CHAPTER 1

Section 1.1 Solutions --

1. Not a function – 0 maps to both -3 and 3.	**2.** Not a function – 2 maps to both -2 and 2, and 5 maps to both -5 and 5.
3. Not a function – 4 maps to both -2 and 2, and 9 maps to both -3 and 3.	**4.** Function
5. Function	**6.** Function
7. Not a function – Since $\left(1,-2\sqrt{2}\right)$ and $\left(1,2\sqrt{2}\right)$ are both on the graph, it does not pass vertical line test.	**8.** Not a function – Since $(1,-1)$ and $(1,1)$ are both on the graph, it does not pass the vertical line test.
9. Not a function – Since $(1,-1)$ and $(1,1)$ are both on the graph, it does not pass the vertical line test.	**10.** Function
11. Function	**12.** Function
13. Not a function – Since $(0,5)$ and $(0,-5)$ are both on the graph, it does not pass the vertical line test.	**14.** Not a function – Since $(0,4)$ and $(0,-4)$ are both on the graph, it does not pass the vertical line test.
15. Function	**16.** Function
17. Not a function – Since $(0,-1)$ and $(0,-3)$ are both on the graph, it does not pass the vertical line test.	**18.** Function
19. **a)** 5 **b)** 1 **c)** -3	**20.** **a)** 1 **b)** -5 **c)** 0
21. **a)** 3 **b)** 2 **c)** 5	**22.** **a)** 0 **b)** 4 **c)** -5
23. **a)** -5 **b)** -5 **c)** -5	**24.** **a)** -2 **b)** -6 **c)** -4
25. **a)** 2 **b)** -8 **c)** -5	**26.** **a)** 2 **b)** 0 **c)** 3
27. 1	**28.** -1.5 and 3
29. 1 and –3	**30.** -7
31. For all *x* in the interval $[-4,4]$	**32.** For all *x* in the set $[-4,0)\cup[4]$
33. 6	**34.** -3
35. $f(-2)=2(-2)-3=\boxed{-7}$	**36.** $G(-3)=(-3)^2+2(-3)-7=\boxed{-4}$
37. $g(1)=5+1=\boxed{6}$	**38.** $F(-1)=4-(-1)^2=\boxed{3}$
39. Using #35 and #37, we see that $f(-2)+g(1)=-7+6=\boxed{-1}$.	**40.** Using #36 and #38, we see that $G(-3)-F(-1)=-4-3=\boxed{-7}$.
41. Using #35 and #37, we see that $3f(-2)-2g(1)=3(-7)-2(6)=\boxed{-33}$.	**42.** Using #36 and #38, we see that $2F(-1)-2G(-3)=2(3)-2(-4)=\boxed{14}$.

43. Using #35 and #37, we see that

$$\frac{f(-2)}{g(1)} = \boxed{-\frac{7}{6}}.$$

44. Using #36 and #38, we see that

$$\frac{G(-3)}{F(-1)} = \boxed{-\frac{4}{3}}.$$

45.

$$\frac{f(0)-f(-2)}{g(1)} = \frac{(2(0)-3)-(-7)}{6} = \frac{-3+7}{6} = \boxed{\frac{2}{3}}$$

46.

$$\frac{G(0)-G(-3)}{F(-1)} = \frac{\left(0^2+2(0)-7\right)-(-4)}{3} = \frac{-7+4}{3} = \boxed{-1}$$

47.

$$\begin{aligned} f(x+1)-f(x-1) &= [2(x+1)-3]-[2(x-1)-3] \\ &= [2x+2-3]-[2x-2-3] \\ &= [2x-1]-[2x-5] \\ &= 2x-1-2x+5 \\ &= \boxed{4} \end{aligned}$$

48.

$$\begin{aligned} F(t+1)-F(t-1) &= \left[4-(t+1)^2\right]-\left[4-(t-1)^2\right] \\ &= \left[4-\left(t^2+2t+1\right)\right]-\left[4-\left(t^2-2t+1\right)\right] \\ &= \left[4-t^2-2t-1\right]-\left[4-t^2+2t-1\right] \\ &= 4-t^2-2t-1-4+t^2-2t+1 \\ &= \boxed{-4t} \end{aligned}$$

49.

$$\begin{aligned} g(x+a)-f(x+a) &= [5+(x+a)]-[2(x+a)-3] \\ &= [5+x+a]-[2x+2a-3] \\ &= 5+x+a-2x-2a+3 \\ &= \boxed{8-x-a} \end{aligned}$$

50.

$$\begin{aligned} G(x+b)+F(b) &= \left[(x+b)^2+2(x+b)-7\right]+\left[4-b^2\right] \\ &= x^2+2bx+b^2+2x+2b-7+4-b^2 \\ &= \boxed{x^2+2bx+2x+2b-3} \end{aligned}$$

51. The domain is $\mathbb{R}$.
This is written using interval notation as $\boxed{(-\infty,\infty)}$.

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55. The domain is the set of all real numbers x such that $x-5 \neq 0$, that is $x \neq 5$.
This is written using interval notation as $\boxed{(-\infty, 5) \cup (5, \infty)}$.

56. The domain is the set of all real numbers t such that $t+3 \neq 0$, that is $t \neq -3$.
This is written using interval notation as $\boxed{(-\infty, -3) \cup (-3, \infty)}$.

57. The domain is the set of all real numbers x such that

$$x^2 - 4 = (x-2)(x+2) \neq 0,$$

that is $x \neq -2, 2$.
This is written using interval notation as $\boxed{(-\infty, -2) \cup (-2, 2) \cup (2, \infty)}$.

58. The domain is the set of all real numbers x such that

$$x^2 - 1 = (x-1)(x+1) \neq 0,$$

that is $x \neq -1, 1$.
This is written using interval notation as $\boxed{(-\infty, -1) \cup (-1, 1) \cup (1, \infty)}$.

59. Since $x^2 + 1 \neq 0$, for every real number x, the domain is $\mathbb{R}$.
This is written using interval notation as $\boxed{(-\infty, \infty)}$.

60. Since $x^2 + 4 \neq 0$, for every real number x, the domain is $\mathbb{R}$.
This is written using interval notation as $\boxed{(-\infty, \infty)}$.

61. The domain is the set of all real numbers x such that

$$7 - x \geq 0,$$

that is $7 \geq x$.
This is written using interval notation as $\boxed{(-\infty, 7]}$.

62. The domain is the set of all real numbers t such that

$$t - 7 \geq 0,$$

that is $t \geq 7$.
This is written using interval notation as $\boxed{[7, \infty)}$.

63. The domain is the set of all real numbers x such that

$$2x + 5 \geq 0,$$

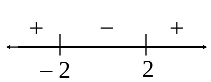
that is $x \geq -\frac{5}{2}$.
This is written using interval notation as $\boxed{\left[-\frac{5}{2}, \infty\right)}$.

64. The domain is the set of all real numbers x such that

$$5 - 2x \geq 0,$$

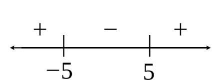
that is $\frac{5}{2} \geq x$.
This is written using interval notation as $\boxed{\left(-\infty, \frac{5}{2}\right]}$.

65. The domain is the set of all real numbers t such that $t^2 - 4 \geq 0$, which is equivalent to $(t-2)(t+2) \geq 0$.
CPs are -2, 2



This is written using interval notation as $\boxed{(-\infty, -2] \cup [2, \infty)}$.

66. The domain is the set of all real numbers x such that $x^2 - 25 \geq 0$, which is equivalent to $(x-5)(x+5) \geq 0$.
CPs are -5, 5



This is written using interval notation as $\boxed{(-\infty, -5] \cup [5, \infty)}$.

67. The domain is the set of all real numbers x such that

$$x-3>0,$$

that is $x>3$.
This is written using interval notation as $\boxed{(3,\infty)}$.

68. The domain is the set of all real numbers x such that

$$5-x>0,$$

that is $5>x$.
This is written using interval notation as $\boxed{(-\infty,5)}$.

69. Since $1-2x$ can be any real number, there is no restriction on x, so that the domain is $\boxed{(-\infty,\infty)}$.

70. Since $7-5x$ can be any real number, there is no restriction on x, so that the domain is $\boxed{(-\infty,\infty)}$.

71. The only restriction is that $x+4\neq 0$, so that $x\neq -4$. So, the domain is $\boxed{(-\infty,-4)\cup(-4,\infty)}$.

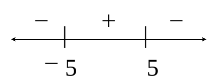
72. The only restriction is that $x^2-9=(x-3)(x+3)\neq 0$, so that $x\neq \pm 3$.
So, the domain is $\boxed{(-\infty,-3)\cup(-3,3)\cup(3,\infty)}$.

73. The domain is the set of all real numbers x such that

$$3-2x>0,$$

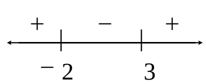
that is $\frac{3}{2}>x$.
This is written using interval notation as $\boxed{\left(-\infty,\frac{3}{2}\right)}$.

74. The domain is the set of all real numbers t such that $25-x^2>0$, which is equivalent to $(5-x)(5+x)>0$.
CPs are -5, 5



This is written using interval notation as $\boxed{(-5,5)}$.

75. The domain is the set of all real numbers t such that $t^2-t-6>0$, which is equivalent to $(t-3)(t+2)>0$.
CPs are -2, 3

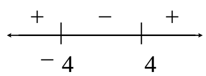


This is written using interval notation as $\boxed{(-\infty,-2)\cup(3,\infty)}$.

76. Since $t^2+9>0$, for all real numbers t, there is no restriction. So, the domain is $\boxed{(-\infty,\infty)}$.

77. The domain is the set of all real numbers *t* such that $x^2 - 16 \geq 0$, which is equivalent to $(x-4)(x+4) \geq 0$.

CPs are $-4,\ 4$



This is written using interval notation as $\boxed{(-\infty,-4] \cup [4,\infty)}$.

78. There is no restriction on *x*. So, the domain is $\boxed{(-\infty,\infty)}$.

79. The function can be written as $r(x) = \dfrac{x^2}{\sqrt{3-2x}}$. So, the domain is the set of real numbers *x* such that $3-2x>0$, that is $\frac{3}{2} > x$. This is written using interval notation as $\boxed{\left(-\infty,\frac{3}{2}\right)}$.

80. The function can be written as $p(x) = \dfrac{(x-1)^2}{\left(x^2-9\right)^{3/5}}$. So, the domain is the set of real numbers *x* such that $x^2 - 9 = (x-3)(x+3) \neq 0$, so that $x \neq \pm 3$. So, the domain is $\boxed{(-\infty,-3) \cup (-3,3) \cup (3,\infty)}$.

81. The domain of any linear function is $\boxed{(-\infty,\infty)}$.

82. The domain of any quadratic function is $\boxed{(-\infty,\infty)}$.

83. Solve $x^2 - 2x - 5 = 3$.

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$\boxed{x = -2, 4}$$

84. Solve $\frac{5}{6}x - \frac{3}{4} = \frac{2}{3}$.

$$10x - 9 = 8$$
$$10x = 17$$
$$\boxed{x = \tfrac{17}{10}}$$

85.

$$2x(x-5)^3 - 12(x-5)^2 = 0$$
$$2(x-5)^2\left[x(x-5) - 6\right] = 0$$
$$2(x-5)^2(x^2 - 5x - 6) = 0$$
$$2(x-5)^2(x-6)(x+1) = 0$$
$$\boxed{x = -1, 5, 6}$$

86.

$$3x(x+3)^2 - 6(x+3)^3 = 0$$
$$3(x+3)^2\left[x - 2(x+3)\right] = 0$$
$$3(x+3)^2\left(-x-6\right) = 0$$
$$\boxed{x = -3, -6}$$

87. <u>Assume</u>: 6am corresponds to $x=6$
noon corresponds to $x=12$

Then, the temperature at 6am is:

$$T(6)=-0.7(6)^2+16.8(6)-10.8=64.8^\circ F$$

The temperature at noon is:

$$\begin{aligned}T(12)&=-0.7(12)^2+16.8(12)-10.8\\&=90^\circ F\end{aligned}$$

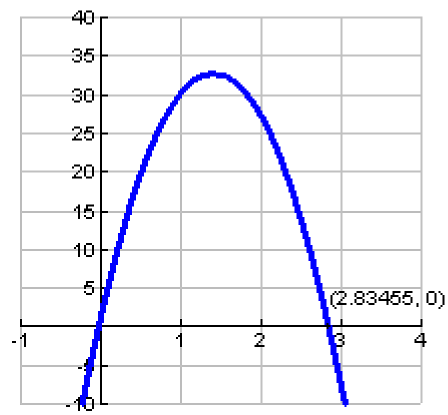
88. 9am corresponds to $x=9$ and 3pm corresponds to $x=15$. So,

$$T(9)=-0.5(9)^2+14.2(9)-2.8=84.5^\circ F$$

$$T(15)=-0.5(15)^2+14.2(15)-2.8=97.7^\circ F$$

89. $h(2)=-16(2)^2+45(2)+1=27\text{ ft}$

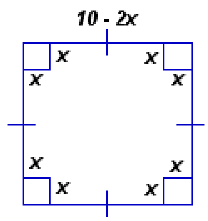
Since height must be nonnegative, only those values of t for which $h(t)\geq 0$ should be included in the domain. As such, we must solve $-16t^2+45t+1\geq 0$. Graphically, we see that



Hence, the domain of h is approximately [0, 2.8].

90. $h(14)=-16(4)^2+128(4)=\boxed{256\text{ ft}}$. The domain is $[0,\infty)$ since we are starting at time $t=0$ sec.

91. Start with a square piece of cardboard with dimensions $10 \text{ in.} \times 10 \text{ in.}$. Then, cut out 4 square corners with dimensions $x \text{ in.} \times x \text{ in.}$, as shown in the diagram:



Upon bending all four corners up, a box of height *x* is formed. Notice that all four sides of the base of the resulting box have length $10-2x$. The volume of the box, $V(x)$, is given by:

$$V(x) = (\text{Length})\cdot(\text{Width})\cdot(\text{Height})$$
$$= (10-2x)(10-2x)(x)$$
$$= x(10-2x)^2$$

The domain is $(0,5)$. (For any other values of *x*, one cannot form a box.)

92. The volume of a right circular cylindrical tank whose base radius is 10 ft and whose height is *h* is given by $V(h) = \pi(10)^2 h = 100\pi h$. If the height is increased by 2 ft, the corresponding volume would be:

$$V(h+2) = \pi(10)^2(h+2) = 100\pi h + 200\pi$$

So, the volume increased by 200π cubic ft, which corresponds to $200\pi \cdot 7.48\,\text{gal} \cong \boxed{4700\,\text{gal}}$.

93. $E(4) \approx 84$ Yen, $E(7) \approx 84$ Yen, $E(8) \approx 83$ Yen

94. a. The number of Japanese Yen to the US Dollar exchange rate increased by approximately 1 Japanese Yen to US Dollar from Week 2 to Week 3.
b. The number of Japanese Yen to the US Dollar exchange rate decreased by approximately 2 Japanese Yen to US Dollar from Week 6 to Week 7.

95. $P(14) = -\frac{1}{4}(14^2) + 7(14) + 180 = 229$ people

96. $P(6) = -\frac{1}{4}(6^2) + 7(6) + 180 = 213$ people

97. (1999, 3000), (2003, 4000), (2007, 5000), (2011, 6000), (2015, 7000)

98. Yes, for every input there corresponds a unique output.

99. a) *F*(50) = number of tons of carbon emitted by natural gas in 1950 = 0
b) *g*(50) = number of tons of coal emitted by natural gas in 1950 = 1000
c) *H*(50) = 2000

100. $F(100) + g(100) + G(100)$ represents the total amount (in millions of metric tons) of carbon emitted in 2000 by natural gas, coal, and petroleum.

101. Should apply the <u>vertical</u> line test to determine if the relationship describes a function. The given relationship IS a function in this case.

102. $H(3) - H(-1) \neq H(3) + H(1)$, in general. You cannot distribute -1 through in this manner.

103.
$f(x+1) \neq f(x) + f(1)$, in general. You cannot distribute the function *f* through the input at which you are evaluating it.

104. There are two mistakes.
One, the computation $3-t>0$ should be $3-t \geq 0$. And two, the statement directly preceding the computation should be, "What can $3-t$ be?"
The domain should be $(-\infty, 3]$.

105. False. Consider the function $f(x) = \sqrt{9-x^2}$ on its domain $[-3,3]$. The vertical line test $x=4$ doesn't intersect the graph, but it still defines a function.

106. False. Consider the function $f(x) = x^2$ on its domain $\mathbb{R}$.

107. True

108. True

109.

$$\begin{aligned} f(1) = A(1)^2 - 3(1) &= -1 \\ A - 3 &= -1 \\ A &= 2 \end{aligned}$$

110. $g(3) = \dfrac{1}{b-3}$ is undefined only if $b=3$.

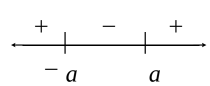
111. $F(-2) = \dfrac{C-(-2)}{D-(-2)} = \dfrac{C+2}{D+2}$ is undefined only if $\boxed{D=-2}$. So,

$F(-1) = \dfrac{C-(-1)}{D-(-1)} = \dfrac{C+1}{D+1} = \dfrac{C+1}{-2+1} = -(C+1) = 4$ implies that $\boxed{C=-5}$.

112. Many functions will work here. The easiest ones to construct are of the form $g(x) = \dfrac{b}{x-5}$. For such a function, certainly $g(5)$ is undefined. In order for $(1,-1)$ to be on the graph, it must be the case that $-1 = \dfrac{b}{1-5} = \dfrac{b}{-4}$, so that $b=4$. So, one function that works is $g(x) = \dfrac{4}{x-5}$.

113. The domain is the set of all real numbers *x* such that $x^2 - a^2 = (x-a)(x+a) \neq 0$, which is equivalent to $x \neq \pm a$. So, the domain is $\boxed{(-\infty,-a) \cup (-a,a) \cup (a,\infty)}$.

114. The domain is the set of all real numbers *x* such that $x^2 - a^2 = (x-a)(x+a) \geq 0$.
CPs: $x = \pm a$



So, the domain is $\boxed{(-\infty,-a] \cup [a,\infty)}$.

115.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[(x+h)^3+(x+h)\right]-\left[x^3+x\right]}{h}$$

$$=\frac{x^3+3x^2h+3xh^2+h^3+x+h-x^3-x}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2+1\right)}{h}=\boxed{3x^2+3xh+h^2+1}$$

So, at $h=0$ we get $\boxed{f'(x)=3x^2+1}$.

116.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[6(x+h)+\sqrt{x+h}\right]-\left[6x+\sqrt{x}\right]}{h}$$

$$=\frac{6h}{h}+\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$=6+\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=6+\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=6+\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

So, at $h=0$ we get $\boxed{f'(x)=6+\frac{1}{2\sqrt{x}}}$.

117.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{x+h-5}{x+h+3}-\frac{x-5}{x+3}}{h}$$

$$=\frac{(x+h-5)(x+3)-(x-5)(x+h+3)}{h(x+h+3)(x+3)}$$

$$=\frac{\left(x^2+xh-5x+3x+3h-15\right)-\left(x^2+hx+3x-5x-5h-15\right)}{h(x+h+3)(x+3)}$$

$$=\frac{8h}{h(x+h+3)(x+3)}=\frac{8}{(x+h+3)(x+3)}$$

So, at $h=0$ we get $\boxed{f'(x)=\frac{8}{(x+3)^2}}$

118.

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{\frac{x+h+7}{5-(x+h)}}-\sqrt{\frac{x+7}{5-x}}}{h}$$

$$=\frac{\sqrt{x+h+7}\sqrt{5-x}-\sqrt{x+7}\sqrt{5-x-h}}{h\sqrt{5-x-h}\sqrt{5-x}}$$

$$=\frac{\sqrt{(x+h+7)(5-x)}-\sqrt{(x+7)(5-x-h)}}{h\sqrt{5-x-h}\sqrt{5-x}}$$

$$=\frac{\sqrt{(x+h+7)(5-x)}-\sqrt{(x+7)(5-x-h)}}{h\sqrt{5-x-h}\sqrt{5-x}}\cdot\frac{\sqrt{(x+h+7)(5-x)}+\sqrt{(x+7)(5-x-h)}}{\sqrt{(x+h+7)(5-x)}+\sqrt{(x+7)(5-x-h)}}$$

$$=\frac{\left(5h+35-x^2-hx-2x\right)-\left(-2x-x^2-hx+35-7h\right)}{h\sqrt{5-x-h}\sqrt{5-x}\left(\sqrt{(x+h+7)(5-x)}+\sqrt{(x+7)(5-x-h)}\right)}$$

$$=\frac{12}{\sqrt{5-x-h}\sqrt{5-x}\left(\sqrt{(x+h+7)(5-x)}+\sqrt{(x+7)(5-x-h)}\right)}$$

So, at $h=0$ we get

$$f'(x)=\frac{12}{\sqrt{5-x}\sqrt{5-x}\left(\sqrt{(x+7)(5-x)}+\sqrt{(x+7)(5-x)}\right)}$$

$$=\frac{6}{(5-x)\sqrt{(x+7)(5-x)}}=\boxed{\frac{6}{(x+7)^{1/2}(5-x)^{3/2}}}$$

Section 1.2 Solutions --

1. $h(-x)=(-x)^2+2(-x)=x^2-2x\neq h(x)$ So, not even. $-h(-x)=-\left(x^2-2x\right)=-x^2+2x\neq h(x)$ So, not odd. Thus, $\boxed{\text{neither}}$.	**2.** $G(-x)=2(-x)^4+3(-x)^3$ $=2x^4-3x^3\neq G(x)$ So, not even. $-G(-x)=-\left(2x^4-3x^3\right)\neq G(x)$ So, not odd. Thus, $\boxed{\text{neither}}$.
3. $h(-x)=(-x)^{1/3}-(-x)$ $=-\left(x^{1/3}-x\right)\neq h(x)$ So, not even. $-h(-x)=-\left(-\left(x^{1/3}-x\right)\right)=x^{1/3}-x=h(x)$ So, $\boxed{\text{odd}}$.	**4.** $g(-x)=(-x)^{-1}+(-x)$ $=-\left(x^{-1}+x\right)\neq g(x)$ So, not even. $-g(-x)=-\left(-\left(x^{-1}+x\right)\right)$ $=x^{-1}+x=g(x)$ So, $\boxed{\text{odd}}$. (Note: $(-x)^{-1}=\frac{1}{-x}=-\frac{1}{x}=-(x)^{-1}$)

5. $f(-x) = |-x| + 5 = |-1||x| + 5$

$= |x| + 5 = f(x)$

So, $\boxed{\text{even}}$. Thus, f cannot be odd.

6. $f(-x) = |-x| + (-x)^2$

$= |-1||x| + x^2 = f(x)$

So, $\boxed{\text{even}}$. Thus, f cannot be odd.

7. $f(-x) = |-x| = |-1||x| = f(x)$

So, $\boxed{\text{even}}$. Thus, f cannot be odd.

8. $f(-x) = \left|(-x)^3\right| = \left|-x^3\right| = |-1|\left|x^3\right| = f(x)$

So, $\boxed{\text{even}}$. Thus, f cannot be odd.

9. $G(-t) = |(-t) - 3| = |-(t+3)|$

$= |t+3| \neq G(t)$

So, not even.

$-G(-t) = -|t+3| \neq G(t)$

So, not odd. Thus, $\boxed{\text{neither}}$.

10. $G(-t) = |(-t) + 2| \neq G(t)$

So, not even.

$-G(-t) = -|(-t) + 2| \neq G(t)$

So, not odd. Thus, $\boxed{\text{neither}}$.

11. $G(-t) = \sqrt{-t-3} = \sqrt{-(t+3)} \neq G(t)$

So, not even.

$-G(-t) = \underbrace{-\sqrt{-(t+3)}}_{\text{Note: Cannot distribute } -1 \text{ here}} \neq G(t)$

So, not odd. Thus, $\boxed{\text{neither}}$.

12. $f(-x) = \sqrt{2-(-x)} = \sqrt{2+x} \neq f(x)$

So, not even.

$-f(-x) = -\sqrt{2+x} \neq f(x)$

So, not odd. Thus, $\boxed{\text{neither}}$.

13. $g(-x) = \sqrt{(-x)^2 + (-x)}$

$= \sqrt{x^2 - x} \neq g(x)$

So, not even.

$-g(-x) = -\sqrt{x^2 - x} \neq g(x)$

So, not odd. Thus, $\boxed{\text{neither}}$.

14. $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

So, $\boxed{\text{even}}$. Thus, f cannot be odd.

15. $h(-x) = \dfrac{1}{-x} + 3 \neq h(x)$

So, not even.

$-h(-x) = -\left(\dfrac{1}{-x} + 3\right) = \dfrac{1}{x} - 3 \neq h(x)$

So, not odd. Thus, $\boxed{\text{neither}}$.

16. $h(-x) = \dfrac{1}{-x} - 2(-x) = -\left(\dfrac{1}{x} - 2x\right) \neq h(x)$

So, not even.

$-h(-x) = -\left(-\left(\dfrac{1}{x} - 2x\right)\right) = \dfrac{1}{x} - 2x = h(x)$

So, $\boxed{\text{odd}}$.

17.

Domain	$(-\infty, \infty)$
Range	$[-1, \infty)$
Increasing	$(-1, \infty)$
Decreasing	$(-3, -2)$
Constant	$(-\infty, -3) \cup (-2, -1)$

d) 0
e) -1
f) 2

18.

Domain	$[-4, \infty)$
Range	$(-\infty, 3]$
Increasing	$(1, 2)$
Decreasing	$(-3, 0) \cup (2, \infty)$
Constant	$[-4, -3) \cup (0, 1)$

d) -1
e) approximately 1.8
f) 1

19.

Domain	$[-7, 2]$
Range	$[-5, 4]$
Increasing	$(-4, 0)$
Decreasing	$(-7, -4) \cup (0, 2)$
Constant	nowhere

d) 4
e) 1
f) -5

20.

Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$
Increasing	$(-\infty, -3) \cup (3, \infty)$
Decreasing	$(-3, 3)$
Constant	nowhere

d) 0
e) 3.5
f) approximately -3.3

21.

Domain	$(-\infty, \infty)$
Range	$(-\infty, \infty)$
Increasing	$(-\infty, -3) \cup (4, \infty)$
Decreasing	nowhere
Constant	$(-3, 4)$

d) 2
e) 2
f) 2

22.

Domain	$(-\infty,\infty)$
Range	$(-\infty,\infty)$
Increasing	nowhere
Decreasing	$(-\infty,\infty)$
Constant	nowhere

d) 0
e) 1
f) -1

23.

Domain	$(-\infty,\infty)$
Range	$[-4,\infty)$
Increasing	$(0,\infty)$
Decreasing	$(-\infty,0)$
Constant	nowhere

d) -4
e) 0
f) 0

24.

Domain	$(-\infty,\infty)$
Range	$[0,\infty)$
Increasing	$(3,\infty)$
Decreasing	$(-\infty,-3)$
Constant	$(-3,3)$

d) 0
e) 0
f) 0

25.

Domain	$(-\infty,0)\cup(0,\infty)$
Range	$(-\infty,0)\cup(0,\infty)$
Increasing	$(-\infty,0)\cup(0,\infty)$
Decreasing	nowhere
Constant	nowhere

d) undefined
e) 3
f) -3

26.

