac{N \cdot s}{m^2}$ $\quad B = 247.8 K$ $\quad C = 140 \cdot K$ $\quad T = 303 \cdot K$

The uncertainty in temperature is $u_T = \frac{0.5 \cdot K}{293 \cdot K}$ $\quad u_T = 0.171 \cdot \%$

Also $\mu(T) = A \cdot 10^{\frac{B}{(T-C)}}$ Evaluating $\mu(293 \cdot K) = 1.005\times 10^{-3} \cdot \frac{N \cdot s}{m^2}$

For the uncertainty

$$\frac{d}{dT}\mu(T) = -\frac{A \cdot B \cdot \ln(10)}{10^{\frac{B}{C-T}} \cdot (C-T)^2}$$

Hence $u_\mu(T) = \left| \frac{T}{\mu(T)} \cdot \frac{d}{dT}\mu(T) \cdot u_T \right| = \frac{\ln(10) \cdot |B \cdot T \cdot u_T|}{(|C-T|)^2}$ Evaluating $u_\mu(T) = 1.11 \cdot \%$

Problem 1.38

[Difficulty: 3]

1.38 An enthusiast magazine publishes data from its road tests on the lateral acceleration capability of cars. The measurements are made using a 150-ft-diameter skid pad. Assume the vehicle path deviates from the circle by ±2 ft and that the vehicle speed is read from a fifth-wheel speed-measuring system to ±0.5 mph. Estimate the experimental uncertainty in a reported lateral acceleration of 0.7 *g*. How would you improve the experimental procedure to reduce the uncertainty?

Given: Lateral acceleration, a = 0.70 g, measured on 150-ft diameter skid pad; Uncertainties in Path deviation ±2 ft; vehicle speed ±0.5 mph

Find: Estimate uncertainty in lateral acceleration; ow could experimental procedure be improved?

Solution: Lateral acceleration is given by $a = V^2/R$.

From Appendix F, $u_a = \pm[(2u_v)^2 + (u_R)^2]^{1/2}$

From the given data, $$V^2 = aR; \quad V = \sqrt{aR} = \sqrt{0.70 \times 32.2 \frac{\text{ft}}{\text{s}^2} \times 75 \text{ ft}} = 41.1 \frac{\text{ft}}{\text{s}}$$

Then $$u_v = \pm \frac{\delta V}{V} = \pm 0.5 \frac{\text{mi}}{\text{hr}} \times \frac{\text{s}}{41.1 \text{ ft}} \times 5280 \frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600 \text{ s}} = \pm 0.0178$$

and $$u_R = \pm \frac{\delta R}{R} = \pm 2 \text{ ft} \times \frac{1}{75 \text{ ft}} = \pm 0.0267$$

so

$$u_a = \pm\left[(2 \times 0.0178)^2 + (0.0267)^2\right]^{1/2} = \pm 0.0445$$

$$u_a = \pm 4.45 \text{ percent}$$

Experimental procedure could be improved by using a larger circle, assuming the absolute errors in measurement are constant.

For

$$D = 400 \text{ ft}; \quad R = 200 \text{ ft}$$

$$V^2 = aR; \quad V = \sqrt{aR} = \sqrt{0.70 \times 32.2 \frac{\text{ft}}{\text{s}^2} \times 200 \text{ ft}} = 67.1 \frac{\text{ft}}{\text{s}} = 45.8 \text{ mph}$$

$$u_V = \pm \frac{0.5}{45.8} = \pm 0.0109; \quad u_R = \pm \frac{2}{200} = \pm 0.0100$$

$$u_a = \pm\left[(2 \times 0.0109)^2 + 0.0100^2\right] = \pm 0.0240 = \pm 2.4\%$$

Problem 1.39

[Difficulty: 4]

1.39 The height of a building may be estimated by measuring the horizontal distance to a point on the ground and the angle from this point to the top of the building. Assuming these measurements are $L = 100 \pm 0.5$ ft and $\theta = 30 \pm 0.2°$, estimate the height H of the building and the uncertainty in the estimate. For the same building height and measurement uncertainties, use *Excel's Solver* to determine the angle (and the corresponding distance from the building) at which measurements should be made to minimize the uncertainty in estimated height. Evaluate and plot the optimum measurement angle as a function of building height for $50 \le H \le 1000$ ft.