Domain	$(-\infty,4)\cup(4,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-\infty,0)\cup(4,\infty)$
Decreasing	$(0,4)$
Constant	nowhere

d) 4
e) approximately 3.5
f) approximately 2.5

27.

Domain	$(-\infty,0)\cup(0,\infty)$
Range	$(-\infty,5)\cup[7]$
Increasing	$(-\infty,0)$
Decreasing	$(5,\infty)$
Constant	$(0,5)$

d) undefined
e) 3
f) 7

28.

Domain	$(-8,0)\cup(0,4]$
Range	$(-4,3]$
Increasing	$(-8,-5)\cup(0,4)$
Decreasing	$(-5,0)$
Constant	nowhere

d) undefined
e) approximately -0.8
f) 0

29.

$$\frac{\left[(x+h)^2-(x+h)\right]-\left[x^2-x\right]}{h}=$$
$$\frac{x^2+2hx+h^2-x-h-x^2+x}{h}=$$
$$\frac{\cancel{h}\left(2x+h-1\right)}{\cancel{h}}=\boxed{2x+h-1}$$

30.

$$\frac{\left[(x+h)^2+2(x+h)\right]-\left[x^2+2x\right]}{h}=$$
$$\frac{x^2+2hx+h^2+2x+2h-x^2-2x}{h}=$$
$$\frac{\cancel{h}\left(2x+h+2\right)}{\cancel{h}}=\boxed{2x+h+2}$$

31.

$$\frac{\left[(x+h)^2+3(x+h)\right]-\left[x^2+3x\right]}{h}=$$

$$\frac{x^2+2hx+h^2+3x+3h-x^2-3x}{h}=$$

$$\frac{\cancel{h}\left(2x+h+3\right)}{\cancel{h}}=\boxed{2x+h+3}$$

32.

$$\frac{\left[-(x+h)^2+5(x+h)\right]-\left[-x^2+5x\right]}{h}=$$

$$\frac{-x^2-2hx-h^2+5x+5h+x^2-5x}{h}=$$

$$\frac{\cancel{h}\left(-2x-h+5\right)}{\cancel{h}}=\boxed{-2x-h+5}$$

33.

$$\frac{\left[(x+h)^2-3(x+h)+2\right]-\left[x^2-3x+2\right]}{h}=$$

$$\frac{x^2+2hx+h^2-3x-3h+2-x^2+3x-2}{h}=$$

$$\frac{\cancel{h}\left(2x+h-3\right)}{\cancel{h}}=\boxed{2x+h-3}$$

34.

$$\frac{\left[(x+h)^2-2(x+h)+5\right]-\left[x^2-2x+5\right]}{h}=$$

$$\frac{x^2+2hx+h^2-2x-2h+5-x^2+2x-5}{h}=$$

$$\frac{\cancel{h}\left(2x+h-2\right)}{\cancel{h}}=\boxed{2x+h-2}$$

35.

$$\frac{\left[-3(x+h)^2+5(x+h)-4\right]-\left[-3x^2+5x-4\right]}{h}=$$

$$\frac{-3x^2-6hx-3h^2+5x+5h-4+3x^2-5x+4}{h}=$$

$$\frac{\cancel{h}\left(-6x-3h+5\right)}{\cancel{h}}=\boxed{-6x-3h+5}$$

36.

$$\frac{\left[-4(x+h)^2+2(x+h)-3\right]-\left[-4x^2+2x-3\right]}{h}=$$

$$\frac{-4x^2-8hx-4h^2+2x+2h-3+4x^2-2x+3}{h}=$$

$$\frac{\cancel{h}\left(-8x-4h+2\right)}{\cancel{h}}=\boxed{-8x-4h+2}$$

37. Note that $\left(a+b\right)^3=a^3+3a^2b+3ab^2+b^3$. As such, we have

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[\left(x+h\right)^3+\left(x+h\right)^2\right]-\left[x^3+x^2\right]}{h}$$

$$=\frac{x^3+3x^2h+3xh^2+h^3+x^2+2xh+h^2-x^3-x^2}{h}$$

$$=\frac{\cancel{h}\left(3x^2+3xh+h^2+2x+h\right)}{\cancel{h}}=\boxed{3x^2+3xh+h^2+2x+h}$$

38. Note that $\left(a-b\right)^4=a^4-4ab^3+6a^2b^2-4a^3b+b^4$. As such, we have

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[(x+h)-1\right]^4-\left[x-1\right]^4}{h}$$

$$=\frac{\left[(x+h)^4-4(x+h)^3+6(x+h)^2-4(x+h)+1\right]-\left[x^4-4x^3+6x^2-4x+1\right]}{h}$$

$$=\frac{\left[x^4-4x^3+6x^2-4x+1+h\left(4x^3+6x^2h+4xh^2+h^3-12x^2-12xh-4h^2+12x+6h-4\right)\right]-x^4+4x^3-6x^2+4x-1}{h}$$

$$=\frac{\cancel{h}\left(4x^3+6x^2h+4xh^2+h^3-12x^2-12xh-4h^2+12x+6h-4\right)}{\cancel{h}}$$

$$=\boxed{4x^3+6x^2h+4xh^2+h^3-12x^2-12xh-4h^2+12x+6h-4}$$

39. $$\frac{f(x+h)-f(x)}{h}=\frac{\frac{2}{x+h-2}-\frac{2}{x-2}}{h}=\frac{2(x-2)-2(x+h-2)}{h(x+h-2)(x-2)}=\frac{2x-4-2x-2h+4}{h(x+h-2)(x-2)}$$ $$=\frac{-2h}{h(x+h-2)(x-2)}=\boxed{\frac{-2}{(x+h-2)(x-2)}}$$
40. $$\frac{f(x+h)-f(x)}{h}=\frac{\frac{x+h+5}{x+h-7}-\frac{x+5}{x-7}}{h}=\frac{(x+h+5)(x-7)-(x+5)(x+h-7)}{h(x+h-7)(x-7)}$$ $$=\frac{\left(x^2+xh+5x-7x-7h-35\right)-\left(x^2+hx-7x+5x+5h-35\right)}{h(x+h-7)(x-7)}$$ $$=\frac{-12h}{h(x+h-7)(x-7)}=\boxed{\frac{-12}{(x+h-7)(x-7)}}$$
41. $$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h}$$ $$=\frac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h}\cdot\frac{\sqrt{1-2(x+h)}+\sqrt{1-2x}}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}$$ $$=\frac{(1-2x-2h)-(1-2x)}{h\left(\sqrt{1-2(x+h)}+\sqrt{1-2x}\right)}=\frac{-2h}{h\left(\sqrt{1-2(x+h)}+\sqrt{1-2x}\right)}$$ $$=\boxed{\frac{-2}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}}$$
42. $$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{(x+h)^2+(x+h)+1}-\sqrt{x^2+x+1}}{h}$$ $$=\frac{\sqrt{(x+h)^2+(x+h)+1}-\sqrt{x^2+x+1}}{h}\cdot\frac{\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}}{\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}}$$ $$=\frac{\left[(x+h)^2+(x+h)+1\right]-\left[x^2+x+1\right]}{h\left(\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}\right)}=\frac{\left[x^2+2hx+h^2+x+h+1\right]-\left[x^2+x+1\right]}{h\left(\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}\right)}$$ $$=\frac{h(2x+h+1)}{h\left(\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}\right)}=\boxed{\frac{2x+h+1}{\sqrt{(x+h)^2+(x+h)+1}+\sqrt{x^2+x+1}}}$$

43.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{4}{\sqrt{x+h}}-\frac{4}{\sqrt{x}}}{h}=\frac{4\sqrt{x}-4\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}=\frac{4\left(\sqrt{x}-\sqrt{x+h}\right)}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$=\frac{4\left(x-(x+h)\right)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\boxed{\frac{-4}{\sqrt{x(x+h)}\left(\sqrt{x}+\sqrt{x+h}\right)}}$$

44.

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{\frac{x+h}{x+h+1}}-\sqrt{\frac{x}{x+1}}}{h}=\frac{\sqrt{x+h}\sqrt{x+1}-\sqrt{x}\sqrt{x+h+1}}{h\sqrt{x+1}\sqrt{x+h+1}}$$

$$=\frac{\sqrt{(x+h)(x+1)}-\sqrt{x(x+h+1)}}{h\sqrt{x+1}\sqrt{x+h+1}}$$

$$=\frac{\sqrt{(x+h)(x+1)}-\sqrt{x(x+h+1)}}{h\sqrt{x+1}\sqrt{x+h+1}}\cdot\frac{\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}}{\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}}$$

$$=\frac{\left(x^2+hx+x+h\right)-\left(x^2+xh+x\right)}{h\sqrt{x+1}\sqrt{x+h+1}\left(\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}\right)}$$

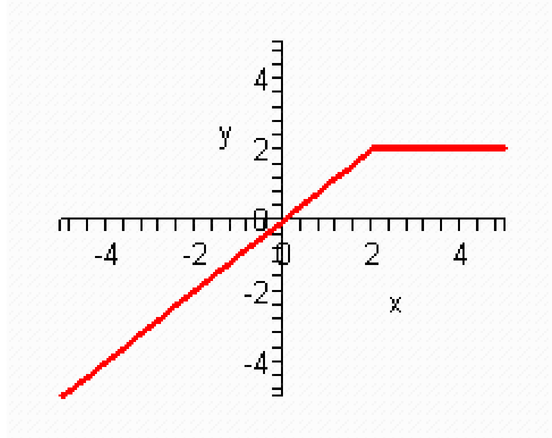
$$=\frac{h}{h\sqrt{x+1}\sqrt{x+h+1}\left(\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}\right)}$$

$$=\boxed{\frac{1}{\sqrt{x+1}\sqrt{x+h+1}\left(\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}\right)}}$$

45. $\frac{3^3-1^3}{3-1}=\frac{27-1}{2}=\boxed{13}$	**46.** $\frac{\frac{1}{3}-\frac{1}{1}}{3-1}=\frac{-\frac{2}{3}}{2}=\boxed{-\frac{1}{3}}$
47. $\frac{\lvert 3\rvert-\lvert 1\rvert}{3-1}=\boxed{1}$	**48.** $\frac{2(3)-2(1)}{3-1}=\frac{4}{2}=\boxed{2}$
49. $\frac{(1-2(3))-(1-2(1))}{3-1}=\frac{-5-(-1)}{2}=\boxed{-2}$	**50.** $\frac{\left(9-3^2\right)-\left(9-1^2\right)}{3-1}=\frac{0-8}{2}=\boxed{-4}$
51. $\frac{\lvert 5-2(3)\rvert-\lvert 5-2(1)\rvert}{3-1}=\frac{\lvert -1\rvert-3}{2}=\boxed{-1}$	**52.** $\frac{\sqrt{3^2-1}-\sqrt{1^2-1}}{3-1}=\frac{\sqrt{8}}{2}=\boxed{\sqrt{2}}$

53.

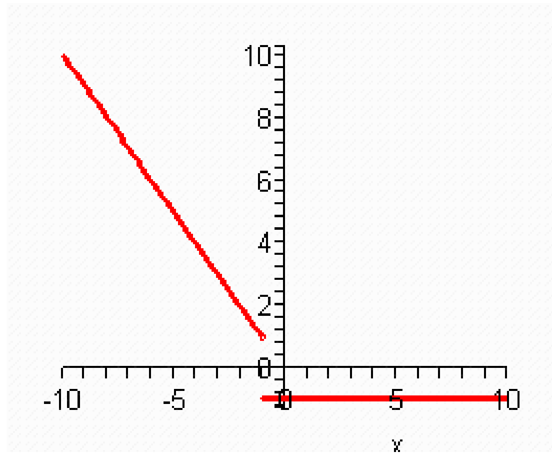
Domain	$(-\infty, \infty)$
Range	$(-\infty, 2]$
Increasing	$(-\infty, 2)$
Decreasing	nowhere
Constant	$(2, \infty)$



54.

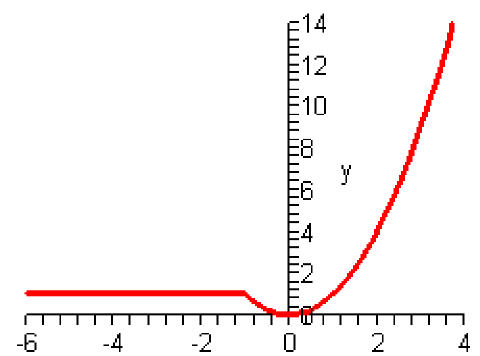
Domain	$(-\infty, \infty)$
Range	$\{-1\} \cup (1, \infty)$
Increasing	nowhere
Decreasing	$(-\infty, -1)$
Constant	$(-1, \infty)$

Notes on Graph: There should be an open hole at $(-1,1)$, and a closed hole at $(-1,-1)$.



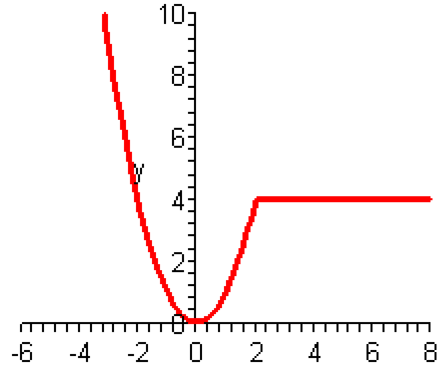
55.

Domain	$(-\infty, \infty)$
Range	$[0, \infty)$
Increasing	$(0, \infty)$
Decreasing	$(-1, 0)$
Constant	$(-\infty, -1)$



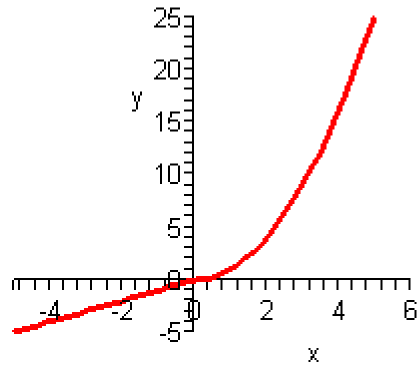
56.

Domain	$(-\infty,\infty)$
Range	$[0,\infty)$
Increasing	$(0,2)$
Decreasing	$(-\infty,0)$
Constant	$(2,\infty)$



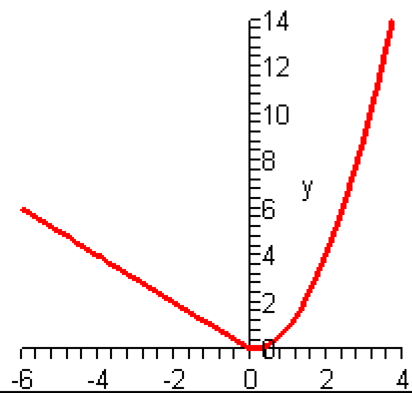
57.

Domain	$(-\infty,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-\infty,\infty)$
Decreasing	nowhere
Constant	nowhere



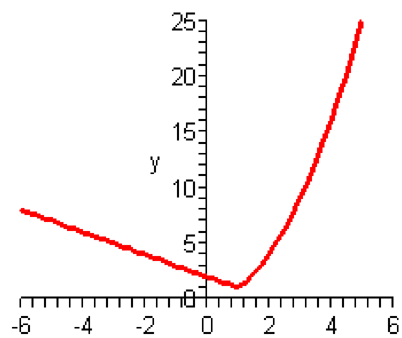
58.

Domain	$(-\infty,\infty)$
Range	$[0,\infty)$
Increasing	$(0,\infty)$
Decreasing	$(-\infty,0)$
Constant	nowhere



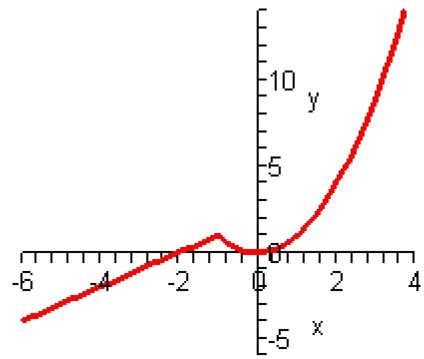
59.

Domain	$(-\infty,\infty)$
Range	$[1,\infty)$
Increasing	$(1,\infty)$
Decreasing	$(-\infty,1)$
Constant	nowhere



60.

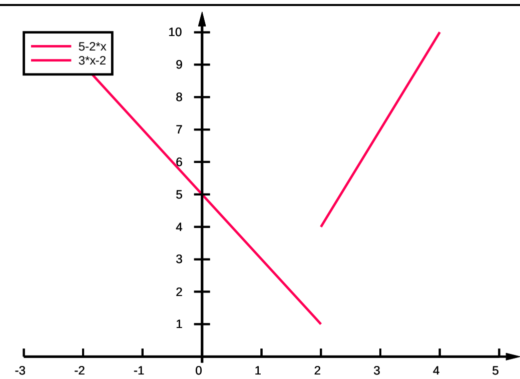
Domain	$(-\infty,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-\infty,-1)\cup(0,\infty)$
Decreasing	$(-1,0)$
Constant	nowhere



61.

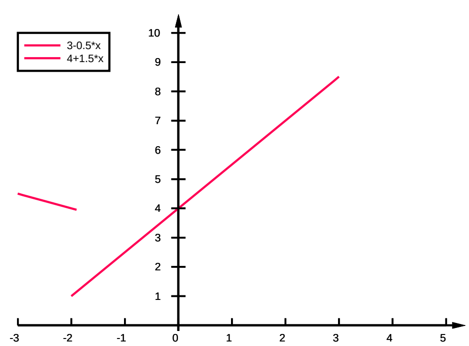
Domain	$(-\infty,2)\cup(2,\infty)$
Range	$(1,\infty)$
Increasing	$(2,\infty)$
Decreasing	$(-\infty,2)$
Constant	nowhere

Notes on Graph: There should be an open hole at(2,1) and at (2,4).



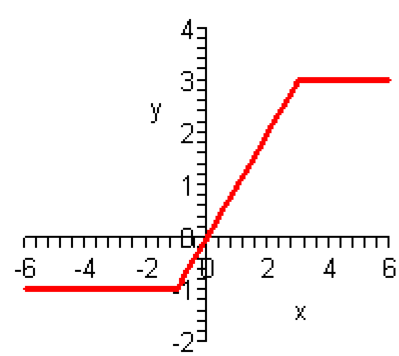
62.

Domain	$(-\infty,-2)\cup(-2,\infty)$
Range	$(1,\infty)$
Increasing	$(-2,\infty)$
Decreasing	$(-\infty,-2)$
Constant	nowhere



63.

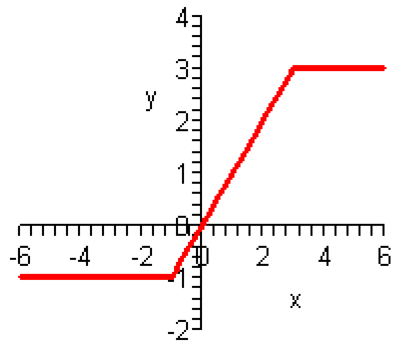
Domain	$(-\infty,\infty)$
Range	$[-1,3]$
Increasing	$(-1,3)$
Decreasing	nowhere
Constant	$(-\infty,-1)\cup(3,\infty)$



64.

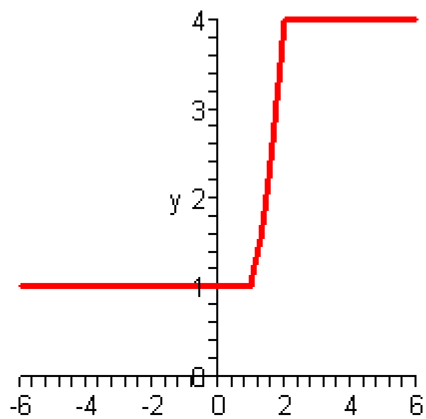
Domain	$(-\infty,-1)\cup(-1,3)\cup(3,\infty)$
Range	$[-1,3]$
Increasing	$(-1,3)$
Decreasing	nowhere
Constant	$(-\infty,-1)\cup(3,\infty)$

Notes on Graph: There should be open holes at $(-1,-1)$ and $(3,3)$.



65.

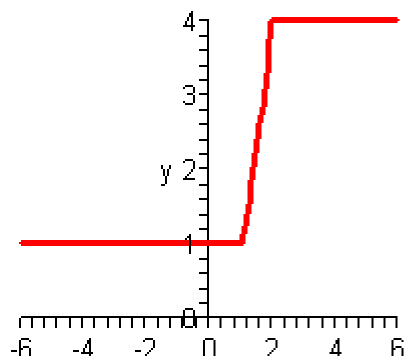
Domain	$(-\infty,\infty)$
Range	$[1,4]$
Increasing	$(1,2)$
Decreasing	nowhere
Constant	$(-\infty,1)\cup(2,\infty)$



66.

Domain	$(-\infty,1)\cup(1,2)\cup(2,\infty)$
Range	$[1,4]$
Increasing	$(1,2)$
Decreasing	nowhere
Constant	$(-\infty,1)\cup(2,\infty)$

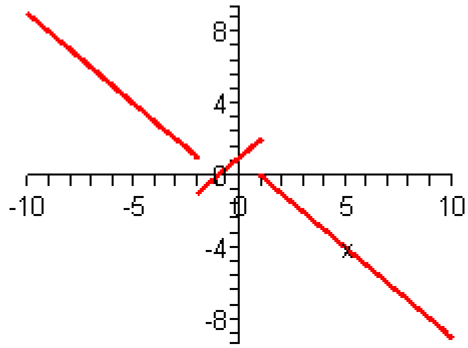
Notes on Graph: There should be open holes at $(1,1)$ and $(2,4)$.



67.

Domain	$(-\infty,-2)\cup(-2,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-2,1)$
Decreasing	$(-\infty,-2)\cup(1,\infty)$
Constant	nowhere

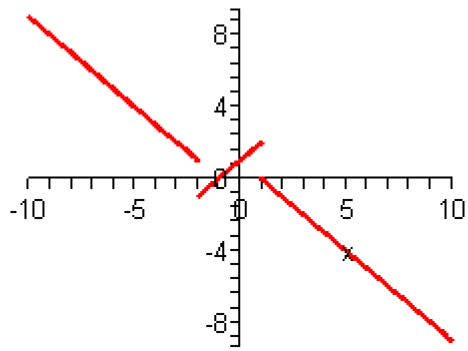
Notes on Graph: There should be open holes at $(-2,1)$, $(-2,-1)$, and $(1,2)$, and a closed hole at $(1,0)$.



68.

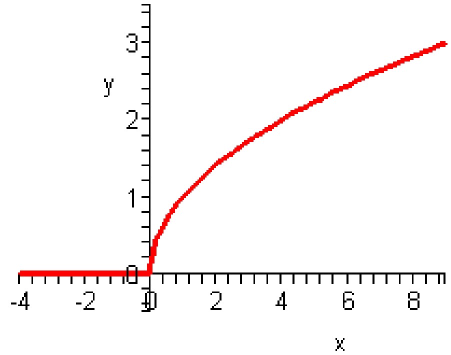
Domain	$(-\infty,1)\cup(1,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-2,1)$
Decreasing	$(-\infty,-2)\cup(1,\infty)$
Constant	nowhere

Notes on Graph: There should be open holes at $(-2,-1)$, $(1,2)$, and $(1,0)$, and a closed hole at $(-2,1)$.



69.

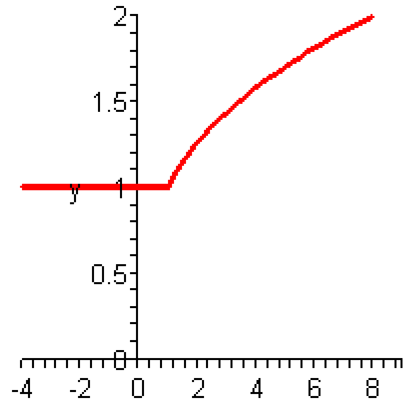
Domain	$(-\infty,\infty)$
Range	$[0,\infty)$
Increasing	$(0,\infty)$
Decreasing	nowhere
Constant	$(-\infty,0)$



70.

Domain	$(-\infty,1)\cup(1,\infty)$
Range	$[1,\infty)$
Increasing	$(1,\infty)$
Decreasing	nowhere
Constant	$(-\infty,1)$

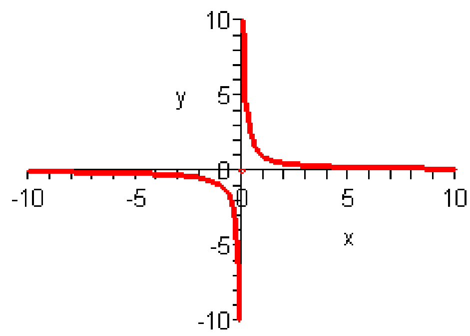
Notes on Graph: There should be an open hole at $(1,1)$.



71.

Domain	$(-\infty,\infty)$
Range	$(-\infty,\infty)$
Increasing	nowhere
Decreasing	$(-\infty,0)\cup(0,\infty)$
Constant	nowhere

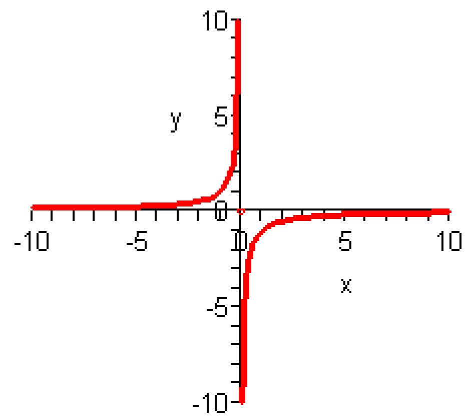
Notes on Graph: There should be a closed hole at (0,0).



72.

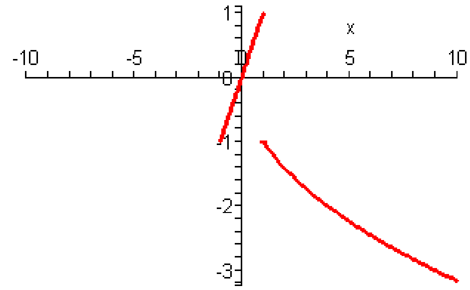
Domain	$(-\infty,\infty)$
Range	$(-\infty,\infty)$
Increasing	$(-\infty,0)\cup(0,\infty)$
Decreasing	nowhere
Constant	nowhere

Notes on Graph: There should be a closed hole at (0,0).



73.

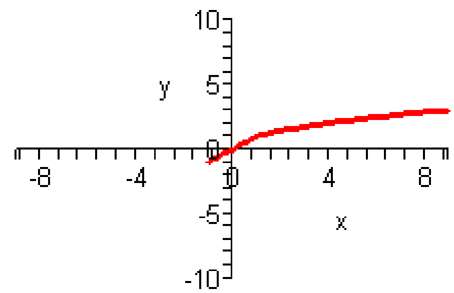
Domain	$(-\infty,1)\cup(1,\infty)$
Range	$(-\infty,-1)\cup(-1,\infty)$
Increasing	$(-1,1)$
Decreasing	$(-\infty,-1)\cup(1,\infty)$
Constant	nowhere



Notes on Graph: There should be open holes at $(-1,-1)$, $(1,1)$ and $(1,-1)$. Also, the graph of $-\sqrt[3]{x}$ should appear on the interval $(-\infty,-1)$ with a closed hole at (-1,1).

74.

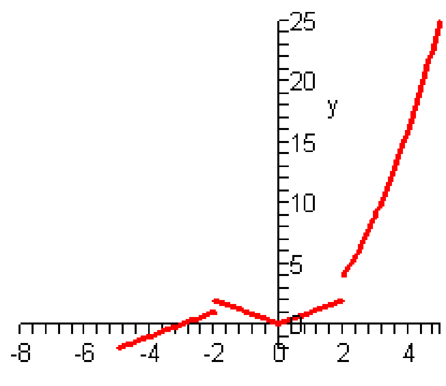
Domain	$(-\infty,1)\cup(1,\infty)$
Range	$(-\infty,1)\cup(1,\infty)$
Increasing	$(1,\infty)$
Decreasing	$(-\infty,-1)$
Constant	nowhere



Notes on Graph: The graph of $-\sqrt[3]{x}$ should appear on the interval $(-\infty,-1)$.

75.

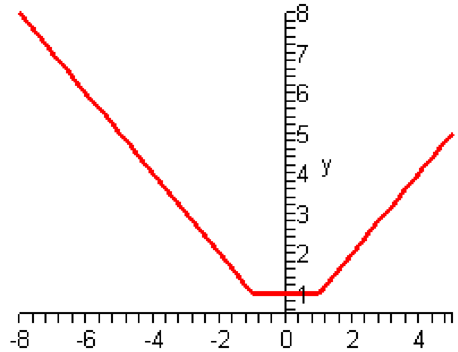
Domain	$(-\infty,\infty)$
Range	$(-\infty,2)\cup[4,\infty)$
Increasing	$(-\infty,-2)\cup(0,2)\cup(2,\infty)$
Decreasing	$(-2,0)$
Constant	nowhere



Notes on Graph: There should be open holes at $(-2,2)$, $(2,2)$ and closed holes at $(-2,1)$, $(2,4)$.

76.

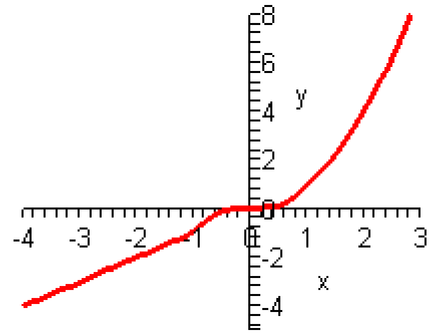
Domain	$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
Range	$[1,\infty)$
Increasing	$(1,\infty)$
Decreasing	$(-\infty,-1)$
Constant	$(-1,1)$



Notes on Graph: There should be open holes at $(-1,1)$, $(1,1)$.

77.

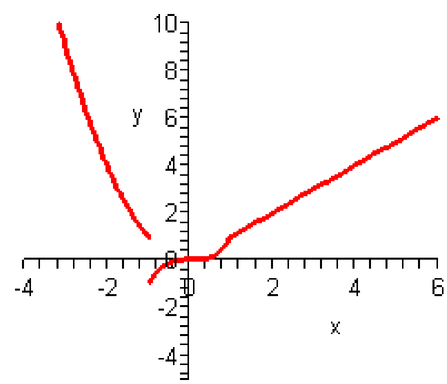
Domain	$(-\infty,1)\cup(1,\infty)$
Range	$(-\infty,1)\cup(1,\infty)$
Increasing	$(-\infty,1)\cup(1,\infty)$
Decreasing	nowhere
Constant	nowhere



Notes on Graph: There should be an open hole at $(1,1)$.

78.

Domain	$(-\infty,\infty)$
Range	$(-1,\infty)$
Increasing	$(-1,\infty)$
Decreasing	$(-\infty,-1)$
Constant	nowhere



Notes on Graph: There should be an open hole at $(-1,-1)$ and a closed hole at $(-1,1)$.

79. Profit is increasing from t = 10 to t = 12, which corresponds to Oct. to Dec. Profit is decreasing from t = 1 to t = 10, which corresponds to Jan to Oct. Profit never remains constant.

80. Cost is increasing from t = 1 to t = 8, which corresponds to Jan to Aug. Cost is decreasing from t = 8 to t = 12, which corresponds to Aug to Dec. Cost never remains constant.

81. Let x = number of T-shirts ordered. The cost function is given by

$$C(x)=\begin{cases}10x, & 0\le x\le 50\\ 9x, & 50<x<100\\ 8x, & x\ge 100\end{cases}$$

82. Let x = number of new uniforms ordered. The cost function is given by

$$C(x)=\begin{cases}176.12x, & 0\le x\le 50\\ 159.73x, & 50<x\le 100\end{cases}$$

83. Let x = number of boats entered. The cost function is given by

$$C(x)=\begin{cases}250x, & 0\le x\le 10\\ \underbrace{2500}_{\text{Cost for first 10 boats}}+175\cdot\underbrace{(x-10)}_{\text{\# of boats beyond first 10}}, & x>10\end{cases}=\begin{cases}250x, & 0\le x\le 10\\ 175x+750, & x>10\end{cases}$$

84. Let x = number of minutes. The cost function is given by

$$C(x)=\begin{cases}0.39x, & 0\le x\le 10\\ \underbrace{3.90}_{\text{Cost for first 10 minutes}}+0.12\cdot\underbrace{(x-10)}_{\text{\# of minutes beyond first 10}}, & x>10\end{cases}=\begin{cases}0.39x, & 0\le x\le 10\\ 0.12x+2.7, & x>10\end{cases}$$

85. Let x = number of books sold.
Since a single book sells for \$20, the amount of money earned for x books is $20x$.
Then, the amount of royalties due to the author (as a function of x) is given by:

$$R(x)=\begin{cases}\underbrace{50,000}_{\text{Amount upfront}}+\underbrace{0.15(20x)}_{\text{Amount from 15\% royalties}}, & 0\le x\le 100,000\\ 50,000+\underbrace{0.15(2,000,000)}_{\text{Royalties from first 100,000 books}}+\underbrace{0.20(20)(x-100,000)}_{\text{20\% royalties on books beyond initial 100,000}}, & 100,000<x\end{cases}$$

Simplifying the terms above yields

$$R(x)=\begin{cases}50,000+3x, & 0\le x\le 100,000\\ -50,000+4x, & x>100,000\end{cases}$$

86. Let x = number of books sold.
Since a single book sells for \$20, the amount of money earned for x books is $20x$.
Then, the amount of royalties due to the author (as a function of x) is given by:

$$R(x)=\begin{cases}\underbrace{35,000}_{\text{Amount upfront}}+\underbrace{0.15(20x)}_{\text{Amount from 15\% royalties}}, & 0\le x\le 100,000\\ 35,000+\underbrace{0.15(2,000,000)}_{\text{Royalties from first 100,000 books}}+\underbrace{0.25(20)(x-100,000)}_{\text{25\% royalties on books beyond initial 100,000}}, & x>100,000\end{cases}$$

Simplifying the terms above yields

$$R(x)=\begin{cases}35,000+3x, & 0\le x\le 100,000\\ -165,000+5x, & x>100,000\end{cases}$$

87. Observe that

$$f(x)=\begin{cases}0.98, & 0<x\le 1\\ 0.98+0.22, & 1<x\le 2\\ 0.98+0.22(2), & 2<x\le 3\\ \vdots & \end{cases}$$

Using the greatest integer function, we have $\boxed{f(x)=0.98+0.22[\![x]\!],\ 0\le x}$.

88. Observe that

$$f(x)=\begin{cases}1.13, & 0<x\le 1\\ 1.13+0.17, & 1<x\le 2\\ 1.13+0.17(2), & 2<x\le 3\\ \vdots & \end{cases}$$

Using the greatest integer function, we have $\boxed{f(x)=1.13+0.17[\![x]\!],\ 0\le x}$.

89. $f(t)=3(-1)^{[\![t]\!]},\ t\ge 0$

90. $f(x)=(-1)^{\left(1+\left[\!\left[\frac{x}{100}\right]\!\right]\right)},\ x\ge 0$

91. **a)** $\frac{1500-500}{1950-1900}=\boxed{20 \text{ per year}}$

b) $\frac{7000-1500}{2000-1950}=\boxed{110 \text{ per year}}$

92. **a)** $\frac{5000-1500}{1975-1950}=\boxed{140 \text{ per year}}$

b) $\frac{7000-5000}{2000-1975}=\boxed{80 \text{ per year}}$

93.

$$\frac{h(2)-h(1)}{2-1}=\frac{\left(-16(2)^2+48(2)\right)-\left(-16(1)^2+48(1)\right)}{2-1}$$

$$\boxed{=0 \text{ ft/sec}}$$

94.

$$\frac{h(3)-h(1)}{3-1}=\frac{\left(-16(3)^2+48(3)\right)-\left(-16(1)^2+48(1)\right)}{3-1}$$

$$\boxed{=-16 \text{ ft/sec}}$$

95. The first quarter starts at t = 1 and ends at t =90. So, the average rate of change in $d(t)$ during the first quarter is

$$\frac{d(90)-d(1)}{90-1}=$$

$$\frac{\left(3\sqrt{90^2+1}-2.75(90)\right)-\left(3\sqrt{1^2+1}-2.75(1)\right)}{89}\approx 0.236$$

So, demand is increasing at an approximate rate of 236 units over the first quarter.

96. The fourth quarter starts at t = 273 and ends at t = 365. So, the average rate of change in $d(t)$ during the fourth quarter is

$$\frac{d(365)-d(273)}{365-273}=$$

$$\frac{\left(3\sqrt{365^2+1}-2.75(365)\right)-\left(3\sqrt{273^2+1}-2.75(273)\right)}{92}$$

≈ 0.250

So, demand is increasing at an approximate rate of 250 units over the fourth quarter.

97. The portion of $C(x)$ for $x>30$ should be: $15+\underbrace{x-30}_{\text{Number minutes beyond first 30}}$

98. The portion of $C(x)$ for $x>10,000$ should be:

$0.02(10,000)+0.04(x-10,000)$

99. False. For instance, $f(x)=x^3$ is always increasing.

100. True.

101. The individual pieces used to form f, namely ax, bx^2, are continuous on $\mathbb{R}$. So, the only x-value with which we need to be concerned regarding the continuity of f is $x=2$. For f to be continuous at 2, we need $a(2)=b(2)^2$, which is the same as $\boxed{a=2b}$.

102. Both $\frac{1}{x}$ and $-\frac{1}{x}$ are undefined at $x=0$. So, for every value of a, either $a>0$ or $a\le 0$. Hence, we would need to evaluate either $\frac{1}{x}$ or $-\frac{1}{x}$ at 0, which is not possible. So, this function cannot be continuous, for any value of a.

103. Since f is already continuous on $(-\infty,-2]\cup[1,\infty)$ (being defined in terms of continuous functions), we need only to focus our attention on the interval $[-2,1]$. In order for f to be continuous at $x=-2$, we need $f(-2)=a(-2)+b$. This is equivalent to

$$\underbrace{-(-2)^2-10(-2)-13}_{=3}=-2a+b$$

In order for f to be continuous at x = 1, we need $f(1)=a(1)+b$. This is equivalent to

$$\underbrace{\sqrt{1-1}-9}_{=-9}=a+b\,.$$

As such, we must solve the system

$$\begin{cases}-2a+b=3 & \textbf{(1)}\\ \quad a+b=-9 & \textbf{(2)}\end{cases}$$

Subtract **(1) – (2)** to eliminate b: $-3a=12$, so that $\boxed{a=-4}$.

Substitute this into **(2)** to find b: $\boxed{b=-5}$.

104. The first two expressions in the definition of f must agree at $x=-2$, and the last two expressions must agree at $x=2$. This yields the system:

$$\begin{cases}-2(-2)-a+2b=\sqrt{-2+a}\\ \sqrt{2+a}=2^2-4(2)+a+4\end{cases}\quad \text{which is equivalent to} \quad \begin{cases}4-a+2b=\sqrt{a-2} & \textbf{(1)}\\ \sqrt{a+2}=a & \textbf{(2)}\end{cases}$$

Solve **(2)** for a:

$$a+2=a^2$$

$$a^2-a-2=0$$

$$(a-2)(a+1)=0$$

$$\boxed{a=2},\ \cancel{-1}$$

Substitute this into **(1)** to find b: $4-2+2b=0$, so that $\boxed{b=-1}$.

105. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{k-k}{h}=0$. So, at $h=0$ we get $\boxed{f'(x)=0}$.

106. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{[m(x+h)+b]-[mx+b]}{h}=\dfrac{mx+mh+b-mx-b}{h}=\dfrac{mh}{h}=m$

So, at $h=0$ we get $\boxed{f'(x)=m}$.

107.

$$\frac{f(x+h)-f(x)}{h} = \frac{\left[a\left(x+h\right)^2 + b\left(x+h\right) + c\right] - \left[ax^2 + bx + c\right]}{h}$$

$$= \frac{ax^2 + 2axh + ah^2 + bx + bh + c - ax^2 - bx - c}{h}$$

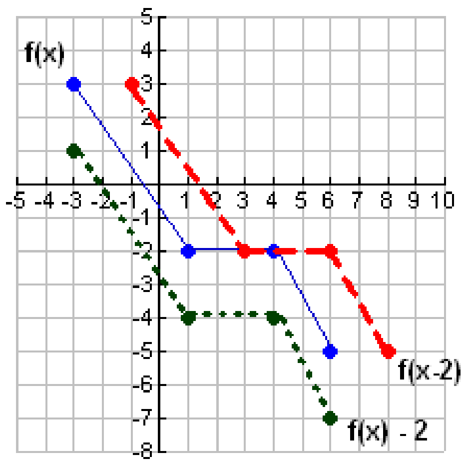
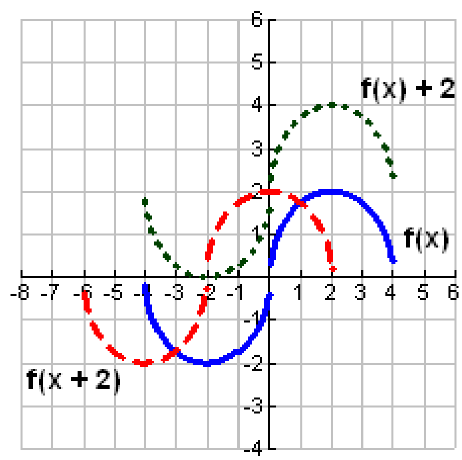
$$= \frac{h\left(2ax + ah + b\right)}{h} = 2ax + ah + b$$

So, at $h = 0$ we get $\boxed{f'(x) = 2ax + b}$.

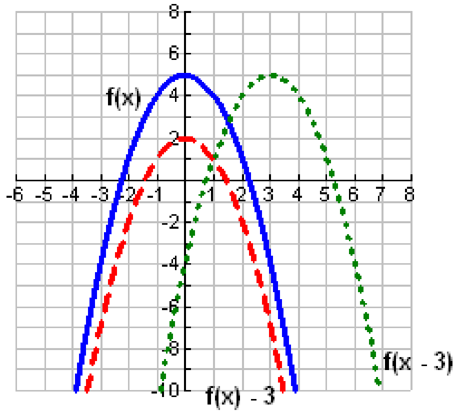
108. Apply the results from problems #117 – 119 on each individual expression (on the OPEN intervals) to obtain

$$f'(x) = \begin{cases} 0, & x < 0, \\ -3, & 0 < x < 4. \\ 2x+4, & x > 4 \end{cases}$$

Section 1.3 Solutions --

1. $y = \lvert x \rvert + 3$	**2.** $y = \lvert x + 4 \rvert$
3. $y = \lvert -x \rvert = \lvert x \rvert$ (since $\lvert -x \rvert = \lvert -1 \rvert \lvert x \rvert = \lvert x \rvert$)	**4.** $y = -\lvert x \rvert$
5. $y = 3\lvert x \rvert$	**6.** $y = \frac{1}{3}\lvert x \rvert$
7. $y = x^3 - 4$	**8.** $y = (x-3)^3$
9. $y = (x+1)^3 + 3$	**10.** $y = -x^3$
11. $y = -x^3$	**12.** $y = x^3$
13. 	**14.** 

15.



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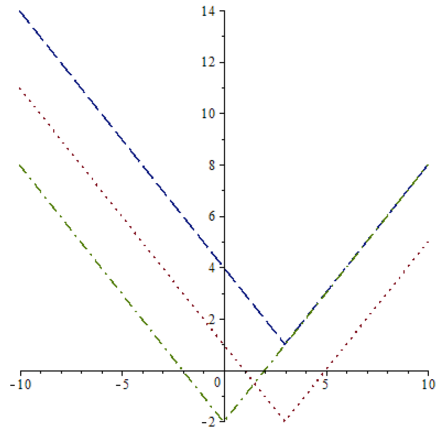
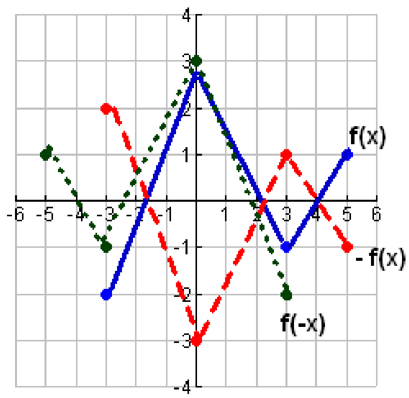
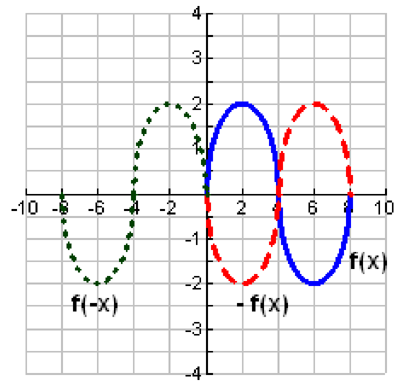


17.



18.



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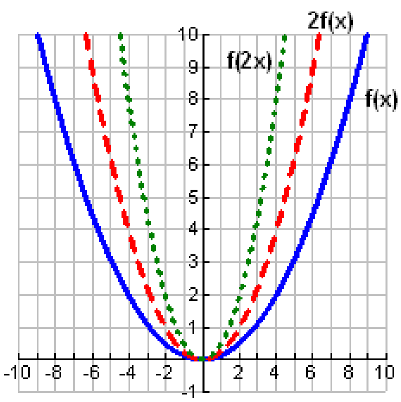

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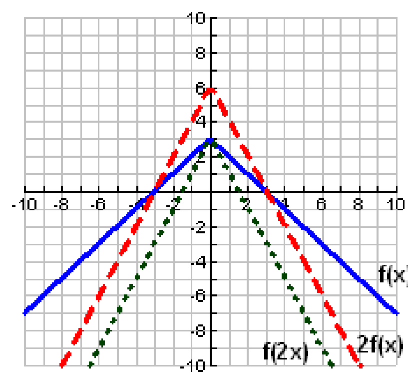

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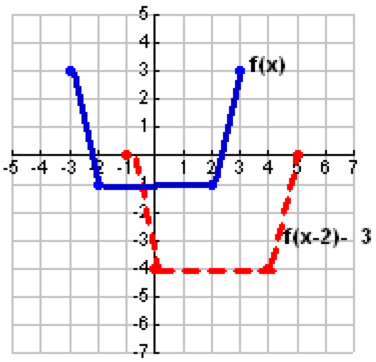

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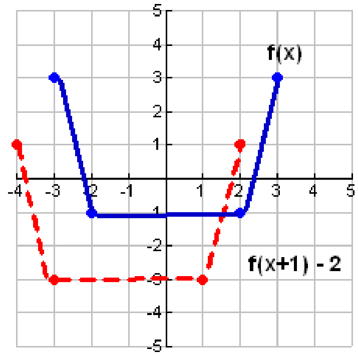

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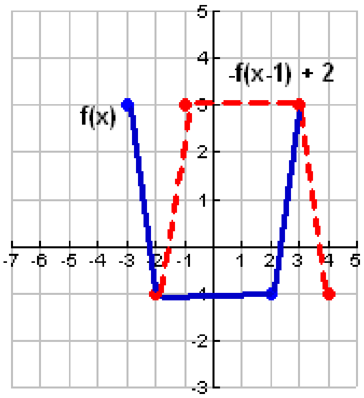

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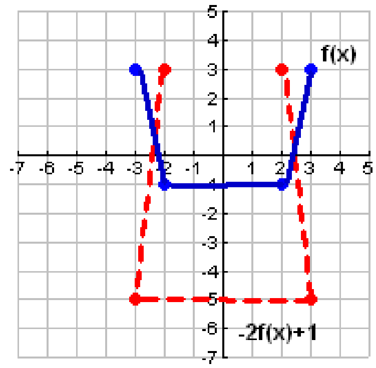

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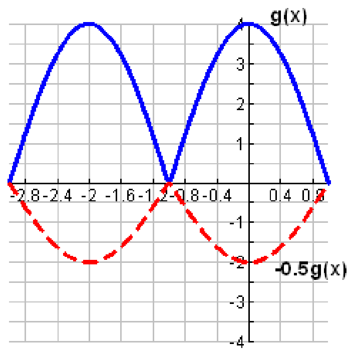

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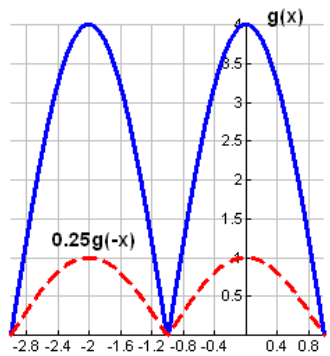
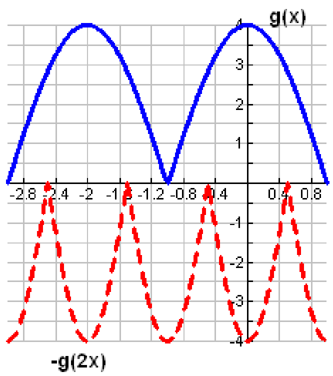
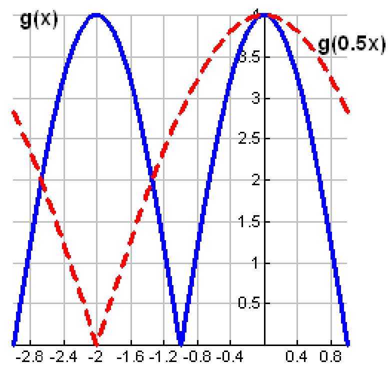
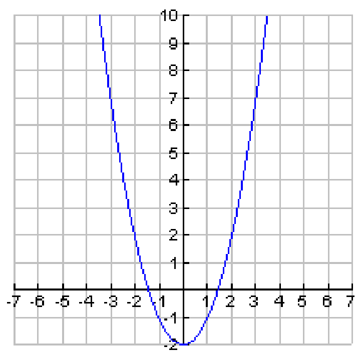
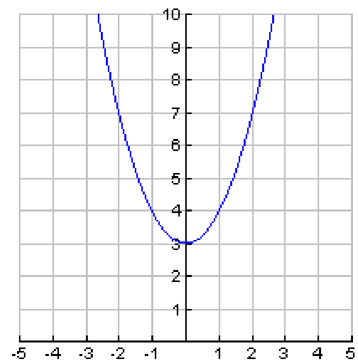
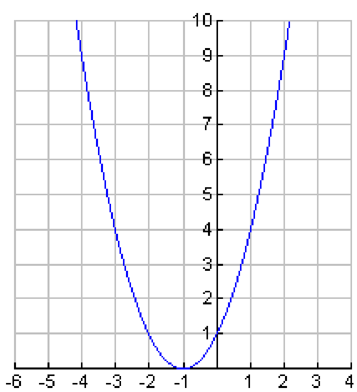
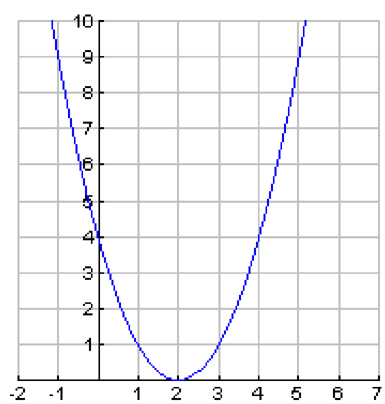
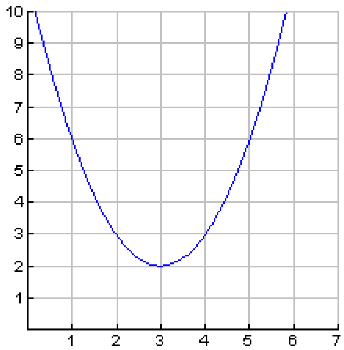
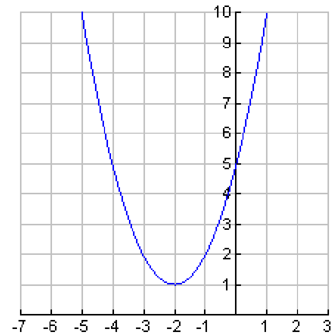


27. 	**28.** 
29. Shift the graph of x^2 down 2 units. 	**30.** Shift the graph of x^2 up 3 units. 
31. Shift the graph of x^2 left 1 unit. 	**32.** Shift the graph of x^2 right 2 units. 

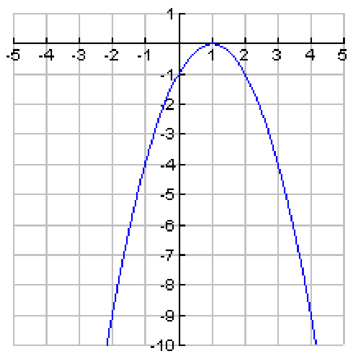
33. Shift the graph of x^2 right 3 units, and up 2 units.



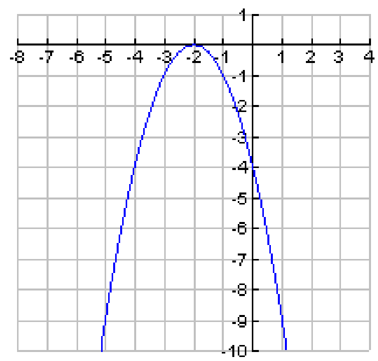
34. Shift the graph of x^2 left 2 units, and up 1 unit.



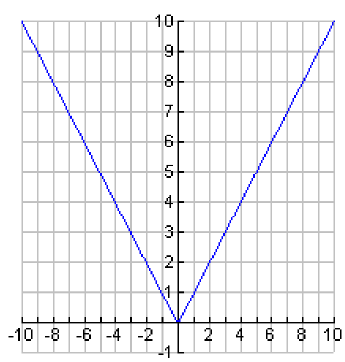
35. Shift the graph of x^2 right 1 unit, and then reflect over *x*-axis.



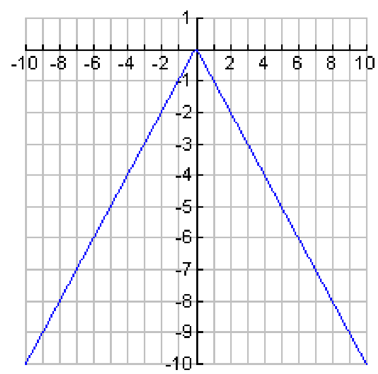
36. Shift the graph of x^2 left 2 units, and then reflect over *x*-axis.



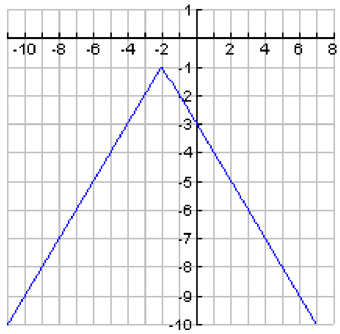
37. Reflect the graph of $|x|$ over *y*-axis. (This yields the same graph as $|x|$ since $|-x| = |-1||x| = |x|$.)