Given: Data on length and angle measurements

Find: Height; Angle for minimum uncertainty in height; Plot

Solution:

The data is: $L = 100\cdot ft$ $\quad \delta L = 0.5\cdot ft$ $\quad \theta = 30\cdot deg$ $\quad \delta\theta = 0.2\cdot deg$

Uncertainties: $u_L = \frac{\delta L}{L}$ $\quad u_L = 0.5\cdot\%$ $\quad u_\theta = \frac{\delta\theta}{\theta}$ $\quad u_\theta = 0.667\cdot\%$

The height is: $H = L\cdot\tan(\theta)$ $\quad H = 57.7\cdot ft$ $\quad$ with uncertainty $\quad u_H = \sqrt{\left(\frac{L}{H}\cdot\frac{\partial}{\partial L}H\cdot u_L\right)^2 + \left(\frac{\theta}{H}\cdot\frac{\partial}{\partial\theta}H\cdot u_\theta\right)^2}$

Hence with $\quad \frac{\partial}{\partial L}H = \tan(\theta)$ $\quad \frac{\partial}{\partial\theta}H = L\cdot\left(1 + \tan(\theta)^2\right)$ $\quad u_H = \sqrt{\left(\frac{L}{H}\cdot\tan(\theta)\cdot u_L\right)^2 + \left[\frac{L\cdot\theta}{H}\cdot\left(1 + \tan(\theta)^2\right)\cdot u_\theta\right]^2}$

Evaluating $\quad u_H = 0.949\cdot\%$ $\quad$ and $\quad \delta H = u_H\cdot H$ $\quad \delta H = 0.548\cdot ft$

The height is then $H = 57.7\cdot ft$ +/- $\delta H = 0.548\cdot ft$

To plot u_H versus θ for a given H we need to replace L, u_L and u_θ with functions of θ. Doing this and simplifying

$$u_H(\theta) = \sqrt{\left(\tan(\theta)\cdot\frac{\delta L}{H}\right)^2 + \left[\frac{\delta\theta}{\tan(\theta)}\cdot\left(1 + \tan(\theta)^2\right)\right]^2}$$

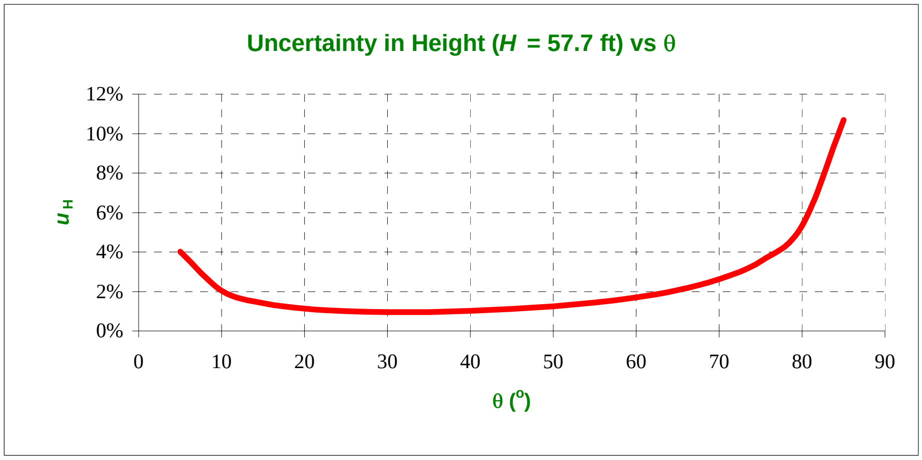
Given data:

H =	57.7	ft
δL =	0.5	ft
$\delta\theta$ =	0.2	deg

For this building height, we are to vary θ (and therefore L) to minimize the uncertainty u_H.