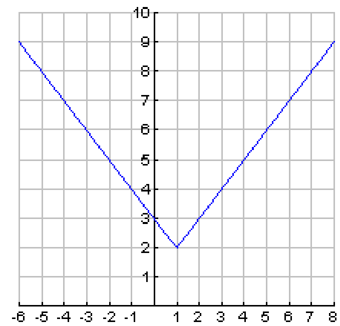
38. Reflect the graph of $|x|$ over *x*-axis.



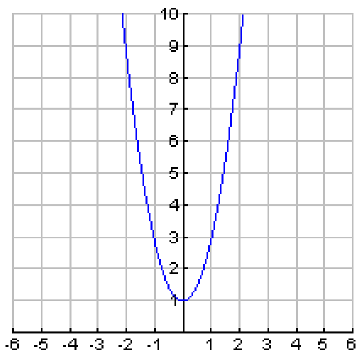
39. Reflect the graph of $|x|$ over x-axis, then shift left 2 units and down 1 unit.



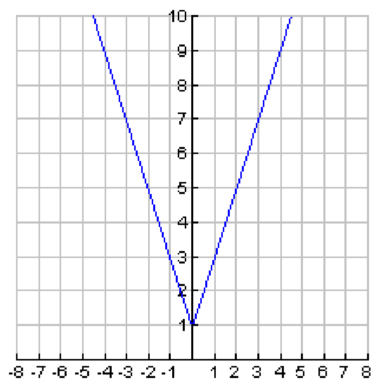
40. Since $|1-x|+2=|x-1|+2$, shift the graph of $|x|$ right 1 unit, and up 2 units.



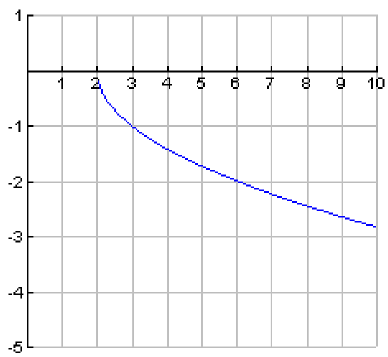
41. Vertically stretch the graph of x^2 by a factor of 2, then shift up 1unit.



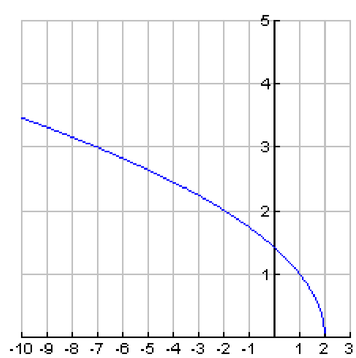
42. Vertically stretch the graph of $|x|$ by a factor of 2, then shift up 1unit.



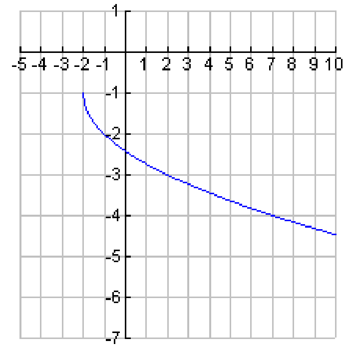
43. Shift the graph of $\sqrt{x}$ right 2 units, then reflect over x-axis.



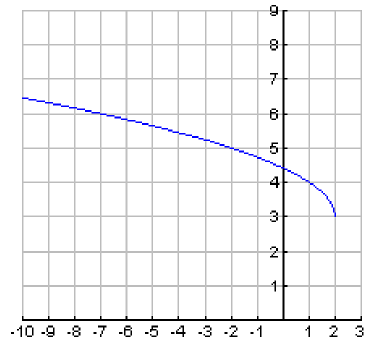
44. Since $\sqrt{2-x}=\sqrt{-(x-2)}$, reflect the graph of $\sqrt{x}$ over y-axis, then shift right 2 units.



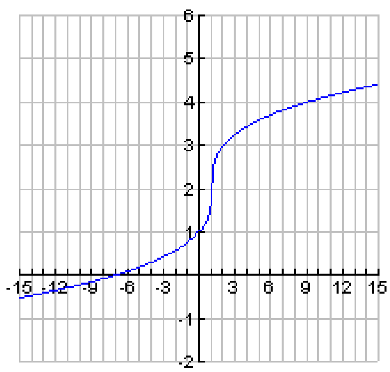
45. Reflect the graph of $\sqrt{x}$ over x-axis, then shift left 2 units and down 1 unit.



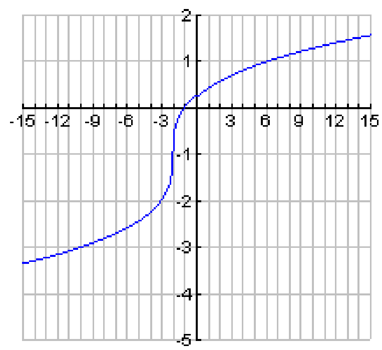
46. Since $\sqrt{2-x}+3=\sqrt{-(x-2)}+3$, reflect the graph of $\sqrt{x}$ over y-axis, then right 2 units and up 3 units.



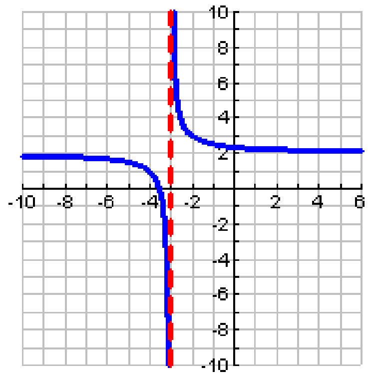
47. Shift the graph of $\sqrt[3]{x}$ right 1 unit, then up 2 units.



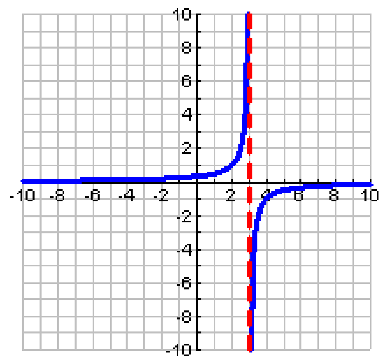
48. Shift the graph of $\sqrt[3]{x}$ left 2 units, then down 1 unit.



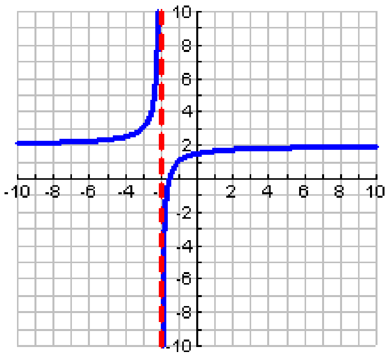
49. Shift the graph of $\frac{1}{x}$ left 3 units, then up 2 units.



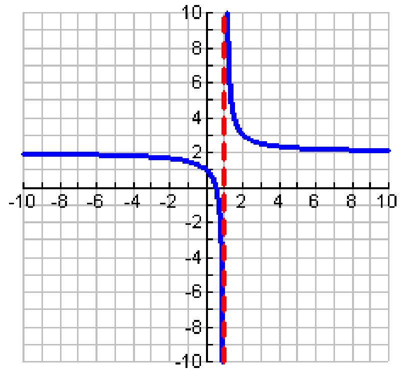
50. Since $\frac{1}{3-x}=-\frac{1}{x-3}$, shift the graph of $\frac{1}{x}$ right 3 units, and then reflect over x-axis.



51. Shift the graph of $\frac{1}{x}$ left 2 units, then reflect over x-axis, and then shift up 2 units.



52. Since $2-\frac{1}{-(x-1)}=2+\frac{1}{x-1}$, shift the graph of $\frac{1}{x}$ right 1 unit, then up 2 units.

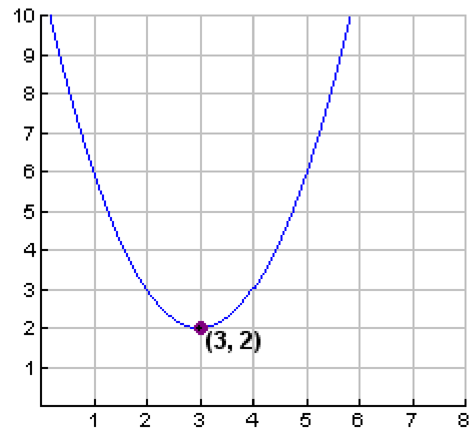


53.

Completing the square yields

$$\begin{aligned} f(x) &= x^2-6x+11 \\ &= \left(x^2-6x+9\right)+11-9 \\ &= (x-3)^2+2 \end{aligned}$$

So, shift the graph of x^2 right 3 units, then up 2 units.

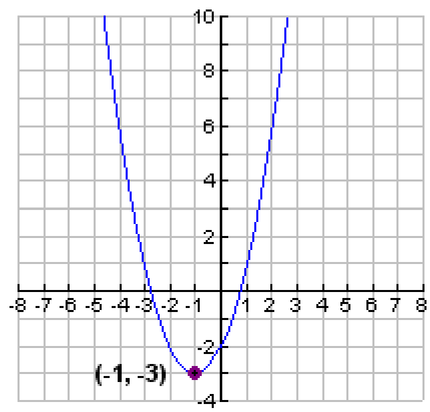


54.

Completing the square yields

$$\begin{aligned} f(x) &= x^2+2x-2 \\ &= \left(x^2+2x+1\right)-2-1 \\ &= (x+1)^2-3 \end{aligned}$$

So, shift the graph of x^2 left 1 unit, then down 3 units.

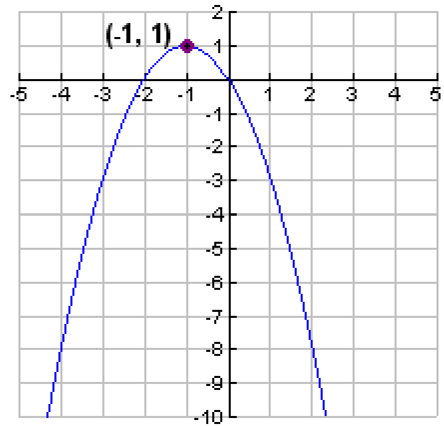


55.

Completing the square yields

$$f(x) = -\left(x^2 + 2x\right)$$
$$= -\left(x^2 + 2x + 1\right) + 1$$
$$= -\left(x + 1\right)^2 + 1$$

So, reflect the graph of x^2 over x-axis, then shift left 1 unit, then up 1 unit.

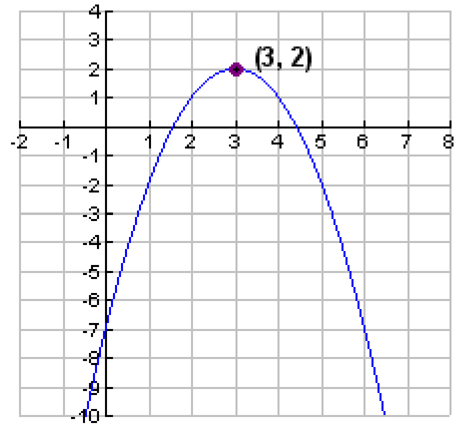


56.

Completing the square yields

$$f(x) = -x^2 + 6x - 7$$
$$= -\left(x^2 - 6x\right) - 7$$
$$= -\left(x^2 - 6x + 9\right) - 7 + 9$$
$$= -\left(x - 3\right)^2 + 2$$

So, reflect the graph of x^2 over x-axis, then shift right 3 units, then up 2 units.

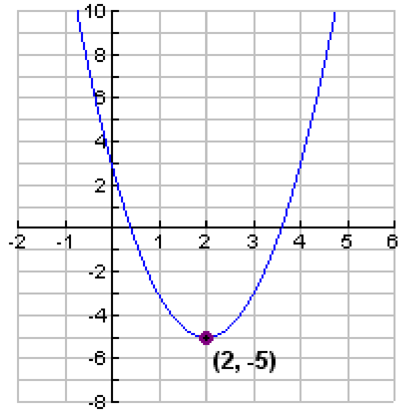


57.

Completing the square yields

$$f(x) = 2x^2 - 8x + 3$$
$$= 2\left(x^2 - 4x\right) + 3$$
$$= 2\left(x^2 - 4x + 4\right) + 3 - 8$$
$$= 2\left(x - 2\right)^2 - 5$$

So, vertically stretch the graph of x^2 by a factor of 2, then shift right 2 units, then down 5 units.

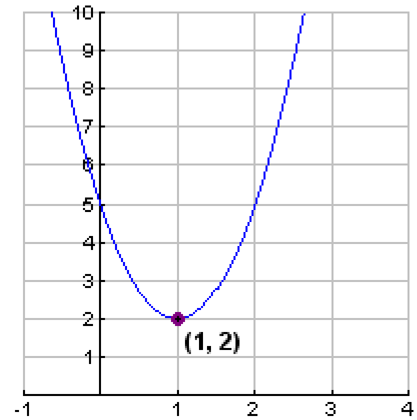


58.

Completing the square yields

$$\begin{aligned} f(x) &= 3x^2 - 6x + 5 \\ &= 3\left(x^2 - 2x\right) + 5 \\ &= 3\left(x^2 - 2x + 1\right) + 5 - 3 \\ &= 3\left(x-1\right)^2 + 2 \end{aligned}$$

So, vertically stretch the graph of x^2 by a factor of 3, then shift right 1 unit, then up 2 units.



59. Let x = number of hours worked per week. Then, the salary is given by $\boxed{S(x) = 10x}$ (in dollars). After 1 year, taking into account the raise, the new salary is $\boxed{S(x) = 10x + 50}$.

60. The profit in a rainy year is given by $P(x) - 10(\text{Cost of } 1)$, where x is the number of pallets sold.
Since they are giving away 10 pallets in a rainy year, they don't make a profit on the first 10. So, the profit would be $P(x-10)$.

61. The 2006 taxes would be:

$$T(x) = 0.33(x - 6500)$$

62. The actual amount administered if the weight is overestimated by 3 ounces is $A(x+3) = \sqrt{x+3} + 2$.

63. This function would be $Q(t) = P(t+50)$.

64. This function would be $Q(t) = P(t+60)$.

65. a. Use h = 162 to get

$$BSA(w) = \sqrt{\frac{162w}{3,600}} = \sqrt{\frac{9w}{200}}.$$

b. If she loses 3 kg, the new function is

$$BSA(w-3) = \sqrt{\frac{9(w-3)}{200}}.$$

66. a. Use h = 180 to get

$$BSA(w) = \sqrt{\frac{180w}{3,600}} = \sqrt{\frac{w}{20}}.$$

b. If he gains 5 kg, the new function is

$$BSA(w+5) = \sqrt{\frac{w+5}{20}}.$$

67. (b) is wrong – shift right 3 units.

68. (b) is wrong and (d) is misplaced. The correct sequence of steps would be:

$$(a) \to (d) \to (*) \to (c),$$

where $(*)$ = reflect over x-axis.

69. True. Since $|-x| = |-1||x| = |x|$.

70. False. $y = \sqrt{-x}$ is the reflection of $y = \sqrt{x}$ over the y-axis.

71. True.

72. True.

73. True

74. False. The shift is a units to the left.

75. The graph of $y = f(x-3) + 2$ is the graph of $y = f(x)$ shifted right 3 units, then up 2 units. So, if the point (a, b) is on the graph of $y = f(x)$, then the point $(a+3, b+2)$ is on the graph of the translation $y = f(x-3) + 2$.

76. The graph of $y=-f(-x)+1$ is the graph of $y=f(x)$ reflected over y-axis, then over x-axis, and then shifted up 1 unit. So, if the point (a,b) is on the graph of $y=f(x)$, then the point $(-a,-b+1)$ is on the graph of the translation $y=-f(-x)+1$.

77. We do this in three steps:

$f(x)$		(a,b)
$f(x+1)$	Shift left 1 unit	$(a-1,b)$
$2f(x+1)$	Multiply all outputs by 2	$(a-1,2b)$
$2f(x+1)-1$	Shift vertically down 1 unit	$(a-1,2b-1)$

So, the point $(a-1,2b-1)$ is guaranteed to lie on the graph.

78. We do this in three steps:

$f(x)$		(a,b)
$f(x-3)$	Shift right 3 units	$(a+3,b)$
$-2f(x-3)$	Multiply all outputs by -2	$(a+3,-2b)$
$-2f(x-3)+4$	Shift vertically up 4 units	$(a+3,-2b+4)$

So, the point $(a+3,-2b+4)$ is guaranteed to lie on the graph.

79.

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-x^2}{h}=\frac{x^2+2hx+h^2-x^2}{h}=\frac{h(2x+h)}{h}=2x+h$$

So, at $h=0$ we get $\boxed{f'(x)=2x}$.

$$\frac{g(x+h)-g(x)}{h}=\frac{(x+h-1)^2-(x-1)^2}{h}=\frac{x^2+hx-x+hx+h^2-h-x-h+1-x^2+2x-1}{h}$$

$$=\frac{h(2x+h-2)}{h}=2x+h-2=2(x-1)+h$$

So, at $h=0$ we get $\boxed{g'(x)=2(x-1)}$.

We observe that the graph of g' is obtained by shifting the graph of f' right 1 unit.

80.

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

So, at $h=0$ we get $\boxed{f'(x)=\frac{1}{2\sqrt{x}}}$.

$$\frac{g(x+h)-g(x)}{h}=\frac{\sqrt{x+h+5}-\sqrt{x+5}}{h}=\frac{\sqrt{x+h+5}-\sqrt{x+5}}{h}\cdot\frac{\sqrt{x+h+5}+\sqrt{x+5}}{\sqrt{x+h+5}+\sqrt{x+5}}$$

$$=\frac{x+h+5-x-5}{h\left(\sqrt{x+h+5}+\sqrt{x+5}\right)}=\frac{1}{\sqrt{x+h+5}+\sqrt{x+5}}$$

So, at $h=0$ we get $\boxed{g'(x)=\frac{1}{2\sqrt{x+5}}}$.

We observe that the graph of g' is obtained by shifting the graph of f' left 5 units.

81. $\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)-2x}{h}=\frac{2h}{h}=2$. So, at $h=0$ we get $\boxed{f'(x)=2}$.

$\frac{g(x+h)-g(x)}{h}=\frac{\left[2(x+h)+7\right]-\left[2x+7\right]}{h}=\frac{2h}{h}=2$. So, at $h=0$ we get $\boxed{g'(x)=2}$.

We observe that the graphs of f' and g' are the same.

82.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[(x+h)^3\right]-\left[x^3\right]}{h}=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2\right)}{h}=\boxed{3x^2+3xh+h^2}$$

So, at $h=0$ we get $\boxed{f'(x)=3x^2}$.

$$\frac{g(x+h)-g(x)}{h}=\frac{\left[(x+h)^3-4\right]-\left[x^3-4\right]}{h}=\frac{x^3+3x^2h+3xh^2+h^3-4-x^3+4}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2\right)}{h}=\boxed{3x^2+3xh+h^2}$$

So, at $h=0$ we get $\boxed{g'(x)=3x^2}$.

We observe that the graphs of f' and g' are the same.

Section 1.4 Solutions ---

1.

$$f(x)+g(x)=(2x+1)+(1-x)$$
$$=x+2$$
$$f(x)-g(x)=(2x+1)-(1-x)$$
$$=2x+1-1+x$$
$$=3x$$
$$f(x)\cdot g(x)=(2x+1)(1-x)$$
$$=2x+1-2x^2-x$$
$$=-2x^2+x+1$$
$$\frac{f(x)}{g(x)}=\frac{2x+1}{1-x}$$

Domains:

$$\left.\begin{matrix} dom(f+g) \\ dom(f-g) \\ dom(fg) \end{matrix}\right\}=(-\infty,\infty)$$
$$dom\left(\frac{f}{g}\right)=(-\infty,1)\cup(1,\infty)$$

2.

$$f(x)+g(x)=(3x+2)+(2x-4)$$
$$=5x-2$$
$$f(x)-g(x)=(3x+2)-(2x-4)$$
$$=3x+2-2x+4$$
$$=x+6$$
$$f(x)\cdot g(x)=(3x+2)\cdot(2x-4)$$
$$=6x^2-12x+4x-8$$
$$=6x^2-8x-8$$
$$\frac{f(x)}{g(x)}=\frac{3x+2}{2x-4}$$

Domains:

$$\left.\begin{matrix} dom(f+g) \\ dom(f-g) \\ dom(fg) \end{matrix}\right\}=(-\infty,\infty)$$
$$dom\left(\frac{f}{g}\right)=(-\infty,2)\cup(2,\infty)$$

3.

$$f(x)+g(x)=(2x^2-x)+(x^2-4)$$
$$=3x^2-x-4$$
$$f(x)-g(x)=(2x^2-x)-(x^2-4)$$
$$=2x^2-x-x^2+4$$
$$=x^2-x+4$$
$$f(x)\cdot g(x)=(2x^2-x)\cdot(x^2-4)$$
$$=2x^4-x^3-8x^2+4x$$
$$\frac{f(x)}{g(x)}=\frac{2x^2-x}{x^2-4}$$

Domains:

$$dom(f+g),\ dom(f-g),\ dom(fg)\}=(-\infty,\infty)$$
$$dom\left(\frac{f}{g}\right)=(-\infty,-2)\cup(-2,2)\cup(2,\infty)$$

4.

$$f(x)+g(x)=(3x+2)+(x^2-25)$$
$$=x^2+3x-23$$
$$f(x)-g(x)=(3x+2)-(x^2-25)$$
$$=3x+2-x^2+25$$
$$=-x^2+3x+27$$
$$f(x)\cdot g(x)=(3x+2)\cdot(x^2-25)$$
$$=3x^3+2x^2-75x-50$$
$$\frac{f(x)}{g(x)}=\frac{3x+2}{x^2-25}$$

Domains:

$$dom(f+g),\ dom(f-g),\ dom(fg)\}=(-\infty,\infty)$$
$$dom\left(\frac{f}{g}\right)=(-\infty,-5)\cup(-5,5)\cup(5,\infty)$$

5.

$$f(x)+g(x)=\tfrac{1}{x}+x=\frac{1+x^2}{x}$$
$$f(x)-g(x)=\tfrac{1}{x}-x=\frac{1-x^2}{x}$$
$$f(x)\cdot g(x)=\tfrac{1}{x}\cdot x=1$$
$$\frac{f(x)}{g(x)}=\frac{\tfrac{1}{x}}{x}=\frac{1}{x^2}$$

Domains:

$$\left.\begin{array}{r} dom(f+g)\\ dom(f-g)\\ dom(fg)\\ dom\left(\dfrac{f}{g}\right)\end{array}\right\}=(-\infty,0)\cup(0,\infty)$$

6.

$$\begin{aligned} f(x)+g(x)&=\frac{2x+3}{x-4}+\frac{x-4}{3x+2}\\ &=\frac{(2x+3)(3x+2)+(x-4)^2}{(x-4)(3x+2)}\\ &=\frac{6x^2+9x+4x+6+x^2-8x+16}{(x-4)(3x+2)}\\ &=\frac{7x^2+5x+22}{(x-4)(3x+2)}\end{aligned}$$

$$\begin{aligned} f(x)-g(x)&=\frac{2x+3}{x-4}-\frac{x-4}{3x+2}\\ &=\frac{(2x+3)(3x+2)-(x-4)^2}{(x-4)(3x+2)}\\ &=\frac{6x^2+9x+4x+6-x^2+8x-16}{(x-4)(3x+2)}\\ &=\frac{5x^2+21x-10}{(x-4)(3x+2)}\end{aligned}$$

$$f(x)\cdot g(x)=\frac{2x+3}{\cancel{x-4}}\cdot\frac{\cancel{x-4}}{3x+2}=\frac{2x+3}{3x+2}$$

$$\frac{f(x)}{g(x)}=\frac{\frac{2x+3}{x-4}}{\frac{x-4}{3x+2}}=\frac{2x+3}{x-4}\cdot\frac{3x+2}{x-4}=\frac{(2x+3)(3x+2)}{(x-4)^2}$$

Domains:

$$\left.\begin{array}{r} dom(f+g)\\ dom(f-g)\\ dom(fg)\\ dom\left(\dfrac{f}{g}\right)\end{array}\right\}=\left(-\infty,-\tfrac{2}{3}\right)\cup\left(-\tfrac{2}{3},4\right)\cup(4,\infty)$$

7.

$$f(x)+g(x)=\sqrt{x}+2\sqrt{x}=3\sqrt{x}$$
$$f(x)-g(x)=\sqrt{x}-2\sqrt{x}=-\sqrt{x}$$
$$f(x)\cdot g(x)=\sqrt{x}\cdot 2\sqrt{x}=2x$$
$$\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{2\sqrt{x}}=\frac{1}{2}$$

Domains:

$$\left.\begin{array}{r} dom(f+g)\\ dom(f-g)\\ dom(fg)\end{array}\right\}=[0,\infty)$$

$$dom\left(\frac{f}{g}\right)=(0,\infty)$$

8.

$$f(x)+g(x)=\sqrt{x-1}+2x^2$$
$$f(x)-g(x)=\sqrt{x-1}-2x^2$$
$$f(x)\cdot g(x)=2x^2\sqrt{x-1}$$
$$\frac{f(x)}{g(x)}=\frac{\sqrt{x-1}}{2x^2}$$

Domains:

Must have both $x-1\geq 0$ and $2x^2\neq 0$. So,

$$\left.\begin{array}{r} dom(f+g)\\ dom(f-g)\\ dom(fg)\\ dom\left(\dfrac{f}{g}\right)\end{array}\right\}=[1,\infty)$$

9.

$$f(x)+g(x)=\sqrt{4-x}+\sqrt{x+3}$$
$$f(x)-g(x)=\sqrt{4-x}-\sqrt{x+3}$$
$$f(x)\cdot g(x)=\sqrt{4-x}\cdot\sqrt{x+3}$$
$$\frac{f(x)}{g(x)}=\frac{\sqrt{4-x}}{\sqrt{x+3}}=\frac{\sqrt{4-x}\sqrt{x+3}}{x+3}$$

Domains:
Must have both $4-x\geq 0$ and $x+3\geq 0$. So,

$$\left.\begin{array}{r}dom(f+g)\\ dom(f-g)\\ dom(fg)\end{array}\right\}=[-3,4].$$

For the quotient, must have both $4-x\geq 0$ and $x+3>0$. So, $dom\left(\frac{f}{g}\right)=(-3,4]$.

10.

$$f(x)+g(x)=\sqrt{1-2x}+\tfrac{1}{x}$$
$$f(x)-g(x)=\sqrt{1-2x}-\tfrac{1}{x}$$
$$f(x)\cdot g(x)=\sqrt{1-2x}\cdot\tfrac{1}{x}$$
$$\frac{f(x)}{g(x)}=\frac{\sqrt{1-2x}}{\frac{1}{x}}=x\sqrt{1-2x}$$

Domains:
Must have both $1-2x\geq 0$ and $x\neq 0$. So,

$$\left.\begin{array}{r}dom(f+g)\\ dom(f-g)\\ dom(fg)\\ dom\left(\frac{f}{g}\right)\end{array}\right\}=(-\infty,0)\cup\left(0,\tfrac{1}{2}\right]$$

11.

$$(f\circ g)(x)=2\left(x^2-3\right)+1=2x^2-6+1=2x^2-5$$
$$(g\circ f)(x)=(2x+1)^2-3=4x^2+4x+1-3=4x^2+4x-2$$

Domains:
$dom(f\circ g)=(-\infty,\infty)=dom(g\circ f)$

12.

$$(f\circ g)(x)=(2-x)^2-1=4-4x+x^2-1=x^2-4x+3$$
$$(g\circ f)(x)=2-\left(x^2-1\right)=2-x^2+1=-x^2+3$$

Domains:
$dom(f\circ g)=(-\infty,\infty)=dom(g\circ f)$

13.

$$(f\circ g)(x)=\frac{1}{(x+2)-1}=\frac{1}{x+1}$$
$$(g\circ f)(x)=\frac{1}{x-1}+2=\frac{1+2(x-1)}{x-1}=\frac{1+2x-2}{x-1}=\frac{2x-1}{x-1}$$

Domains:
$dom(f\circ g)=(-\infty,-1)\cup(-1,\infty)$, $\quad dom(g\circ f)=(-\infty,1)\cup(1,\infty)$

14.

$$(f\circ g)(x)=\frac{2}{(2+x)-3}=\frac{2}{x-1}$$
$$(g\circ f)(x)=2+\frac{2}{x-3}=\frac{2(x-3)+2}{x-3}=\frac{2x-6+2}{x-3}=\frac{2x-4}{x-3}$$

Domains:

$dom(f\circ g)=(-\infty,1)\cup(1,3)\cup(3,\infty)$, $dom(g\circ f)=(-\infty,3)\cup(3,\infty)$

15.

$$(f\circ g)(x)=\left|\frac{1}{x-1}\right|=\frac{1}{|x-1|}$$
$$(g\circ f)(x)=\frac{1}{|x|-1}$$

Domains:

$dom(f\circ g)=(-\infty,1)\cup(1,\infty)$

$dom(g\circ f)=(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

16.

$$(f\circ g)(x)=\left|\frac{1}{x}-1\right|=\left|\frac{1-x}{x}\right|$$
$$(g\circ f)(x)=\frac{1}{|x-1|}$$

Domains:

$dom(f\circ g)=(-\infty,0)\cup(0,\infty)$

$dom(g\circ f)=(-\infty,1)\cup(1,\infty)$

17.

$$(f\circ g)(x)=\sqrt{(x+5)-1}=\sqrt{x+4}$$
$$(g\circ f)(x)=\sqrt{x-1}+5$$

Domains:

$dom(f\circ g)$: Must have $x+4\ge 0$. So, $dom(f\circ g)=[-4,\infty)$.

$dom(g\circ f)$: Must have $x-1\ge 0$. So, $dom(g\circ f)=[1,\infty)$.

18.

$$(f\circ g)(x)=\sqrt{2-\left(x^2+2\right)}=\sqrt{2-x^2-2}=\sqrt{-x^2}$$
$$(g\circ f)(x)=\left(\sqrt{2-x}\right)^2+2=2-x+2=4-x$$

Domains:

$dom(f\circ g)=[0]$ since $-x^2\ge 0$ only when $x=0$. $dom(g\circ f)=(-\infty,2]$

19.

$$(f\circ g)(x)=\left[(x-4)^{1/3}\right]^3+4=x-4+4=x$$
$$(g\circ f)(x)=\left[\left(x^3+4\right)-4\right]^{1/3}=\left[x^3\right]^{1/3}=x$$

Domains:

$dom(f\circ g)=(-\infty,\infty)=dom(g\circ f)$

20.

$$(f\circ g)(x)=\sqrt[3]{\left(x^{2/3}+1\right)^2-1}=\sqrt[3]{x^{4/3}+2x^{2/3}+1-1}=\sqrt[3]{x^{4/3}+2x^{2/3}}=\sqrt[3]{x^{2/3}\left(x^{2/3}+2\right)}$$

$$(g\circ f)(x)=\left(\sqrt[3]{x^2-1}\right)^{2/3}+1=\left(x^2-1\right)^2+1=x^4-2x^2+1+1=x^4-2x^2+2$$

<u>Domains</u>:

$dom(f\circ g)=\left(-\infty,\infty\right)=dom(g\circ f)$

21.

$$\begin{aligned}(f+g)(2)&=f(2)+g(2)\\&=\left[2^2+10\right]+\sqrt{2-1}\\&=14+1=\boxed{15}\end{aligned}$$

22.

$$\begin{aligned}(f+g)(10)&=f(10)+g(10)\\&=\left[10^2+10\right]+\sqrt{10-1}\\&=110+3=\boxed{113}\end{aligned}$$

23.

$$\begin{aligned}(f-g)(2)&=f(2)-g(2)\\&=\left[2^2+10\right]-\sqrt{2-1}\\&=14-1=\boxed{13}\end{aligned}$$

24.

$$\begin{aligned}(f-g)(5)&=f(5)-g(5)\\&=\left[5^2+10\right]-\sqrt{5-1}\\&=35-2=\boxed{33}\end{aligned}$$

25.

$$\begin{aligned}(f\cdot g)(4)&=f(4)\cdot g(4)\\&=\left[4^2+10\right]\cdot\sqrt{4-1}\\&=\boxed{26\sqrt{3}}\end{aligned}$$

26.

$$\begin{aligned}(f\cdot g)(5)&=f(5)\cdot g(5)\\&=\left[5^2+10\right]\cdot\sqrt{5-1}\\&=35(2)=\boxed{70}\end{aligned}$$

27.

$$\left(\frac{f}{g}\right)(10)=\frac{f(10)}{g(10)}=\frac{10^2+10}{\sqrt{10-1}}=\boxed{\tfrac{110}{3}}$$

28.

$$\left(\frac{f}{g}\right)(2)=\frac{f(2)}{g(2)}=\frac{2^2+10}{\sqrt{2-1}}=\boxed{14}$$

29.

$$f(g(2))=f\left(\underbrace{\sqrt{2-1}}_{=1}\right)=1^2+10=\boxed{11}$$

30.

$$f(g(1))=f\left(\underbrace{\sqrt{1-1}}_{=0}\right)=0^2+10=\boxed{10}$$

31.

$$g(f(-3))=g\left(\underbrace{(-3)^2+10}_{=19}\right)=\sqrt{19-1}=\boxed{3\sqrt{2}}$$

32.

$$g(f(4))=g\left(\underbrace{4^2+10}_{=26}\right)=\sqrt{26-1}=\boxed{5}$$

33. 0 is not in the domain of g, so that $g(0)$ is not defined. Hence, $f(g(0))$ is $\boxed{\text{undefined}}$.

34.

$$g(f(0))=g\left(\underbrace{0^2+10}_{=10}\right)=\sqrt{10-1}=\boxed{3}$$

35. $f(g(-3))$ is not defined since $g(-3)$ in not defined.

36.

$$\begin{aligned}g\left(f\left(\sqrt{7}\right)\right)&=g\left(\left(\sqrt{7}\right)^2+10\right)\\&=g(17)=\sqrt{17-1}=\boxed{4}\end{aligned}$$

37. $(f\circ g)(4)=f(g(4))=f\left(\sqrt{4-1}\right)$ $=f\left(\sqrt{3}\right)=\left(\sqrt{3}\right)^2+10=\boxed{13}$	**38.** $(g\circ f)(-3)=g(f(-3))=g\left((-3)^2+10\right)$ $=g(19)=\sqrt{19-1}=\boxed{3\sqrt{2}}$
39. $f(g(1))=f\left(\underbrace{2(1)+1}_{=3}\right)=\boxed{\tfrac{1}{3}}$ $g(f(2))=g\left(\tfrac{1}{2}\right)=2\left(\tfrac{1}{2}\right)+1=\boxed{2}$	**40.** $f(g(1))=f\left(\underbrace{\dfrac{1}{2-1}}_{=1}\right)=1^2+1=\boxed{2}$ $g(f(2))=g\left(\underbrace{2^2+1}_{=5}\right)=\dfrac{1}{2-5}=\boxed{-\tfrac{1}{3}}$
41. $f(g(1))=f\left(\underbrace{1^2+2}_{=3}\right)$ Since 3 is not in the domain of f, this is $\boxed{\text{undefined}}$. Likewise, $g(f(2))$ is $\boxed{\text{undefined}}$ since 2 is not in the domain of *f*.	**42.** $f(g(1))=f\left(\underbrace{1^2+1}_{=2}\right)=\sqrt{3-2}=\boxed{1}$ $g(f(2))=g\left(\underbrace{\sqrt{3-2}}_{=1}\right)=1^2+1=\boxed{2}$
43. $f(g(1))=f\left(\underbrace{1+3}_{=4}\right)=\dfrac{1}{\lvert 4-1\rvert}=\boxed{\tfrac{1}{3}}$ $g(f(2))=g\left(\underbrace{\dfrac{1}{\lvert 2-1\rvert}}_{=1}\right)=1+3=\boxed{4}$	**44.** $f(g(1))=f\left(\underbrace{\lvert 2(1)-3\rvert}_{=1}\right)=\dfrac{1}{1}=\boxed{1}$ $g(f(2))=g\left(\tfrac{1}{2}\right)=\left\lvert 2\left(\tfrac{1}{2}\right)-3\right\rvert=\boxed{2}$
45. $f(g(1))=f\left(\underbrace{1^2+5}_{=6}\right)=\sqrt{6-1}=\boxed{\sqrt{5}}$ $g(f(2))=g\left(\underbrace{\sqrt{2-1}}_{=1}\right)=1^2+5=\boxed{6}$	**46.** $f(g(1))=f\left(\underbrace{\dfrac{1}{1-3}}_{=-2}\right)=\sqrt[3]{-\tfrac{1}{2}-3}=\boxed{\sqrt[3]{-\tfrac{7}{2}}}$ $g(f(2))=g\left(\underbrace{\sqrt[3]{2-3}}_{=-1}\right)=\dfrac{1}{-1-3}=\boxed{-\tfrac{1}{4}}$
47. $f(g(1))$ is $\boxed{\text{undefined}}$ since $g(1)$ is not defined. $g(f(2))=g\left(\dfrac{1}{2^2-3}\right)=g(1)$, which is not defined. So, this is also $\boxed{\text{undefined}}$.	**48.** $f(g(1))=f\left(4-1^2\right)=f(3)=\dfrac{3}{2-3}=\boxed{-3}$ $g(f(2))$ is $\boxed{\text{undefined}}$ since $f(2)$ is not defined.

49. $f(g(1)) = f\left(1^2+2(1)+1\right) = f(4)$ $= (4-1)^{1/3} = \boxed{\sqrt[3]{3}}$ $g(f(2)) = g\left((2-1)^{1/3}\right) = g(1)$ $= 1^2+2(1)+1 = \boxed{4}$	**50.** $f(g(1)) = f\left((1-3)^{1/3}\right) = f\left((-2)^{1/3}\right)$ $= \left(1-\left((-2)^{1/3}\right)^2\right)^{1/2} = \left(\underbrace{1-2^{2/3}}_{<0}\right)^{1/2}$, which is $\boxed{\text{undefined}}$ $g(f(2))$ is $\boxed{\text{undefined}}$ since $f(2)$ is not defined.
51. $f(g(x)) = \cancel{2}\left(\frac{x-1}{\cancel{2}}\right)+1 = x-1+1 = x$ $g(f(x)) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$	**52.** $f(g(x)) = \frac{(3x+2)-2}{3} = \frac{3x}{3} = x$ $g(f(x)) = \cancel{3}\left(\frac{x-2}{\cancel{3}}\right)+2 = x-2+2 = x$
53. $f(g(x)) = \sqrt{\left(x^2+1\right)-1} = \sqrt{x^2} = \underbrace{\lvert x\rvert = x}_{\text{Since } x\geq 1}$ $g(f(x)) = \left(\sqrt{x-1}\right)^2+1 = (x-1)+1 = x$	**54.** $f(g(x)) = 2-\left(\sqrt{2-x}\right)^2 = 2-(2-x)$ $= 2-2+x = x$ $g(f(x)) = \sqrt{2-\left(2-x^2\right)} = \sqrt{2-2+x^2} = \sqrt{x^2} = x$
55. $f(g(x)) = \frac{1}{\frac{1}{x}} = x \qquad g(f(x)) = \frac{1}{\frac{1}{x}} = x$	**56.** $f(g(x)) = \left[5-\left(5-x^3\right)\right]^{1/3} = \left[5-5+x^3\right]^{1/3}$ $= \left[x^3\right]^{1/3} = x$ $g(f(x)) = 5-\left[(5-x)^{1/3}\right]^3 = 5-(5-x)$ $= 5-5+x = x$

57.

$$f(g(x)) = 4\left(\frac{\sqrt{x+9}}{2}\right)^2-9 = 4\left(\frac{x+9}{4}\right)-9 = x$$

$$g(f(x)) = \frac{\sqrt{\left(4x^2-9\right)+9}}{2} = \frac{\sqrt{4x^2}}{2} = \frac{2x}{2} = x$$

58.

$$f(g(x)) = \sqrt[3]{8\left(\frac{x^3+1}{8}\right)-1} = \sqrt[3]{x^3} = x$$

$$g(f(x)) = \frac{\left(\sqrt[3]{8x-1}\right)^3+1}{8} = \frac{8x-1+1}{8} = x$$

59. $f(g(x)) = \frac{1}{\frac{x+1}{x}-1} = \frac{1}{\frac{x+1-x}{x}} = \frac{1}{\frac{1}{x}} = x \qquad g(f(x)) = \frac{\frac{1}{x-1}+1}{\frac{1}{x-1}} = \frac{\frac{1+x-1}{x-1}}{\frac{1}{x-1}} = \frac{\frac{x}{\cancel{x-1}}}{\frac{1}{\cancel{x-1}}} = x$

60. $f(g(x)) = g(f(x)) = \sqrt{25-\left(\sqrt{25-x^2}\right)^2} = \sqrt{25-\left(25-x^2\right)} = \sqrt{x^2} = x$ since $x \ge 0$.	
61. $f(x) = 2x^2 + 5x \quad g(x) = 3x - 1$	**62.** The most natural pairs are: $f(x) = \frac{1}{x} \quad g(x) = x^2 + 1$; $f(x) = \frac{1}{x+1} \quad g(x) = x^2$
63. $f(x) = \frac{2}{\lvert x \rvert} \quad g(x) = x - 3$	**64.** $f(x) = \sqrt{x} \quad g(x) = 1 - x^2$
65. $f(x) = \dfrac{3}{\sqrt{x}-2} \quad g(x) = x + 1$	**66.** $f(x) = \dfrac{x}{3x+2} \quad g(x) = \sqrt{x}$

67. $F(C(K)) = \frac{9}{5}\left(K - 273.15\right) + 32$

68. We need to calculate the composition function $(K \circ C)(F)$.

Solve $F = \frac{9}{5}C + 32$ for C: $\quad C = \frac{5}{9}(F - 32)$

Solve $C = K - 273.15$ for K: $\quad K = C + 273.15$

So, $(K \circ C)(F) = K(C(F)) = K\left(\frac{5}{9}(F-32)\right) = \frac{5}{9}(F-32) + 273.15 = \boxed{\frac{5}{9}F + 255.37}$.

Thus, $32^\circ F$ corresponds to $\frac{5}{9}(32) + 255.37 = 273.15K$, and

$212^\circ F$ corresponds to $\frac{5}{9}(212) + 255.37 = 373.15K$.

69. First, solve $p = 3000 - \frac{1}{2}x$ for x: $\quad x = 2(3000 - p) = 6000 - 2p$

a. $C(x(p)) = C\left(6000 - 2p\right) = 2000 + 10(6000 - 2p) = 62,000 - 20p$

b. $R(x(p)) = 100(6000 - 2p) = 600,000 - 200p$

c. Profit $P = R - C$. So,

$$\begin{aligned} P(x(p)) &= R(x(p)) - C(x(p)) \\ &= \left(600,000 - 200p\right) - \left(62,000 - 20p\right) \\ &= 538,000 - 180p \end{aligned}$$

70. First, solve $p = 10,000 - \frac{1}{4}x$ for x: $\quad x = 4(10,000 - p) = 40,000 - 4p$

a. $C(x(p)) = C\left(40,000 - 4p\right) = 30,000 + 5(40,000 - 4p) = 230,000 - 20p$

b. $R(x(p)) = 1000(40,000 - 4p) = 40,000,000 - 4000p$

c. Profit $P = R - C$. So,

$$\begin{aligned} P(x(p)) &= R(x(p)) - C(x(p)) \\ &= \left(40,000,000 - 4000p\right) - \left(230,000 - 20p\right) \\ &= 39,770,000 - 3,980p \end{aligned}$$

71. a. $\left(C \circ n\right)(t) = C(n(t)) = 10\left(50t - t^2\right) + 1375 = -10t^2 + 500t + 1375$

b. $\left(C \circ n\right)(16) = C(n(16)) = -10(16)^2 + 500(16) + 1375 = 6815$

This is the cost of production on a day when the assembly line was running for 16 hours; this cost is $6,815,000.

72. a. $(C \circ n)(t) = C(n(t)) = 8\left(100t - 4t^2\right) + 2375 = -32t^2 + 800t + 2375$ **b.** $(C \circ n)(24) = C(n(24)) = -32(24)^2 + 800(24) + 2375 = 3143$ This is the cost of production on a day when the assembly line was running for 24 hours; this cost is $3,143,000.	
73. a. $A(r(t)) = \pi\left(10t - 0.2t^2\right)^2$ **b.** $A(r(7)) = \pi\left(10(7) - 0.2(7)^2\right)^2 = 11{,}385$ square miles	
74. a. $A(r(t)) = \pi\left(8t - 0.1t^2\right)^2$ **b.** $A(r(5)) = \pi\left(8(5) - 0.1(5)^2\right)^2 = 4{,}418$ square miles	
75. Must exclude -2 from the domain. .	
76. Must also exclude -2 from the domain.	
77. $(f \circ g)(x) = f(g(x))$, not $f(x) \cdot g(x)$	**78.** Didn't distribute " – " to all parts of $g(x)$. Should have been: $f(x) - g(x) = (x+2) - \left(x^2 - 4\right)$ $= x + 2 - x^2 + 4$ $= -x^2 + x + 6$
79. The mistake made was that $(f+g)$ was multiplied by 2 when it ought to have been evaluated at 2.	**80.** Domain is $[3, \infty)$
81. False. The domain of the sum, difference, or product of two functions is the <u>intersection</u> of their domains; the domain of the quotient is the set obtained by intersecting the two domains and then excluding all values where the denominator equals 0.	**82.** False. For example, consider the functions $f(x) = x+1,\ g(x) = 3$. Observe that $f(g(4)) = f(3) = 4$ $g(f(4)) = g(5) = 3$
83. True	**84.** False
85. $(g \circ f)(x) = \dfrac{1}{(x+a)-a} = \dfrac{1}{x}$ <u>Domain</u>: $x \neq 0$	

86.

$$(g\circ f)(x)=\frac{1}{\left(ax^2+bx+c\right)-c}=\frac{1}{ax^2+bx}$$
$$=\frac{1}{x(ax+b)}$$

Domain: $x\neq 0,\ -\frac{b}{a},\ c$

87.

$(g\circ f)(x)=\left(\sqrt{x+a}\right)^2-a=x+a-a=x$

Domain: Must have $x+a\geq 0,$ so that $x\geq -a$. So, domain is $\left[-a,\infty\right)$.

88. $(g\circ f)(x)=\dfrac{1}{\left(\frac{1}{x^a}\right)^b}=\dfrac{1}{\frac{1}{x^{ab}}}=x^{ab}$

Domain: $\left(0,\infty\right)$.

89. Observe that $\dfrac{F(x+h)-F(x)}{h}=\dfrac{(x+h)-x}{h}=\dfrac{h}{h}=1$**.** So, at $h=0$ we get $F'(x)=1$.

Also, $\dfrac{G(x+h)-G(x)}{h}=\dfrac{\left(x+h\right)^2-x^2}{h}=\dfrac{x^2+2hx+h^2-x^2}{h}=\dfrac{h\left(2x+h\right)}{h}=2x+h$

So, at $h=0$ we get $G'(x)=2x$.

Finally, observe that

$$\frac{H(x+h)-H(x)}{h}=\frac{\left[\left(x+h\right)+\left(x+h\right)^2\right]-\left[x+x^2\right]}{h}$$
$$=\frac{(x+h)-x}{h}+\frac{x^2+2hx+h^2-x^2}{h}=\frac{h}{h}+\frac{h\left(2x+h\right)}{h}=1+2x+h$$

So, at $h=0$ we get $H'(x)=1+2x=F'(x)+G'(x)$. As such, we conclude that it appears as though $H'(x)=F'(x)+G'(x)$.

90. Observe that

$$\frac{F(x+h)-F(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

So, at $h=0$ we get $F'(x)=\frac{1}{2\sqrt{x}}$.

Also,

$$\frac{G(x+h)-G(x)}{h}=\frac{\left[(x+h)^3+1\right]-\left[x^3+1\right]}{h}=\frac{x^3+3x^2h+3xh^2+1+h^3-x^3-1}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2\right)}{h}=\boxed{3x^2+3xh+h^2}$$

So, at $h=0$ we get $G'(x)=3x^2$.

Finally, observe that

$$\frac{H(x+h)-H(x)}{h}=\frac{\left(\sqrt{x+h}-\left[(x+h)^3+1\right]\right)-\left(\sqrt{x}-\left[x^3+1\right]\right)}{h}$$

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}-\frac{\left[(x+h)^3+1\right]-\left[x^3+1\right]}{h}$$

$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}-\left(3x^2+3xh+h^2\right)$$

So, at $h=0$, we get $H'(x)=\frac{1}{2\sqrt{x}}-3x^2=F'(x)-G'(x)$. As such, we conclude that it appears as though $H'(x)=F'(x)-G'(x)$.

91. Observe that $\frac{F(x+h)-F(x)}{h}=\frac{5-5}{h}=0$. So, at $h=0$ we get $F'(x)=0$. Also,

$$\frac{G(x+h)-G(x)}{h}=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$$

$$=\frac{x+h-1-x+1}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$$

So, at $h=0$ we get $G'(x)=\frac{1}{2\sqrt{x-1}}$. Finally, observe that

$$\frac{H(x+h)-H(x)}{h}=\frac{5\sqrt{x+h-1}-5\sqrt{x-1}}{h}=5\left[\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\right]=\frac{5}{\sqrt{x+h-1}+\sqrt{x-1}}.$$

Hence, $H'(x)=\frac{5}{2\sqrt{x-1}}\neq F'(x)G'(x)$. So, we conclude that it appears as though $H'(x)\neq F'(x)G'(x)$.

92. Observe that $\frac{F(x+h)-F(x)}{h}=\frac{(x+h)-x}{h}=\frac{h}{h}=1$. So, at $h=0$ we get $F'(x)=1$.

Also,

$$\frac{G(x+h)-G(x)}{h}=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$=\frac{x+h-1-x+1}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}=\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

So, at $h=0$ we get $G'(x)=\frac{1}{2\sqrt{x+1}}$. Finally, observe that

$$\frac{H(x+h)-H(x)}{h}=\frac{\frac{x+h}{\sqrt{x+h+1}}-\frac{x}{\sqrt{x+1}}}{h}=\frac{(x+h)\sqrt{x+1}-x\sqrt{x+h+1}}{h\sqrt{x+1}\sqrt{x+h+1}}$$

$$=\frac{(x+h)\sqrt{x+1}-x\sqrt{x+h+1}}{h\sqrt{x+1}\sqrt{x+h+1}}\cdot\frac{(x+h)\sqrt{x+1}+x\sqrt{x+h+1}}{(x+h)\sqrt{x+1}+x\sqrt{x+h+1}}$$

$$=\frac{(x+h)^2(x+1)-x^2(x+h+1)}{h\sqrt{x+1}\sqrt{x+h+1}\left[(x+h)\sqrt{x+1}+x\sqrt{x+h+1}\right]}$$

$$=\frac{x^3+2hx^2+h^2x+x^2+2hx+h^2-x^3-x^2h-x^2}{h\sqrt{x+1}\sqrt{x+h+1}\left[(x+h)\sqrt{x+1}+x\sqrt{x+h+1}\right]}$$

$$=\frac{\not{h}\left(x^2+hx+2x+h\right)}{\not{h}\sqrt{x+1}\sqrt{x+h+1}\left[(x+h)\sqrt{x+1}+x\sqrt{x+h+1}\right]}$$

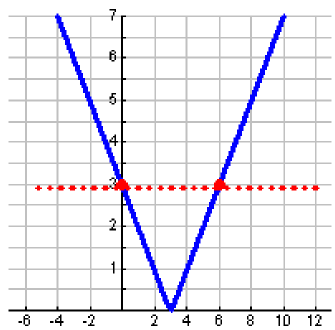
So, $H'(x)=\frac{x^2+2x}{(x+1)\left[2x\sqrt{x+1}\right]}=\frac{x(x+2)}{2x(x+1)\sqrt{x+1}}=\frac{x+2}{2(x+1)\sqrt{x+1}}\neq\frac{F'(x)}{G'(x)}$.

Section 1.5 Solutions ---

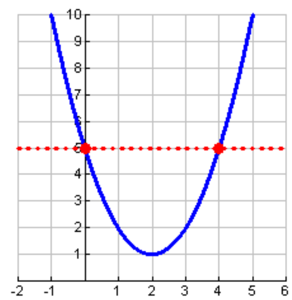
1. Not a function since 4 maps to both 2 and -2.	**2.** Is a function. Not one-to-one 0,1,2,3 all map to 1 in the range.	**3.** Is a function. Not one-to-one $0, 2, -2$ all map to 1 in the range, for instance.
4. Is a function. One-to-one	**5.** Is a function. Not one-to-one Doesn't pass the horizontal line test. Both $(-1,1)$, $(0,1)$ are on the graph.	**6.** Is a function. Not one-to-one Doesn't pass the horizontal line test.
7. Is a function. One-to-one	**8.** Is a function. One-to-one	**9.** Is a function. Not one-to-one Doesn't pass the horizontal line test.

10. Is a function. One-to-one

11. Not one-to-one. Both $(0,3)$, $(6,3)$ lie on the graph.



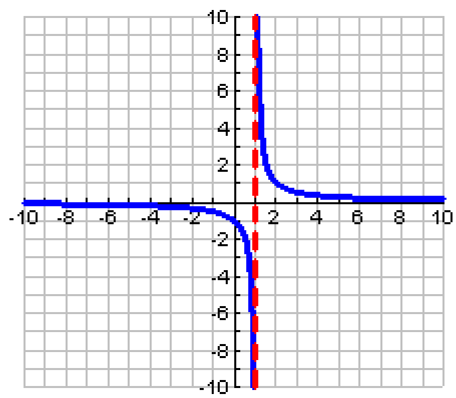
12. Not one-to-one. Both $(0,5)$, $(4,5)$ lie on the graph.