Plotting u_H vs θ

θ (deg)	u_H
5	4.02%
10	2.05%
15	1.42%
20	1.13%
25	1.00%
30	0.95%
35	0.96%
40	1.02%
45	1.11%
50	1.25%
55	1.44%
60	1.70%
65	2.07%
70	2.62%
75	3.52%
80	5.32%
85	10.69%

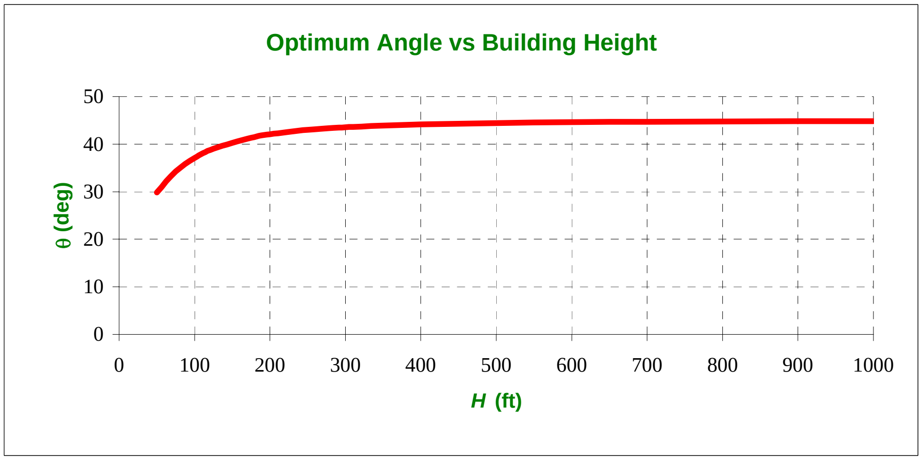


Optimizing using *Solver*

θ (deg)	u_H
31.4	0.947%

To find the optimum θ as a function of building height *H* we need a more complex *Solver*

H (ft)	θ (deg)	u_H
50	29.9	0.992%
75	34.3	0.877%
100	37.1	0.818%
125	39.0	0.784%
175	41.3	0.747%
200	42.0	0.737%
250	43.0	0.724%
300	43.5	0.717%
400	44.1	0.709%
500	44.4	0.705%
600	44.6	0.703%
700	44.7	0.702%
800	44.8	0.701%
900	44.8	0.700%
1000	44.9	0.700%



Use *Solver* to vary ALL θ's to minimize the total u_H!

Total u_H's: 11.3%

Problem 1.40

[Difficulty: 4]

1.40 An American golf ball is described in Problem 1.32 Assuming the measured mass and its uncertainty as given, determine the precision to which the diameter of the ball must be measured so the density of the ball may be estimated within an uncertainty of ±1 percent.

Given: American golf ball, m = 1.62 ± 0.01 oz, D = 1.68 in.

Find: Precision to which D must be measured to estimate density within uncertainty of ± 1percent.

Solution: Apply uncertainty concepts

Definition: Density, $\rho \equiv \frac{m}{\forall} \quad \forall = \frac{4}{3}\pi R^3 = \frac{\pi D^3}{6}$

Computing equation: $$u_R = \pm\left[\left(\frac{x_1}{R}\frac{\partial R}{\partial x_1}u_{x_1}\right)^2 + \cdots\right]^{\frac{1}{2}}$$

From the definition,

$$\rho = \frac{m}{\pi D^{3/6}} = \frac{6\,m}{\pi D^3} = \rho(m, D)$$

Thus $\frac{m}{\rho}\frac{\partial\rho}{\partial m} = 1$ and $\frac{D}{\rho}\frac{\partial\rho}{\partial D} = 3$, so

$$u_\rho = \pm[(1\,u_m)^2 + (3\,u_D)^2]^{\frac{1}{2}}$$

$$u_\rho^2 = u_m{}^2 + 9\,u_D^2$$

Solving,

$$u_D = \pm\tfrac{1}{3}[u_\rho{}^2 - u_m^2]^{\frac{1}{2}}$$

From the data given,

$$u_\rho = \pm 0.0100$$

$$u_m = \frac{\pm 0.01\,\text{oz}}{1.62\,\text{oz}} = \pm 0.00617$$

$$u_D = \pm\frac{1}{3}[(0.0100)^2 - (0.00617)^2]^{\frac{1}{2}} = \pm 0.00262 \text{ or } \pm 0.262\%$$

Since $u_D = \pm\frac{\delta D}{D}$, then

$$\delta D = \pm D\;u_D = \pm 1.68\,\text{in.}_x\;0.00262 = \pm 0.00441\,\text{in.}$$

The ball diameter must be measured to a precision of ± 0.00441 in.(± 0.112 mm) or better to estimate density within ± 1percent. A micrometer or caliper could be used.