13.

$$f(x_1) = f(x_2) \Rightarrow \frac{1}{x_1 - 1} = \frac{1}{x_2 - 1}$$
$$\Rightarrow x_2 - 1 = x_1 - 1$$
$$\Rightarrow x_2 = x_1$$

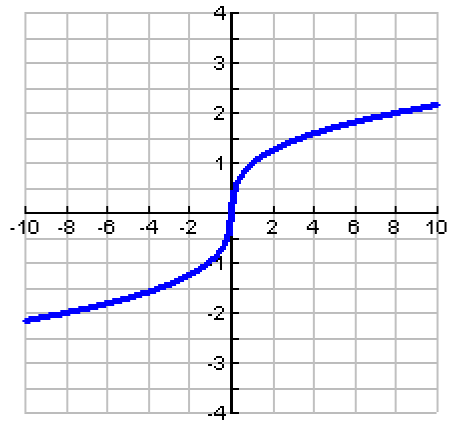
One-to-one



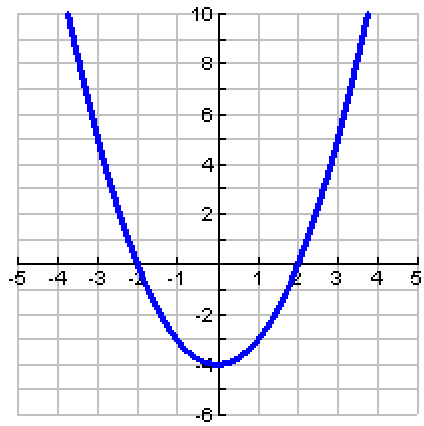
14.

$$f(x_1) = f(x_2) \Rightarrow \sqrt[3]{x_1} = \sqrt[3]{x_2}$$
$$\Rightarrow \left(\sqrt[3]{x_1}\right)^3 = \left(\sqrt[3]{x_2}\right)^3$$
$$\Rightarrow x_1 = x_2$$

One-to-one



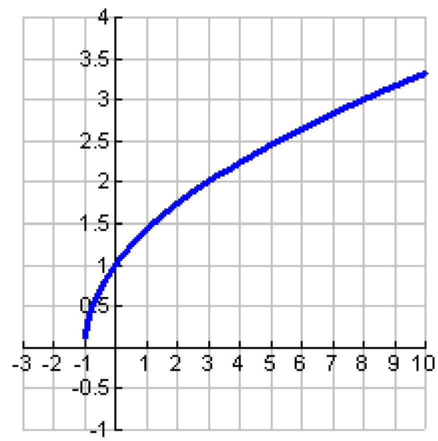
15. f is not one-to-one since, for example, $f(-1) = f(1) = -3$.



16.

$$f(x_1) = f(x_2) \Rightarrow \sqrt{x_1 + 1} = \sqrt{x_2 + 1}$$
$$\Rightarrow x_1 + 1 = x_2 + 1$$
$$\Rightarrow x_1 = x_2$$

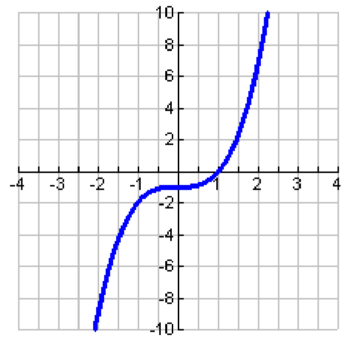
One-to-one



17.

$$f(x_1) = f(x_2) \Rightarrow x_1^3 - 1 = x_2^3 - 1$$
$$\Rightarrow x_1^3 = x_2^3$$
$$\Rightarrow x_1 = x_2$$

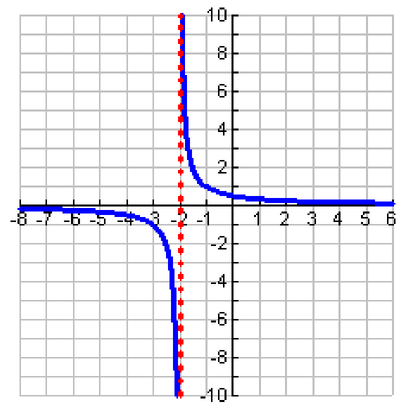
One-to-one



18.

$$f(x_1) = f(x_2) \Rightarrow \frac{1}{x_1 + 2} = \frac{1}{x_2 + 2}$$
$$\Rightarrow x_2 + 2 = x_1 + 2$$
$$\Rightarrow x_2 = x_1$$

One-to-one

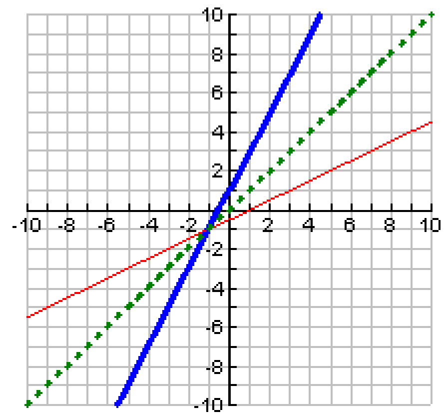


19.

Given: $f(x) = 2x + 1, \quad f^{-1}(x) = \dfrac{x-1}{2}$

$$f\left(f^{-1}(x)\right) = \cancel{2}\left(\frac{x-1}{\cancel{2}}\right) + 1 = x - 1 + 1 = x$$

$$f^{-1}\left(f(x)\right) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$$



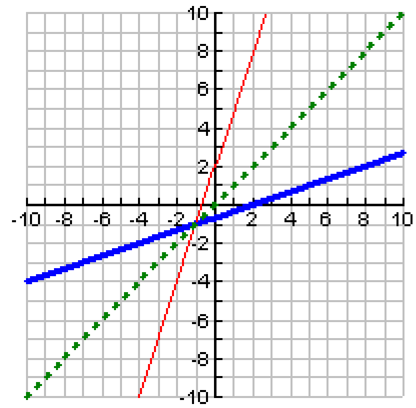
Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y = x$.

20.

Given: $f(x)=\dfrac{x-2}{3},\quad f^{-1}(x)=3x+2$

$$f\left(f^{-1}(x)\right)=\frac{(3x+2)-2}{3}=\frac{3x}{3}=x$$

$$f^{-1}\left(f(x)\right)=\cancel{3}\left(\frac{x-2}{\cancel{3}}\right)+2=x-2+2=x$$



Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

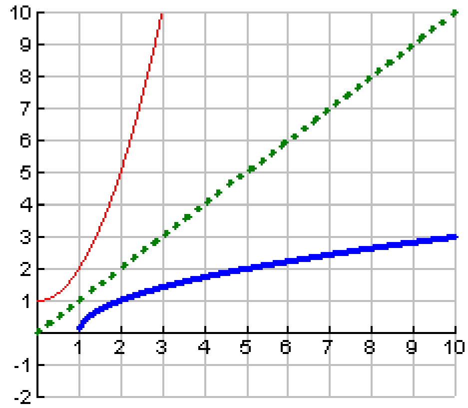
21.

Given: $f(x)=\sqrt{x-1},\quad x\ge 1$

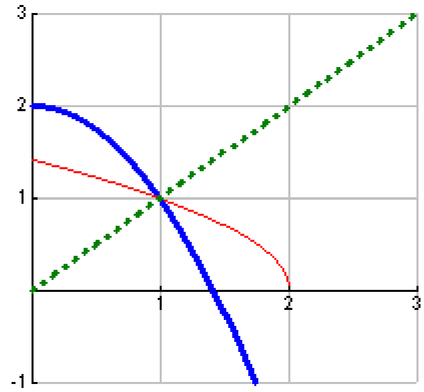
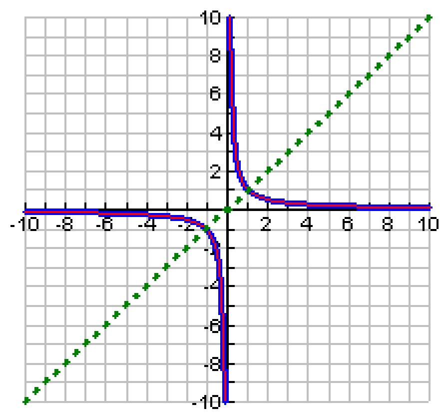
$f^{-1}(x)=x^2+1,\quad x\ge 0$

$$f\left(f^{-1}(x)\right)=\sqrt{\left(x^2+1\right)-1}=\sqrt{x^2}=\underbrace{|x|=x}_{\text{Since } x\ge 0}$$

$$f^{-1}\left(f(x)\right)=\left(\sqrt{x-1}\right)^2+1=x-1+1=x$$



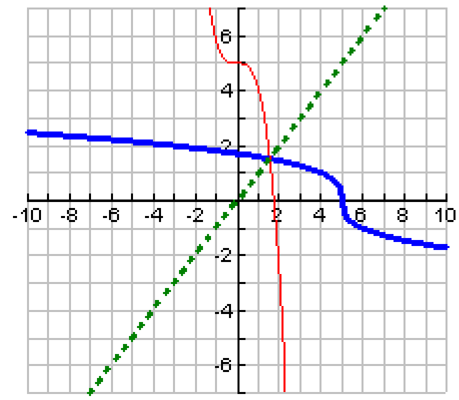
Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

22. Given: $f(x) = 2 - x^2, \ x \geq 0$ $f^{-1}(x) = \sqrt{2-x}, \ x \leq 2$ $f\left(f^{-1}(x)\right) = 2 - \left(\sqrt{2-x}\right)^2 = 2 - (2-x) = x$ $f^{-1}\left(f(x)\right) = \sqrt{2 - \left(2 - x^2\right)} = \sqrt{x^2} = \underbrace{\lvert x \rvert = x}_{\text{Since } x \geq 0}$	 Notes on the Graphs: Thick, solid curve is the graph of *f*. Thin, solid curve is the graph of f^{-1}. Thick, dotted curve is the graph of $y = x$.
23. Given: $f(x) = \frac{1}{x}, \ f^{-1}(x) = \frac{1}{x}$ $f\left(f^{-1}(x)\right) = \frac{1}{\frac{1}{x}} = x$ $f^{-1}\left(f(x)\right) = \frac{1}{\frac{1}{x}} = x$	 Notes on the Graphs: Thick, solid curve is the graph of *f*. Thin, solid curve is the graph of f^{-1}. (Note: These curves overlap since the functions are the same.) Thick, dotted curve is the graph of $y = x$.

24.

Given: $f(x)=(5-x)^{1/3}$, $f^{-1}(x)=5-x^3$

$$f\left(f^{-1}(x)\right)=(5-(5-x^3))^{1/3}=(5-5+x^3)^{1/3}$$
$$=(x^3)^{1/3}=x$$
$$f^{-1}\left(f(x)\right)=5-\left[(5-x)^{1/3}\right]^3=5-(5-x)=x$$

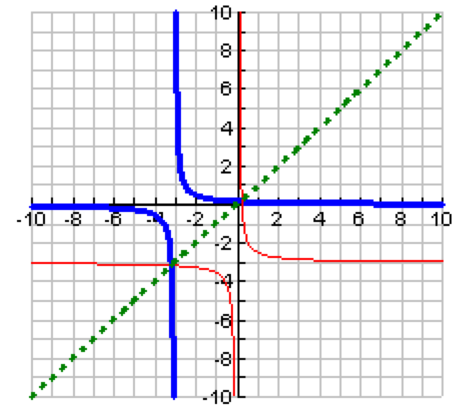


Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

25.

Given: $f(x)=\dfrac{1}{2x+6}$, $f^{-1}(x)=\dfrac{1}{2x}-3$

$$f\left(f^{-1}(x)\right)=\frac{1}{2\left(\frac{1}{2x}-3\right)+6}=\frac{1}{\frac{1}{x}-6+6}$$
$$=\frac{1}{\frac{1}{x}}=x$$
$$f^{-1}\left(f(x)\right)=\frac{1}{2\left[\frac{1}{2x+6}\right]}-3=\frac{1}{\frac{1}{x+3}}-3$$
$$=x+3-3=x$$



Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

26.

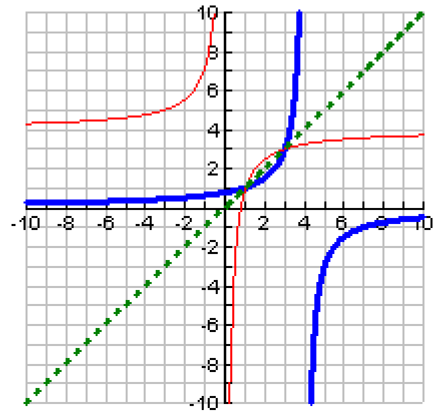
Given: $f(x)=\dfrac{3}{4-x}$, $f^{-1}(x)=4-\dfrac{3}{x}$

$$f\left(f^{-1}(x)\right)=\frac{3}{4-\left(4-\frac{3}{x}\right)}=\frac{3}{4-4+\frac{3}{x}}$$

$$=3\cdot\frac{x}{3}=x$$

$$f^{-1}\left(f(x)\right)=4-\frac{3}{\frac{3}{4-x}}=4-\cancel{3}\cdot\frac{4-x}{\cancel{3}}$$

$$=4-4+x=x$$



Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

27.

Given: $f(x)=\dfrac{x+3}{x+4}$, $f^{-1}(x)=\dfrac{3-4x}{x-1}$

$$f\left(f^{-1}(x)\right)=\frac{\frac{3-4x}{x-1}+3}{\frac{3-4x}{x-1}+4}=\frac{\frac{3-4x+3(x-1)}{x-1}}{\frac{3-4x+4(x-1)}{x-1}}$$

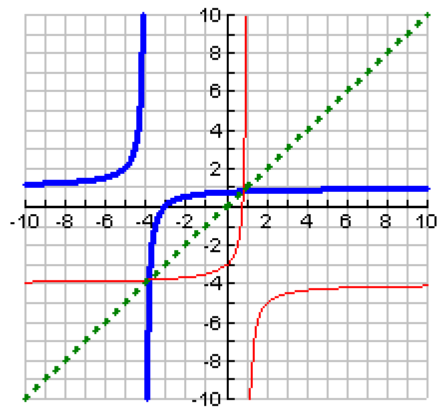
$$=\frac{\frac{\cancel{3}-4x+3x-\cancel{3}}{x-1}}{\frac{3-\cancel{4x}+\cancel{4x}-4}{x-1}}=\frac{\frac{-x}{x-1}}{\frac{-1}{x-1}}$$

$$=\frac{-x}{\cancel{x-1}}\cdot\frac{\cancel{x-1}}{-1}=x$$

$$f^{-1}\left(f(x)\right)=\frac{3-4\left(\frac{x+3}{x+4}\right)}{\left(\frac{x+3}{x+4}\right)-1}=\frac{3-\frac{4x+12}{x+4}}{\frac{x+3}{x+4}-1}$$

$$=\frac{\frac{3x+\cancel{12}-4x-\cancel{12}}{x+4}}{\frac{\cancel{x}+3-\cancel{x}-4}{x+4}}=\frac{\frac{-x}{x+4}}{\frac{-1}{x+4}}$$

$$=\frac{-x}{\cancel{x+4}}\cdot\frac{\cancel{x+4}}{-1}=x$$



Notes on the Graphs:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

28.

<u>Given</u>: $f(x)=\frac{x-5}{3-x},\quad f^{-1}(x)=\frac{3x+5}{x+1}$

$$f\left(f^{-1}(x)\right)=\frac{\frac{3x+5}{x+1}-5}{3-\frac{3x+5}{x+1}}=\frac{\frac{3x+5-5(x+1)}{x+1}}{\frac{3x+3-(3x+5)}{x+1}}$$

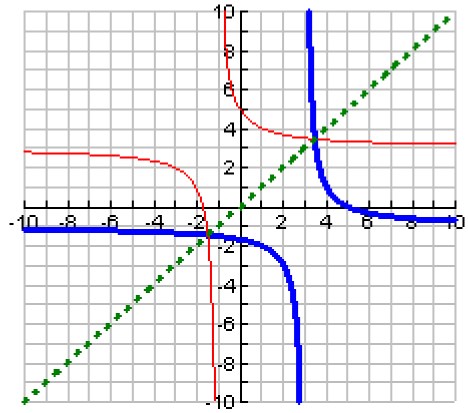
$$=\frac{\frac{3x+\cancel{5}-5x-\cancel{5}}{x+1}}{\frac{\cancel{3x}+3-\cancel{3x}-5}{x+1}}=\frac{\frac{-2x}{x+1}}{\frac{-2}{x+1}}$$

$$=\frac{-2x}{\cancel{x+1}}\cdot\frac{\cancel{x+1}}{-2}=x$$

$$f^{-1}\left(f(x)\right)=\frac{3\left(\frac{x-5}{3-x}\right)+5}{\left(\frac{x-5}{3-x}\right)+1}=\frac{\frac{3x-15}{3-x}+5}{\frac{x-5}{3-x}+1}$$

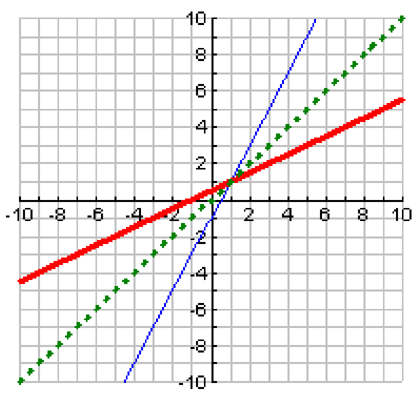
$$=\frac{\frac{3x-15+5(3-x)}{3-x}}{\frac{x-5+3-x}{3-x}}=\frac{\frac{3x-\cancel{15}+\cancel{15}-5x}{3-x}}{\frac{\cancel{x}-5+3-\cancel{x}}{3-x}}$$

$$=\frac{-2x}{\cancel{3-x}}\cdot\frac{\cancel{3-x}}{-2}=x$$



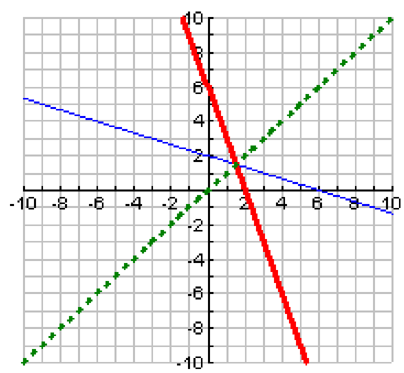
<u>Notes on the Graphs</u>:
Thick, solid curve is the graph of *f*.
Thin, solid curve is the graph of f^{-1}.
Thick, dotted curve is the graph of $y=x$.

29.



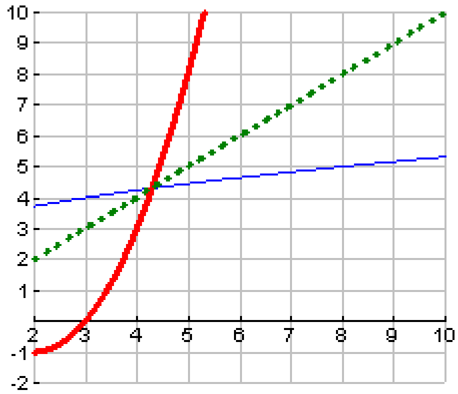
<u>Notes on the Graphs</u>:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y=x$.
Thick, solid curve is the graph of f^{-1}.

30.



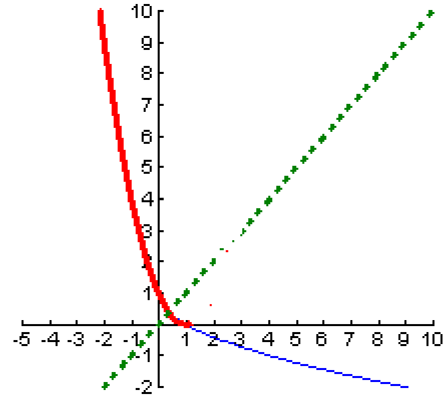
<u>Notes on the Graphs</u>:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y=x$.
Thick, solid curve is the graph of f^{-1}.

31.



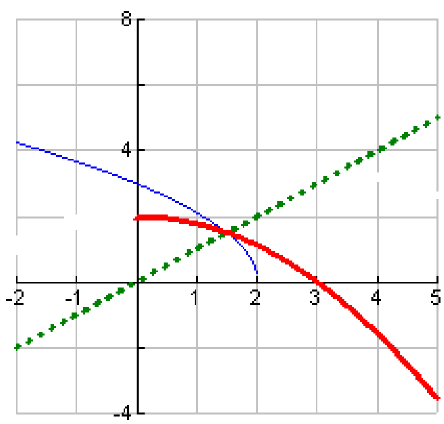
Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

32.



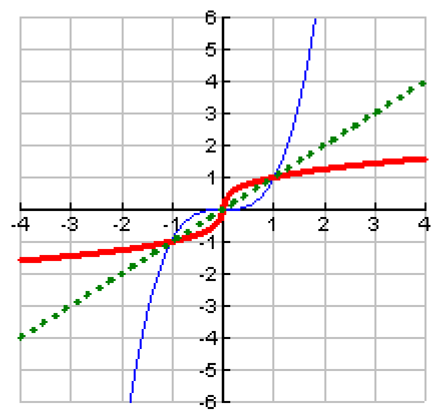
Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

33.



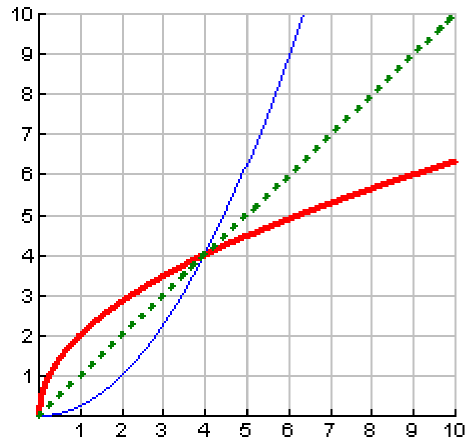
Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

34.



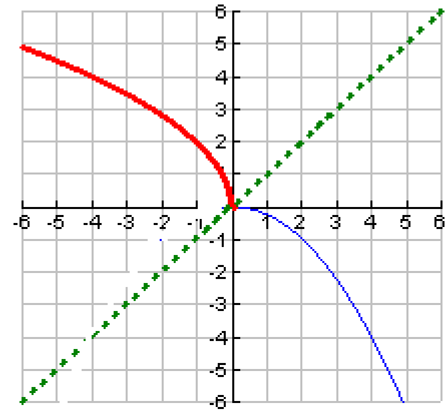
Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

35.



Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

36.



Notes on the Graphs:
Thin, solid curve is the graph of *f*.
Thick, dotted curve is the graph of $y = x$.
Thick, solid curve is the graph of f^{-1}.

37. Solve $y = -3x + 2$ for *x*:

$$x = -\tfrac{1}{3}(y-2)$$

Thus, $f^{-1}(x) = -\tfrac{1}{3}(x-2) = -\tfrac{1}{3}x + \tfrac{2}{3}$.

Domains:
$dom(f) = rng(f^{-1}) = (-\infty, \infty)$
$rng(f) = dom(f^{-1}) = (-\infty, \infty)$

38. Solve $y = 2x + 3$ for *x*:

$$x = \tfrac{1}{2}(y-3)$$

Thus, $f^{-1}(x) = \tfrac{1}{2}(x-3)$.

Domains:
$dom(f) = rng(f^{-1}) = (-\infty, \infty)$
$rng(f) = dom(f^{-1}) = (-\infty, \infty)$

39. Solve $y = x^3 + 1$ for *x*:

$$x = \sqrt[3]{y-1}$$

Thus, $f^{-1}(x) = \sqrt[3]{x-1}$.

Domains:
$dom(f) = rng(f^{-1}) = (-\infty, \infty)$
$rng(f) = dom(f^{-1}) = (-\infty, \infty)$

40. Solve $y = x^3 - 1$ for *x*:

$$x = \sqrt[3]{y+1}$$

Thus, $f^{-1}(x) = \sqrt[3]{x+1}$.

Domains:
$dom(f) = rng(f^{-1}) = (-\infty, \infty)$
$rng(f) = dom(f^{-1}) = (-\infty, \infty)$

41. Solve $y = \sqrt{x-3}$ for *x*:

$$x = y^2 + 3$$

Thus, $f^{-1}(x) = x^2 + 3$.

Domains:
$dom(f) = rng(f^{-1}) = [3, \infty)$
$rng(f) = dom(f^{-1}) = [0, \infty)$

42. Solve $y = \sqrt{3-x}$ for *x*:

$$x = 3 - y^2$$

Thus, $f^{-1}(x) = 3 - x^2$.

Domains:
$dom(f) = rng(f^{-1}) = (-\infty, 3]$
$rng(f) = dom(f^{-1}) = [0, \infty)$

43. Solve $y = x^2 - 1$ for x: $x = \sqrt{y+1}$ Thus, $f^{-1}(x) = \sqrt{x+1}$.	Domains: $dom(f) = rng(f^{-1}) = [0,\infty)$ $rng(f) = dom(f^{-1}) = [-1,\infty)$
44. Solve $y = 2x^2 + 1$ for x: $2x^2 = y - 1$ $x = +\sqrt{\frac{y-1}{2}}$ (since $x \geq 0$) Thus, $f^{-1}(x) = \sqrt{\frac{x-1}{2}}$.	Domains: $dom(f) = rng(f^{-1}) = [0,\infty)$ $rng(f) = dom(f^{-1}) = [1,\infty)$
45. Solve $y = (x+2)^2 - 3$ for x: $y + 3 = (x+2)^2$ $\sqrt{y+3} = x + 2$ (since $x \geq -2$) $-2 + \sqrt{y+3} = x$ Thus, $f^{-1}(x) = -2 + \sqrt{x+3}$.	Domains: $dom(f) = rng(f^{-1}) = [-2,\infty)$ $rng(f) = dom(f^{-1}) = [-3,\infty)$
46. Solve $y = (x-3)^2 - 2$ for x: $y + 2 = (x-3)^2$ $\sqrt{y+2} = x - 3$ (since $x \geq 3$) $3 + \sqrt{y+2} = x$ Thus, $f^{-1}(x) = 3 + \sqrt{x+2}$.	Domains: $dom(f) = rng(f^{-1}) = [3,\infty)$ $rng(f) = dom(f^{-1}) = [-2,\infty)$
47. Solve $y = \frac{2}{x}$ for x: $xy = 2$ $x = \frac{2}{y}$ Thus, $f^{-1}(x) = \frac{2}{x}$.	Domains: $dom(f) = rng(f^{-1}) = (-\infty,0)\cup(0,\infty)$ $rng(f) = dom(f^{-1}) = (-\infty,0)\cup(0,\infty)$
48. Solve $y = -\frac{3}{x}$ for x: $yx = -3$ $x = -\frac{3}{y}$ Thus, $f^{-1}(x) = -\frac{3}{x}$.	Domains: $dom(f) = rng(f^{-1}) = (-\infty,0)\cup(0,\infty)$ $rng(f) = dom(f^{-1}) = (-\infty,0)\cup(0,\infty)$
49. Solve $y = \frac{2}{3-x}$ for x: $(3-x)y = 2$ $3y - xy = 2$ $xy = 3y - 2$ $x = \frac{3y-2}{y}$ Thus, $f^{-1}(x) = \frac{3x-2}{x} = 3 - \frac{2}{x}$.	Domains: $dom(f) = rng(f^{-1}) = (-\infty,3)\cup(3,\infty)$ $rng(f) = dom(f^{-1}) = (-\infty,0)\cup(0,\infty)$

50. Solve $y = \frac{7}{x+2}$ for x:

$$\begin{aligned} (x+2)y &= 7 \\ 2y + xy &= 7 \\ xy &= 7 - 2y \\ x &= \tfrac{7-2y}{y} \end{aligned}$$

Thus, $f^{-1}(x) = \frac{7-2x}{x}$.

Domains:

$dom(f) = rng(f^{-1}) = (-\infty, -2) \cup (-2, \infty)$

$rng(f) = dom(f^{-1}) = (-\infty, 0) \cup (0, \infty)$

51. Solve $y = \frac{7x+1}{5-x}$ for x:

$$\begin{aligned} y(5-x) &= 7x+1 \\ 5y - xy &= 7x+1 \\ -7x - xy &= 1-5y \\ -x(7+y) &= 1-5y \\ x &= \tfrac{5y-1}{7+y} \end{aligned}$$

Thus, $f^{-1}(x) = \frac{5x-1}{x+7}$.

Domains:

$dom(f) = rng(f^{-1}) = (-\infty, 5) \cup (5, \infty)$

$rng(f) = dom(f^{-1}) = (-\infty, -7) \cup (-7, \infty)$

52. Solve $y = \frac{2x+5}{7+x}$ for x:

$$\begin{aligned} y(7+x) &= 2x+5 \\ xy + 7y &= 2x+5 \\ -2x + xy &= 5-7y \\ x(y-2) &= 5-7y \\ x &= \tfrac{5-7y}{y-2} \end{aligned}$$

Thus, $f^{-1}(x) = \frac{5-7x}{x-2}$.

Domains:

$dom(f) = rng(f^{-1}) = (-\infty, -7) \cup (-7, \infty)$

$rng(f) = dom(f^{-1}) = (-\infty, 2) \cup (2, \infty)$

53. Solve $y = \dfrac{1}{\sqrt{x}}$ for x:

$$\begin{aligned} y &= \frac{1}{\sqrt{x}} \\ y\sqrt{x} &= 1 \\ \sqrt{x} &= \frac{1}{y},\ y > 0 \\ x &= \frac{1}{y^2},\ y > 0 \end{aligned}$$

Thus, $f^{-1}(x) = \dfrac{1}{x^2}$

Domains:

$dom(f) = rng(f^{-1}) = (0, \infty)$

$rng(f) = dom(f^{-1}) = (0, \infty)$

54. Solve $y = \frac{x}{\sqrt{x+1}}$, $x > -1$, for x:

$$y = \frac{x}{\sqrt{x+1}}$$
$$y\sqrt{x+1} = x$$
$$y^2(x+1) = x^2$$
$$y^2 x + y^2 = x^2$$
$$x^2 - xy^2 - y^2 = 0$$
$$x = \frac{y^2 \pm \sqrt{y^4 + 4y^2}}{2}$$
$$x = \frac{y^2 \pm y\sqrt{y^2 + 4}}{2}$$

Since $y\sqrt{y^2+4} > y^2$, we know that eventually $y^2 - y\sqrt{y^2+4} \le -1$, which cannot occur because of the initial restriction on *x*. Hence,

$$f^{-1}(x) = \frac{x^2 \pm x\sqrt{x^2+4}}{2}$$

Domains:

$dom(f) = rng(f^{-1}) = (-1, \infty)$

$rng(f) = dom(f^{-1}) = (-\infty, \infty)$

55. Solve $y = \sqrt{\frac{x+1}{x-2}}$ for x:

$$y = \sqrt{\frac{x+1}{x-2}}$$
$$y^2 = \frac{x+1}{x-2}$$
$$y^2(x-2) = x+1$$
$$x(y^2 - 1) = 1 + 2y^2$$
$$x = \frac{2y^2+1}{y^2-1}$$

So, $f^{-1}(x) = \frac{2x^2+1}{x^2-1}$.

Note: We must have $\frac{x+1}{x-2} \ge 0$ for domain of *f*.

Domains:

$dom(f) = rng(f^{-1}) = (-\infty, -1] \cup (2, \infty)$

$rng(f) = dom(f^{-1}) = [0, 1) \cup (1, \infty)$

56. Solve $y=\sqrt{x^2-1},\ x\geq 1$ for x:

$$y=\sqrt{x^2-1},\ x\geq 1$$
$$y^2=x^2-1$$
$$y^2+1=x^2$$
$$x=\pm\sqrt{y^2+1}$$

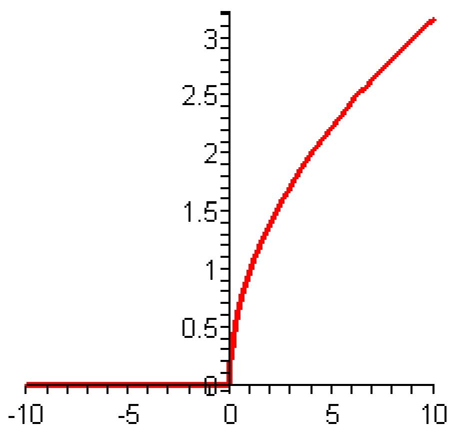
Since $y\geq 0$, we conclude that $f^{-1}(x)=\sqrt{x^2+1}$.

<u>Domains</u>:

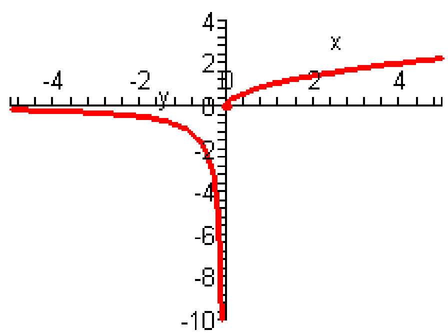
$dom(f)=rng(f^{-1})=[1,\infty)$

$rng(f)=dom(f^{-1})=[0,\infty)$

57. Not one-to-one



58. One-to-one



Calculate the inverse function piecewise:

For $x<0$: Solve $y=\frac{1}{x}$ for x:

$$y=\frac{1}{x}$$
$$xy=1$$
$$x=\frac{1}{y}$$

So, $G^{-1}(x)=\frac{1}{x}$ on $(-\infty,0)$.

For $x\geq 0$: Solve $y=\sqrt{x}$ for x:

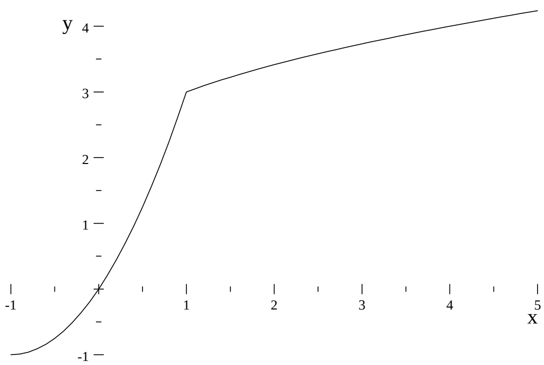
$$y=\sqrt{x}$$
$$x=y^2$$

So, $G^{-1}(x)=x^2$ on $(0,\infty)$.

Thus, the inverse function is given by:

$$G^{-1}(x)=\begin{cases}\frac{1}{x}, & x<0\\ x^2, & x\geq 0\end{cases}$$

59. One-to-one



Calculate the inverse piecewise:

For $x \le -1$: Solve $y = \sqrt[3]{x}$ for *x*.

$$y = \sqrt[3]{x}$$
$$x = y^3$$

So, $f^{-1}(x) = x^3$ on $(-\infty, -1)$.

For $-1 < x \le 1$: Solve $y = x^2 + 2x$ for *x*.

$$y = x^2 + 2x$$
$$y = x^2 + 2x + 1 - 1$$
$$y = (x+1)^2 - 1$$
$$y + 1 = (x+1)^2$$
$$\pm\sqrt{y+1} = x + 1$$
$$x = -1 \pm \sqrt{y+1}$$

Since *rng*(*f*) = dom(f^{-1}) = (-1, 3) on this portion, we see that $f^{-1}(x) = -1 + \sqrt{x+1}$ on (-1, 1).

For *x* > 1: Solve $y = \sqrt{x} + 2$ for *x*.

$$y = \sqrt{x} + 2$$
$$y - 2 = \sqrt{x} \quad .$$
$$x = (y-2)^2$$

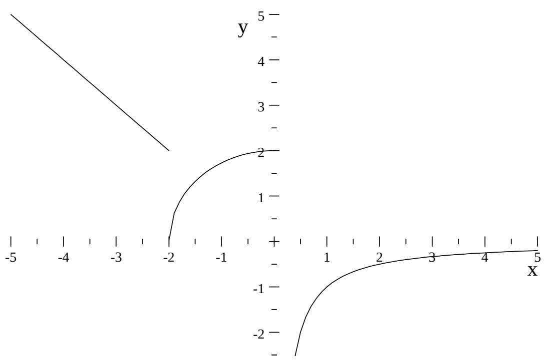
So, $f^{-1}(x) = (x-2)^2$ on $(1, \infty)$.

Thus,

$$f^{-1}(x) = \begin{cases} x^3, & x \le -1, \\ -1 + \sqrt{x+1}, & -1 < x \le 1, \\ (x-2)^2, & x > 1 \end{cases}$$

60.

One-to-one



Calculate the inverse piecewise:

For $x < -2$: Solve $y = -x$ for x.

$$y = -x$$
$$x = -y$$

So, $f^{-1}(x) = -x$ on $(2, \infty)$.

For $-2 < x \leq 0$: Solve $y = \sqrt{4 - x^2}$ for x.

$$y = \sqrt{4 - x^2}$$
$$y^2 = 4 - x^2$$
$$x^2 = 4 - y^2$$
$$x = \pm\sqrt{4 - y^2}$$

Since *rng*(*f*) = dom(f^{-1}) = [-2, 0] on this portion, we see that $f^{-1}(x) = -\sqrt{4 - x^2}$ on [0,2].

For $x > 0$: Solve $y = -\frac{1}{x}$ for x.

$$y = -\tfrac{1}{x}$$
$$x = -\tfrac{1}{y}.$$

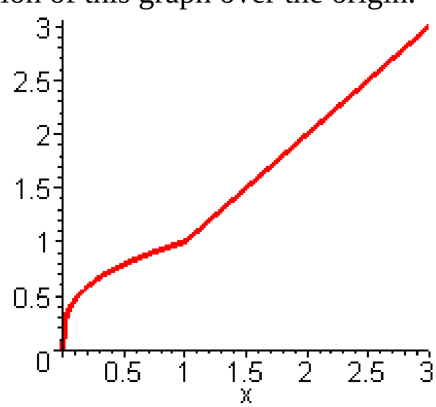
So, $f^{-1}(x) = -\frac{1}{x}$ on $(-\infty, 0)$.

Thus,

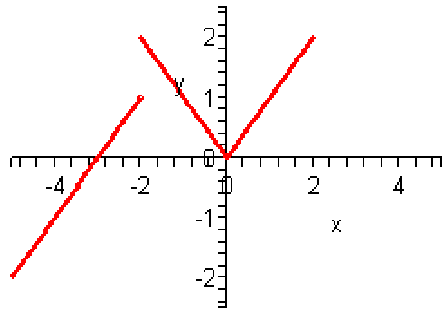
$$f^{-1}(x) = \begin{cases} -x, & x > 2, \\ -\sqrt{4 - x^2}, & 0 \leq x \leq 2, \\ -\frac{1}{x}, & x < 0 \end{cases}$$

61. One-to-one

The portion of the graph for non-negative *x* values is as follows. The graph for negative *x*-values is merely a reflection of this graph over the origin.



62. Not one-to-one



(Note: Should also have the graph of $y = x^2, x \geq 2$, above. The present curve would have an open hole at (2,2), and this newly-added piece would have a closed hole at (2,4) and extend upward to the right quadratically.)

63. Solve $y = \frac{9}{5}x + 32$ for *x*:

$$y - 32 = \frac{9}{5}x$$
$$\frac{5}{9}(y - 32) = x$$

So, $f^{-1}(x) = \frac{5}{9}(x - 32)$.

The inverse function represents the conversion from degrees Fahrenheit to degrees Celsius.

64. Solve $y = \frac{5}{9}(x - 32)$ for *x*:

$$\frac{9}{5}y = x - 32$$
$$\frac{9}{5}y + 32 = x$$

So, $C^{-1}(x) = \frac{9}{5}x + 32$. The inverse function represents the conversion from degrees Celsius to degrees Fahrenheit.

65. Let *x* = number of boats entered. The cost function is

$$C(x) = \begin{cases} 250x, & 0 \leq x \leq 10 \\ \underbrace{2500 + 175(x - 10)}_{= 175x + 750}, & x > 10 \end{cases}$$

To calculate $C^{-1}(x)$, we calculate the inverse of each piece separately:

For $0 \leq x \leq 10$: Solve $y = 250x$ for *x*: $x = \frac{y}{250}$. So, $C^{-1}(x) = \frac{x}{250}$, for $0 \leq x \leq 2500$.

For $x > 10$: Solve $y = 175x + 750$ for *x*: $x = \frac{y - 750}{175}$. So, $C^{-1}(x) = \frac{x - 750}{175}$, for $x > 2500$.

Thus, the inverse function is given by:

$$C^{-1}(x) = \begin{cases} \frac{x}{250}, & 0 \leq x \leq 2500 \\ \frac{x - 750}{175}, & x > 2500 \end{cases}$$

66. Let x = number of long-distance minutes. The cost function is

$$C(x)=\begin{cases} 0.39x, & 0\le x\le 10 \\ \underbrace{3.9+0.12(x-10)}_{=\,0.12x+2.7}, & x>10 \end{cases}$$

To calculate $C^{-1}(x)$, we calculate the inverse of each piece separately:

For $0\le x\le 10$: Solve $y=0.39x$ for x: $x=\frac{y}{0.39}$. So, $C^{-1}(x)=\frac{x}{0.39}$, for $0\le x\le 3.9$.

For $x>10$: Solve $y=0.12x+2.7$ for x: $x=\frac{y-2.7}{0.12}$. So, $C^{-1}(x)=\frac{x-2.7}{0.12}$, for $x>3.9$.

Thus, the inverse function is given by:

$$C^{-1}(x)=\begin{cases} \frac{x}{0.39}, & 0\le x\le 3.9 \\ \frac{x-2.7}{0.12}, & x>3.9 \end{cases}$$

67. Let x = number of hours worked. Then, the take home pay is given by

$$E(x)=\underbrace{7x}_{\substack{\$7\text{ per hour,}\\ \text{for } x \text{ hours}}}-\underbrace{0.25(7x)}_{\substack{\text{Amount withheld}\\ \text{for taxes}}}=5.25x.$$

To calculate E^{-1}, solve $y=5.25x$ for x: $x=\frac{y}{5.25}$. So, $E^{-1}(x)=\frac{x}{5.25}$, $x\ge 0$.

The inverse function tells you how many hours you need to work to attain a certain take home pay.

68. Let x = number of hours worked.

Since the hourly rate for overtime pay is 1.5(8) = 12 dollars per hour, we see that the weekly earnings are described by the following function:

$$E(x)=\begin{cases} 8x, & 0\le x\le 40 \\ \underbrace{320}_{\substack{\text{Pay for first}\\ \text{40 hours}}}+\underbrace{12(x-40)}_{\substack{\text{Amount of overtime}\\ \text{pay}}}, & x>40 \end{cases}=\begin{cases} 8x, & 0\le x\le 40 \\ 12x-160, & x>40 \end{cases}$$

To calculate $E^{-1}(x)$, we calculate the inverse of each piece separately:

For $0\le x\le 40$: Solve $y=8x$ for x: $x=\frac{y}{8}$. So, $E^{-1}(x)=\frac{x}{8}$, for $0\le x\le 320$.

For $x>40$: Solve $y=12x-160$ for x: $x=\frac{y+160}{12}$. So, $E^{-1}(x)=\frac{x+160}{12}$, for $x>320$.

Thus, the inverse function is given by:

$$E^{-1}(x)=\begin{cases} \frac{x}{8}, & 0\le x\le 320 \\ \frac{x+160}{12}, & x>320 \end{cases}$$

The inverse function tells you how many hours you need to work to attain a certain take home pay.

69. The domain is [0, 24] since the function measures temperature for one day.

Since the function is increasing, the range is [$T(0)$, $T(24)$], which is [97.5528, 101.70].

70. Solve the following expression for t: $y = 0.0003(t-24)^3 + 101.70$:

$$y = 0.0003(t-24)^3 + 101.70$$
$$y - 101.70 = 0.0003(t-24)^3$$
$$\frac{y-101.70}{0.0003} = (t-24)^3$$
$$\sqrt[3]{\frac{y-101.70}{0.0003}} = t - 24$$
$$t = 24 + \sqrt[3]{\frac{y-101.70}{0.0003}}$$

Thus, $T^{-1}(t) = 24 + \sqrt[3]{\frac{t-101.70}{0.0003}} = 24 + \sqrt[3]{\frac{t-101.70}{\frac{3}{10,000}}} = 24 + \sqrt[3]{\frac{10,000\left(t-101.70\right)}{3}}$.

71. Domain of T^{-1} = Range of T = [97.5528, 101.70] (from #69)
Range of T^{-1} = Domain of T = [0, 24] (from #69)

72. $T^{-1}(99.5) = 24 + \sqrt[3]{\frac{99.5-101.70}{0.0003}} \approx 4.57$. So, this occurs at around 4:30am.

73. Not a function since the graph does not pass the vertical line test.

74. $dom\left(f^{-1}\right) = rng(f) = \left[0,\infty\right)$, not $\left[2,\infty\right)$.

75. False. In fact, no even function can be one-to-one since the condition $f(x) = f(-x)$ implies that the horizontal line test is violated.

76. False. The function $f(x) = 0$ is odd, but not one-to-one.

77. False. Consider $f(x) = x$. Then, $f^{-1}(x) = x$ also.

78. True. $dom(f)$ is inside $\left(-\infty,0\right)$ and $rng(f)$ is inside $\left(0,\infty\right)$. Since they are switched for f^{-1}, $dom(f^{-1})$ is inside $\left(0,\infty\right)$ and $rng(f^{-1})$ is inside $\left(-\infty,0\right)$. Thus, the graph of f^{-1} is in Quadrant IV.

79. $(b,0)$ since the x and y coordinates of all points on the graph of f are switched to get the corresponding points on the graph of f^{-1}.

80. If $(a,0)$ is on the graph of f, then $(0,a)$ is on the graph of f^{-1}. This is its y-intercept.

81. The equation of the unit circle is $x^2 + y^2 = 1$. The portion in Quadrant I is given by

$$y = \sqrt{1-x^2},\ 0 \le x \le 1 .$$

To calculate the inverse of this function, solve for x:

$$y^2 = 1 - x^2, \text{ which gives us } x = \sqrt{1-y^2}$$

So, $f^{-1}(x) = \sqrt{1-x^2}$, $0 \le x \le 1$. The domain and range of both are $[0,1]$.

82. Let $f(x) = \frac{c}{x},\ c \neq 0$. To calculate the inverse of this function, solve for x:

$$y = \tfrac{c}{x} \Rightarrow x = \tfrac{c}{y} \Rightarrow yx = c \Rightarrow y = \tfrac{c}{x}.$$

Thus, $f(x) = f^{-1}(x),\ x \neq 0$.

83. As long as $m \neq 0$ (that is, while the graph of *f* is not a horizontal line), it is one-to-one.

84. Assume $m \neq 0$. Then, solving $y = mx + b$ for *x* yields: $x = \frac{y-b}{m}$. So, the inverse of $f(x) = mx + b$ is $f^{-1}(x) = \frac{x-b}{m}$.

85. We know from earlier problems that rational functions of the form $g(x) = \dfrac{1}{x+b}$ are one-to-one. Further, if a rational function possesses two vertical asymptotes, it is impossible for it to be one-to-one. As such, we choose *a* such that the numerator cancels with one of the factors in the denominator. The choice of *a* that does this is *a* = 4.

Then, $f(x) = \dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x-2)(x+2)} = \dfrac{1}{x+2}$. Observe also that

$$dom\left(f\right) = rng\left(f^{-1}\right) = (-\infty,-2) \cup (-2,\infty)$$
$$dom\left(f^{-1}\right) = rng\left(f\right) = (-\infty,0) \cup (0,\infty)$$

To calculate $f^{-1}(x)$, solve $y = \dfrac{1}{x+2}$ for x:

$$y = \frac{1}{x+2} \Rightarrow y(x+2) = 1 \Rightarrow yx + 2y = 1 \Rightarrow x = \frac{1-2y}{y}$$

So, we conclude that $f^{-1}(x) = \dfrac{1-2x}{x}$.

86. The only point guaranteed to be on the graph of $f^{-1}(x)$ is (*b*, *a*).

87. **a.** Solve $y = 2x+1$ for x: $y = 2x+1 \Rightarrow x = \dfrac{y-1}{2}$. So, $f^{-1}(x) = \dfrac{x-1}{2}$.

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\left(2(x+h)+1\right) - (2x+1)}{h} = \dfrac{2x+2h+1-2x-1}{h} = 2$.

So, evaluating at $h = 0$ yields $f'(x) = 2$.

c. $\dfrac{f^{-1}(x+h) - f^{-1}(x)}{h} = \dfrac{\frac{1}{2}(x+h-1) - \frac{1}{2}(x-1)}{h} = \dfrac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{2} - \frac{1}{2}x + \frac{1}{2}}{h} = \frac{1}{2}$.

So, evaluating at $h = 0$ yields $\left(f^{-1}\right)'(x) = \frac{1}{2}$.

d. $\dfrac{1}{f'\left(f^{-1}(x)\right)} = \dfrac{1}{f'(\frac{1}{2})} = \dfrac{x-1}{2} = \left(f^{-1}\right)'(x)$

88. a. Solve $y = x^2,\ x > 0$ for x: $x = y^2 \Rightarrow x = \pm\sqrt{y}$. So, $f^{-1}(x) = \sqrt{x}$.

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2 - x^2}{h} = \dfrac{x^2 + 2hx + h^2 - x^2}{h} = \dfrac{h(2x+h)}{h} = 2x + h$.

So, evaluating at $h = 0$ yields $f'(x) = 2x$.

c.

$$\frac{f^{-1}(x+h)-f^{-1}(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

So, evaluating at $h = 0$ yields $\left(f^{-1}\right)'(x) = \frac{1}{2\sqrt{x}}$.

d. $\dfrac{1}{f'\left(f^{-1}(x)\right)} = \dfrac{1}{f'(\sqrt{x})} = \dfrac{1}{2\sqrt{x}} = \left(f^{-1}\right)'(x)$

89. a. Solve $y = \sqrt{x+2},\ x > -2$ for x: $y = \sqrt{x+2} \Rightarrow y^2 = x + 2 \Rightarrow x = y^2 - 2$. So, $f^{-1}(x) = x^2 - 2,\ x \geq 0$.

b.

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} = \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$= \frac{x+h+2-x-2}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} = \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

So, evaluating at $h = 0$ yields $f'(x) = \frac{1}{2\sqrt{x+2}}$.

c.

$$\frac{\left(f^{-1}\right)(x+h)-\left(f^{-1}\right)(x)}{h} = \frac{\left[(x+h)^2 - 2\right]-\left(x^2-2\right)}{h} = \frac{x^2 + 2hx + h^2 - 2 - x^2 + 2}{h}$$

$$= \frac{h(2x+h)}{h} = 2x + h$$

So, evaluating at $h = 0$ yields $\left(f^{-1}\right)'(x) = 2x$.

d. $\dfrac{1}{f'\left(f^{-1}(x)\right)} = \dfrac{1}{f'(x^2-2)} = \dfrac{1}{\frac{1}{2\sqrt{\left(x^2-2\right)+2}}} = \dfrac{1}{\frac{1}{2x}} = 2x = \left(f^{-1}\right)'(x)$

90. **a.** Solve $y=\dfrac{1}{x+1},\ x>-1$ for *x*:

$$y=\frac{1}{x+1} \Rightarrow y(x+1)=1 \Rightarrow yx+y=1 \Rightarrow x=\frac{1-y}{y},\ y>0.\ \text{So, } f^{-1}(x)=\frac{1-x}{x},\ x>0.$$

b.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h}=\frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}=\frac{-h}{h(x+h+1)(x+1)}=\frac{-1}{(x+h+1)(x+1)}$$

So, evaluating at $h=0$ yields $f'(x)=-\frac{1}{(x+1)^2}$.

c.

$$\frac{(f^{-1})(x+h)-(f^{-1})(x)}{h}=\frac{\frac{1-(x+h)}{x+h}-\frac{1-x}{x}}{h}=\frac{[1-(x+h)]x-(1-x)(x+h)}{hx(x+h)}$$

$$=\frac{x-x^2-hx-x-h+x^2+xh}{hx(x+h)}=-\frac{1}{x(x+h)}$$

So, evaluating at $h=0$ yields $\left(f^{-1}\right)'(x)=\dfrac{-1}{x^2}$.

d. $$\frac{1}{f'\left(f^{-1}(x)\right)}=\frac{1}{f'\left(\frac{1-x}{x}\right)}=\frac{1}{\frac{-1}{\left(\frac{1-x}{x}+1\right)^2}}=-\left(\frac{1-x}{x}+1\right)^2=-\left(\frac{1}{x}\right)^2=\left(f^{-1}\right)'(x)$$

Chapter 1 Review Solutions ---

1. Yes		**2.** Yes	
3. No, since both $(0,6)$ and $(0,-6)$ satisfy the equation, so that the graph fails the vertical line test.		**4.** No, since the graph fails the vertical line test.	
5. Yes	**6.** Yes	**7.** No, since the graph fails the vertical line test.	**8.** Yes
9. **(a)** 2 **(b)** 4 **(c)** when $x=-3,\ 4$		**10.** **(a)** 2 **(b)** -2 **(c)** when $x\approx-2,3.2$	
11. **(a)** 0 **(b)** -2 **(c)** when $x\approx-5,2$		**12.** **(a)** 7 **(b)** -3 **(c)** never	
13. $f(3)=4(3)-7=\boxed{5}$		**14.** $F(4)=4^2+4(4)-3=\boxed{29}$	
15. $f(-7)\cdot g(3)=\left(4(-7)-7\right)\cdot\left|3^2+2(3)+4\right|$ $=-35\left|19\right|=\boxed{-665}$		**16.** $\dfrac{F(0)}{g(0)}=\boxed{-\dfrac{3}{4}}$	

17.

$$\frac{f(2)-F(2)}{g(0)} = \frac{\left(4(2)-7\right)-\left(2^2+4(2)-3\right)}{4}$$

$$\frac{1-9}{4} = \boxed{-2}$$

18. $f(3+h) = 4(3+h)-7 = \boxed{5+4h}$

19.

$$\frac{f(3+h)-f(3)}{h} = \frac{\left(4(3+h)-7\right)-\left(4(3)-7\right)}{h}$$

$$= \frac{5+4h-5}{h} = \boxed{4}$$

20.

$$\frac{F(t+h)-F(t)}{h} = \frac{\left((t+h)^2+4(t+h)-3\right)-\left(t^2+4t-3\right)}{h}$$

$$= \frac{t^2+2ht+h^2+4t+4h-3-t^2-4t+3}{h}$$

$$= \frac{2ht+h^2+4h}{h} = \frac{h\left(2t+h+4\right)}{h} = \boxed{2t+h+4}$$

21. $(-\infty,\infty)$

22. $(-\infty,\infty)$

23. $(-\infty,-4)\cup(-4,\infty)$

24. $(-\infty,\infty)$

25. We need $x-4\geq 0$, so the domain is $[4,\infty)$.

26. We need $2x-6>0$, so the domain is $(3,\infty)$.

27. Solve $2 = f(5) = \dfrac{D}{5^2-16}$ for D: $2=\frac{D}{9}$, so that $\boxed{D=18}$.

28. There are many such functions. The most natural one to construct has the form $f(x) = \dfrac{D}{(x+3)(x-2)}$. Since $(0,-4)$ is to lie on the graph of *f*, we substitute this point into the equation for the function to find the corresponding value of *D* that will ensure this: $-4 = \dfrac{D}{(0+3)(0-2)} = \dfrac{D}{-6}$, so that $D=24$. Hence, one such function is given by:

$$f(x) = \frac{24}{(x+3)(x-2)}.$$

29.

$h(-x) = (-x)^3-7(-x) = -\left(x^3-7x\right) \neq h(x)$

So, not even.

$-h(-x) = -\left(-\left(x^3-7x\right)\right) = x^3-7x = h(x)$

So, $\boxed{\text{odd}}$.

30.

$f(-x) = (-x)^4+3(-x)^2 = x^4+3x^2 = f(x)$

So, $\boxed{\text{even}}$.

Hence, cannot be odd.

31.

$$f(-x)=\frac{1}{(-x)^3}+3(-x)=-\left(\frac{1}{x^3}+3x\right)\neq f(x)$$

So, not even.

$$-f(-x)=-\left(-\left(\frac{1}{x^3}+3x\right)\right)=\frac{1}{x^3}+3x=f(x)$$

So, odd.

32.

$$f(-x)=\frac{1}{(-x)^2}+3(-x)^4+|-x|=f(x)$$

So, even.

Hence, f cannot be odd.

33.

Domain	$[-5,\infty)$
Range	$[-3,\infty)$
Increasing	$(-5,-3)\cup(3,\infty)$
Decreasing	$(-1,1)$
Constant	$(-3,1)\cup(1,3)$

d) 2
e) 3
f) 1

34.

Domain	$(-\infty,\infty)$
Range	$[-4,\infty)$
Increasing	$(-2,\infty)$
Decreasing	$(-\infty,-2)$
Constant	nowhere

d) 0
e) -3
f) 3

35.

Domain	$[-6,6]$
Range	$[0,3]\cup\{-3,-2,-1\}$
Increasing	$(0,3)$
Decreasing	$(3,6)$
Constant	$(-6,-4)\cup(-4,-2)\cup(-2,0)$

d) -1
e) -2
f) 3

36.

Domain	$[-6,6]$
Range	$[-3,1]$
Increasing	$(-3,1)\cup(2,3)\cup(4,5)$
Decreasing	$(-6,-3)\cup(1,2)\cup\ldots$ $\ldots(3,4)\cup(5,6)$
Constant	nowhere

d) 0
e) -3
f) 1

37.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[(x+h)^3-1\right]-\left[x^3-1\right]}{h}=\frac{x^3+3x^2h+3xh^2+h^3-1-x^3+1}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2\right)}{h}=\boxed{3x^2+3xh+h^2}$$

38.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{x+h-1}{x+h+2}-\frac{x-1}{x+2}}{h}=\frac{(x+h-1)(x+2)-(x-1)(x+h+2)}{h(x+h+2)(x+2)}$$

$$=\frac{\left(x^2+xh-x+2x+2h-2\right)-\left(x^2+hx+2x-x-h-2\right)}{h(x+h+2)(x+2)}$$

$$=\frac{3h}{h(x+h+2)(x+2)}=\boxed{\frac{3}{(x+h+2)(x+2)}}$$

39.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left(x+h+\frac{1}{x+h}\right)-\left(x+\frac{1}{x}\right)}{h}=\frac{h+\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=1+\frac{x-(x+h)}{hx(x+h)}=\boxed{1-\frac{1}{x(x+h)}}$$

40.

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{\frac{x+h}{x+h+1}}-\sqrt{\frac{x}{x+1}}}{h}$$

$$=\frac{\sqrt{x+h}\sqrt{x+1}-\sqrt{x}\sqrt{x+h+1}}{h\sqrt{x+h+1}\sqrt{x+1}}$$

$$=\frac{\sqrt{(x+h)(x+1)}-\sqrt{x(x+h+1)}}{h\sqrt{x+h+1}\sqrt{x+1}}$$

$$=\frac{\sqrt{(x+h)(x+1)}-\sqrt{x(x+h+1)}}{h\sqrt{x+h+1}\sqrt{x+1}}\cdot\frac{\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}}{\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}}$$

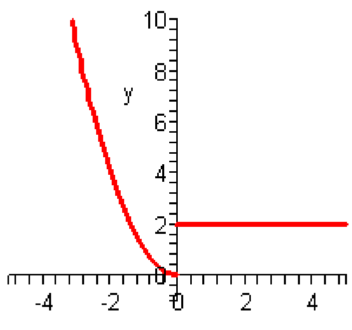
$$=\frac{x^2+x+h+xh-x^2-xh-x}{h\sqrt{x+h+1}\sqrt{x+1}\left(\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}\right)}$$

$$=\boxed{\frac{1}{\sqrt{x+h+1}\sqrt{x+1}\left(\sqrt{(x+h)(x+1)}+\sqrt{x(x+h+1)}\right)}}$$

41. $\dfrac{(4-2^2)-(4-0^2)}{2}=\boxed{-2}$

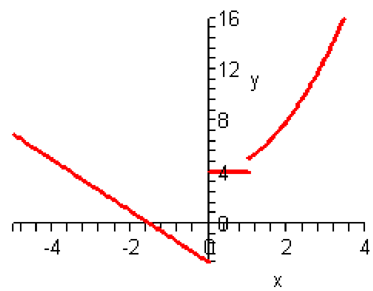
42. $\dfrac{|2(5)-1|-|2(1)-1|}{5-1}=\dfrac{9-1}{4}=\boxed{2}$

43. Domain: $(-\infty,\infty)$ Range: $(0,\infty)$



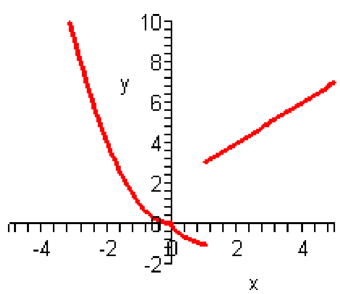
Notes on the graph: There is an open hole at (0,0), and a closed hole at (0,2).

44. Domain: $(-\infty,\infty)$ Range: $[-3,\infty)$



Notes on the graph: There are open holes at (0,4) and (1,5), and closed holes at (1,4) and $(0,-3)$.

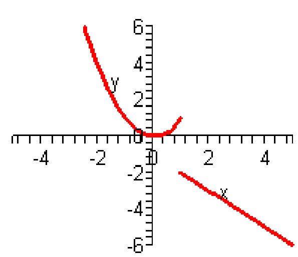
45. Domain: $(-\infty,\infty)$ Range: $[-1,\infty)$



Notes on the graph: There is an open hole at (1,3), and a closed hole at $(1,-1)$.

46. Domain: $(-\infty,0)\cup(0,\infty)$

Range: $(-\infty,-2]\cup(0,\infty)$

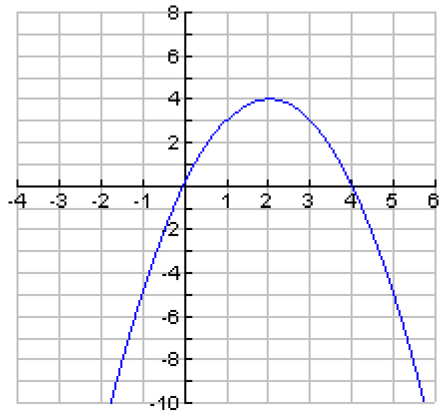


Notes on the graph: There are open holes at (0,0), (1,1), and a closed hole at $(1,-2)$.

47. $\frac{280,000-135,000}{5-0}=\frac{145,000}{5}=\boxed{\$29,000 \text{ per year}}$

48. $\frac{64-38}{10-0}=\frac{26}{10}=\boxed{2.6\%}$

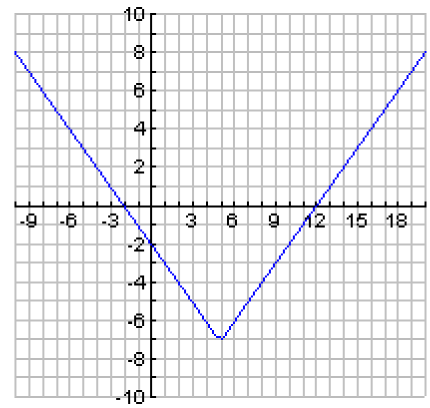
49. Reflect the graph of x^2 over x-axis, then shift right 2 units and then up 4 units.



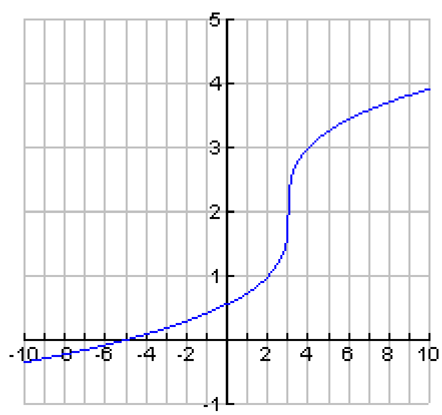
50. First, note that

$$|-x+5|-7=|-(x-5)|-7=|x-5|-7$$

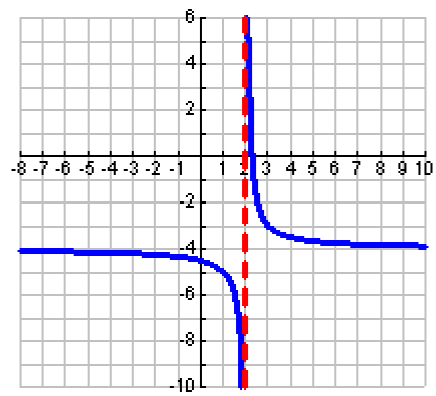
So, shift the graph of $|x|$ right 5 units and then down 7 units.



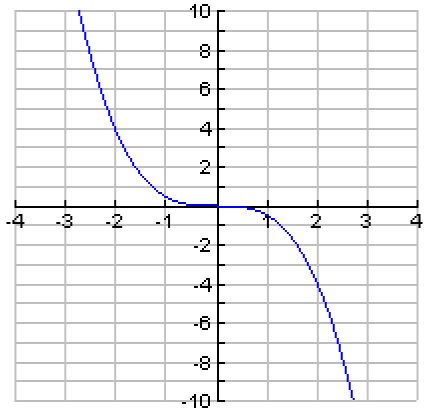
51. Shift the graph of $\sqrt[3]{x}$ right 3 units, and then up 2 units.



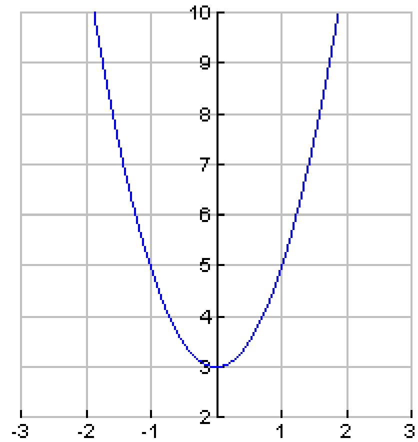
52. Shift the graph of $\frac{1}{x}$ right 2 units, and then down 4 units.



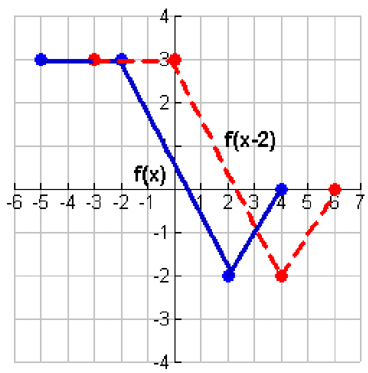
53. Reflect the graph of x^3 over x-axis, and then contract vertically by a factor of 2.



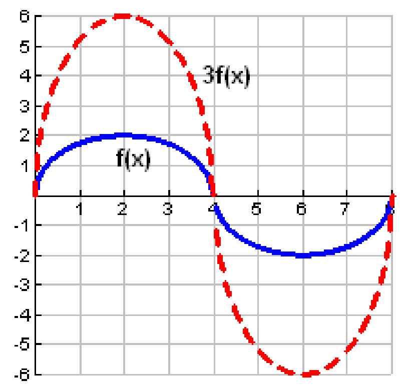
54. Expand the graph of x^2 vertically by a factor of 2, and then shift up 3 units.



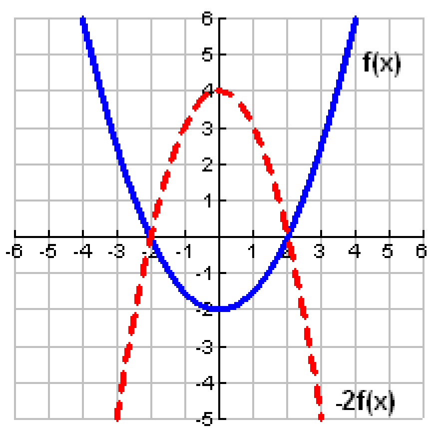
55.



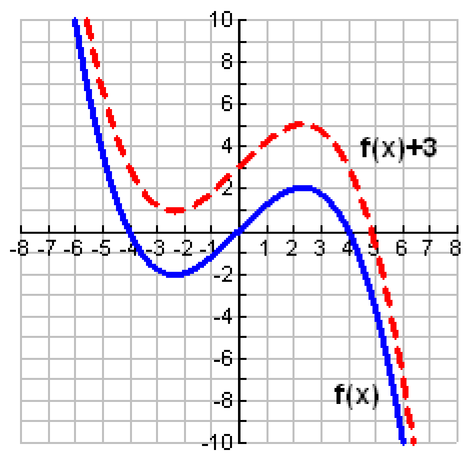
56.



57.



58.



59. $y=\sqrt{x+3}$ Domain: $[-3,\infty)$

60. $y=\sqrt{x}-4$ Domain: $[0,\infty)$

61. $y=\sqrt{x-2}+3$ Domain: $[2,\infty)$

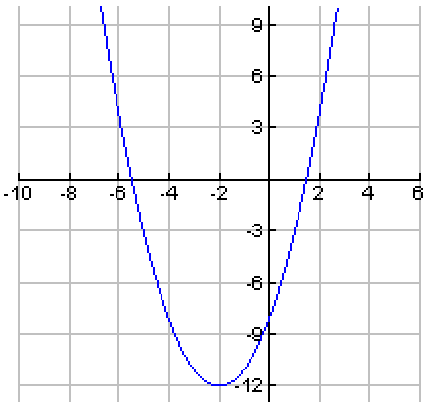
62. $y=\sqrt{-x}$ Domain: $(-\infty,0]$

63. $y=5\sqrt{x}-6$ Domain: $[0,\infty)$

64. $y=\frac{1}{2}\sqrt{x}+3$ Domain: $[0,\infty)$

65. $y=\left(x^2+4x+4\right)-8-4=\left(x+2\right)^2-12$

Domain: $\mathbb{R}$ or $(-\infty,\infty)$

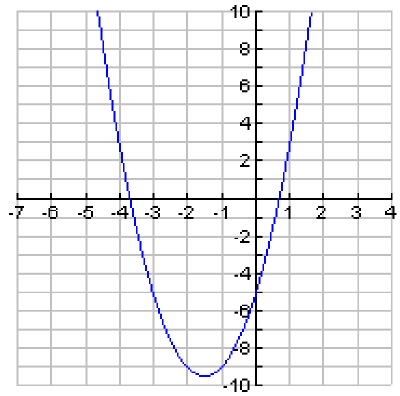


66.

$y=2\left(x^2+3x\right)-5=2\left(x^2+3x+\frac{9}{4}\right)-5-\frac{9}{2}$

$=2\left(x+\frac{3}{2}\right)^2-\frac{19}{2}$

Domain: $\mathbb{R}$ or $(-\infty,\infty)$



67.

$g(x)+h(x)=(-3x-4)+(x-3)=-2x-7$

$g(x)-h(x)=(-3x-4)-(x-3)=-4x-1$

$g(x)\cdot h(x)=(-3x-4)\cdot(x-3)=-3x^2+5x+12$

$\dfrac{g(x)}{h(x)}=\dfrac{-3x-4}{x-3}$

Domains:

$$\left.\begin{array}{r} dom(g+h) \\ dom(g-h) \\ dom(gh) \end{array}\right\}=(-\infty,\infty)$$

$$dom\left(\frac{g}{h}\right)=(-\infty,3)\cup(3,\infty)$$

68.

$g(x)+h(x)=(2x+3)+\left(x^2+6\right)=x^2+2x+9$

$g(x)-h(x)=(2x+3)-\left(x^2+6\right)=-x^2+2x-3$

$g(x)\cdot h(x)=(2x+3)\cdot\left(x^2+6\right)$

$=2x^3+3x^2+12x+18$

$\dfrac{g(x)}{h(x)}=\dfrac{2x+3}{x^2+6}$

Domains:

$$\left.\begin{array}{r} dom(g+h) \\ dom(g-h) \\ dom(gh) \\ dom\left(\frac{g}{h}\right) \end{array}\right\}=(-\infty,\infty)$$

69.

$$g(x)+h(x)=\tfrac{1}{x^2}+\sqrt{x}$$
$$g(x)-h(x)=\tfrac{1}{x^2}-\sqrt{x}$$
$$g(x)\cdot h(x)=\tfrac{1}{x^2}\cdot\sqrt{x}=\tfrac{1}{x^{3/2}}$$
$$\frac{g(x)}{h(x)}=\frac{\frac{1}{x^2}}{\sqrt{x}}=\frac{1}{x^{5/2}}$$

<u>Domains</u>:

$$\left.\begin{array}{r} dom(g+h) \\ dom(g-h) \\ dom(gh) \\ dom\left(\dfrac{g}{h}\right) \end{array}\right\}=(0,\infty)$$

70.

$$g(x)+h(x)=\frac{x+3}{2(x-2)}+\frac{3x-1}{x-2}$$
$$=\frac{(x+3)+2(3x-1)}{(2x-4)}$$
$$=\frac{7x+1}{2(x-2)}$$
$$g(x)-h(x)=\frac{x+3}{2(x-2)}-\frac{3x-1}{x-2}$$
$$=\frac{(x+3)-2(3x-1)}{(2x-4)}$$
$$=\frac{-5x+5}{2(x-2)}$$

$$g(x)\cdot h(x)=\frac{x+3}{2(x-2)}\cdot\frac{3x-1}{x-2}=\frac{(x+3)\cdot(3x-1)}{2(x-2)^2}$$

$$\frac{g(x)}{h(x)}=\frac{\frac{x+3}{2(x-2)}}{\frac{3x-1}{x-2}}=\frac{x+3}{2\cancel{(x-2)}}\cdot\frac{\cancel{x-2}}{3x-1}=\frac{x+3}{2(3x-1)}$$

<u>Domains</u>:

$$\left.\begin{array}{r} dom(g+h) \\ dom(g-h) \\ dom(gh) \end{array}\right\}=(-\infty,2)\cup(2,\infty)$$

$$dom\left(\frac{f}{g}\right)=\left(-\infty,\tfrac{1}{3}\right)\cup\left(\tfrac{1}{3},2\right)\cup(2,\infty)$$

71.

$$g(x)+h(x)=\sqrt{x-4}+\sqrt{2x+1}$$
$$g(x)-h(x)=\sqrt{x-4}-\sqrt{2x+1}$$
$$g(x)\cdot h(x)=\sqrt{x-4}\cdot\sqrt{2x+1}$$
$$\frac{g(x)}{h(x)}=\frac{\sqrt{x-4}}{\sqrt{2x+1}}$$

<u>Domains</u>:

Must have both $x-4\geq 0$ and $2x+1\geq 0$. So,

$$\left.\begin{array}{r} dom(f+g) \\ dom(f-g) \\ dom(fg) \end{array}\right\}=[4,\infty).$$

For the quotient, must have both $x-4\geq 0$ and $2x+1>0$. So,

$$dom\left(\frac{f}{g}\right)=[4,\infty).$$

72.

$$g(x)+h(x)=(x^2-4)+(x+2)=x^2+x-2$$
$$g(x)-h(x)=(x^2-4)-(x+2)=x^2-x-6$$
$$g(x)\cdot h(x)=(x^2-4)\cdot(x+2)$$
$$=x^3+2x^2-4x-8$$
$$\frac{g(x)}{h(x)}=\frac{x^2-4}{x+2}=x-2,\quad x\neq -2$$

<u>Domains</u>:

$$\left.\begin{array}{r} dom(g+h) \\ dom(g-h) \\ dom(gh) \end{array}\right\}=(-\infty,\infty)$$

$$dom\left(\frac{g}{h}\right)=(-\infty,-2)\cup(-2,\infty)$$

73.

$$(f\circ g)(x)=3\left(2x+1\right)-4=6x-1$$
$$(g\circ f)(x)=2\left(3x-4\right)+1=6x-7$$

Domains:

$dom(f\circ g)=\left(-\infty,\infty\right)=dom(g\circ f)$

74.

$$(f\circ g)(x)=\left(x+3\right)^3+2\left(x+3\right)-1$$
$$=x^3+9x^2+29x+32$$
$$(g\circ f)(x)=\left(x^3+2x-1\right)+3=x^3+2x+2$$

Domains:

$dom(f\circ g)=\left(-\infty,\infty\right)=dom(g\circ f)$

75.

$$(f\circ g)(x)=\frac{2}{\frac{1}{4-x}+3}=\frac{2}{\frac{1+3(4-x)}{4-x}}$$
$$=\frac{2(4-x)}{13-3x}=\frac{8-2x}{13-3x}$$
$$(g\circ f)(x)=\frac{1}{4-\frac{2}{x+3}}=\frac{1}{\frac{4(x+3)-2}{x+3}}=\frac{x+3}{4x+10}$$

Domains:

$dom(f\circ g)=\left(-\infty,4\right)\cup\left(4,\tfrac{13}{3}\right)\cup\left(\tfrac{13}{3},\infty\right)$

$dom(g\circ f)=\left(-\infty,-3\right)\cup\left(-3,-\tfrac{5}{2}\right)\cup\left(-\tfrac{5}{2},\infty\right)$

76.

$$(f\circ g)(x)=\sqrt{2\left(\sqrt{x+6}\right)^2-5}=\sqrt{2(x+6)-5}$$
$$=\sqrt{2x+7}$$
$$(g\circ f)(x)=\sqrt{\sqrt{2x^2-5}+6}$$

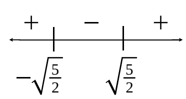
Domains:

$\underline{dom(f\circ g)}$: Need both $x+6\geq 0$ and $2x+7\geq 0$. Thus, $dom(f\circ g)=\left[-\tfrac{7}{2},\infty\right)$.

$\underline{dom(g\circ f)}$: Note $\sqrt{2x^2-5}+6\geq 0$, for all values of x for which $\sqrt{2x^2-5}$ is defined. This is true when $2x^2-5\geq 0$. So, solving this inequality yields:

$$2x^2-5\geq 0$$
$$x^2-\tfrac{5}{2}\geq 0$$
$$\left(x-\sqrt{\tfrac{5}{2}}\right)\left(x+\sqrt{\tfrac{5}{2}}\right)\geq 0$$

CPs: $\pm\sqrt{\tfrac{5}{2}}$



So, $dom(g\circ f)=\left(-\infty,-\sqrt{\tfrac{5}{2}}\right)\cup\left(\sqrt{\tfrac{5}{2}},\infty\right)$.

77.

$(f \circ g)(x) = \sqrt{x^2 - 4 - 5} = \sqrt{(x-3)(x+3)}$

$(g \circ f)(x) = \left(\sqrt{x-5}\right)^2 - 4 = x - 9$

Domains:

$dom(f \circ g)$: Need $(x-3)(x+3) \geq 0$.

CPs: ± 3

$+ \quad - \quad +$

$-3 \quad 3$

So, $dom(g \circ f) = (-\infty, -3] \cup [3, \infty)$.

$dom(g \circ f)$: Need $x - 5 \geq 0$. Thus,

$dom(g \circ f) = [5, \infty)$.

78.

$$(f \circ g)(x) = \frac{1}{\sqrt{\frac{1}{x^2-4}}} = \sqrt{x^2-4}$$

$$(g \circ f)(x) = \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^2 - 4} = \frac{1}{\frac{1}{x} - 4}$$

$$= \frac{1}{\frac{1-4x}{x}} = \frac{x}{1-4x}$$

Domains:

$dom(f \circ g)$: Need $(x-2)(x+2) > 0$.

CPs: ± 2

$+ \quad - \quad +$

$-2 \quad 2$

So, $dom(f \circ g) = (-\infty, -2) \cup (2, \infty)$.

$dom(g \circ f)$: Need $1 - 4x \neq 0$, so that $x \neq \frac{1}{4}$. So, $dom(g \circ f) = \left(0, \frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$.

79.

$g(3) = 6(3) - 3 = 15$

$f(g(3)) = f(15) = 4(15)^2 - 3(15) + 2 = 857$

$f(-1) = 4(-1)^2 - 3(-1) + 2 = 9$

$g(f(-1)) = g(9) = 6(9) - 3 = 51$

80.

$g(3) = 3^2 + 5 = 14$, but $f(g(3)) = f(14)$ is not defined.

$f(-1) = \sqrt{4 - (-1)} = \sqrt{5}$

$g(f(-1)) = g\left(\sqrt{5}\right) = \left(\sqrt{5}\right)^2 + 5 = 10$

81.

$g(3) = |5(3) + 2| = 17$

$$f(g(3)) = f(17) = \frac{17}{|2(17) - 3|} = \frac{17}{31}$$

$$f(-1) = \frac{-1}{|2(-1) - 3|} = -\frac{1}{5}$$

$$g(f(-1)) = g\left(-\frac{1}{5}\right) = \left|5\left(-\frac{1}{5}\right) + 2\right| = 1$$

82.

$g(3) = 3^2 - 1 = 8$

$$f(g(3)) = f(8) = \frac{1}{8-1} = \frac{1}{7}$$

$$f(-1) = \frac{1}{-1-1} = -\frac{1}{2}$$

$$g(f(-1)) = g\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - 1 = -\frac{3}{4}$$

83.

$$f(g(3)) = \left(\sqrt[3]{3-4}\right)^2 - \left(\sqrt[3]{3-4}\right) + 10$$

$$= (-1)^2 - (-1) + 10 = \boxed{12}$$

$$g(f(-1)) = g\left((-1)^2 + 1 + 10\right)$$

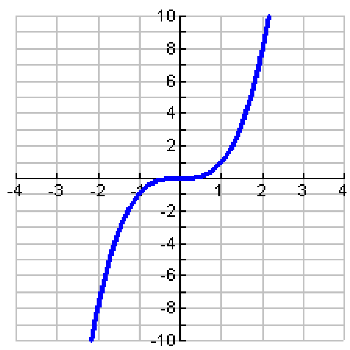
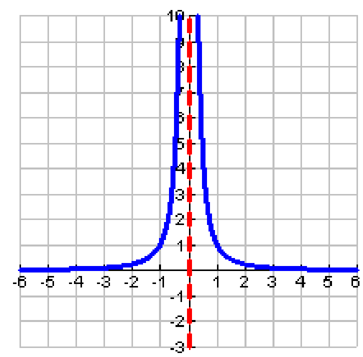
$$= \sqrt[3]{12 - 4} = \boxed{2}$$

84. $f(g(3))$ is $\boxed{\text{undefined}}$ since $g(3)$ is not defined.

$$g(f(-1)) = g\left(\frac{4}{(-1)^2 - 2}\right) = g(-4) = \boxed{\tfrac{1}{7}}$$

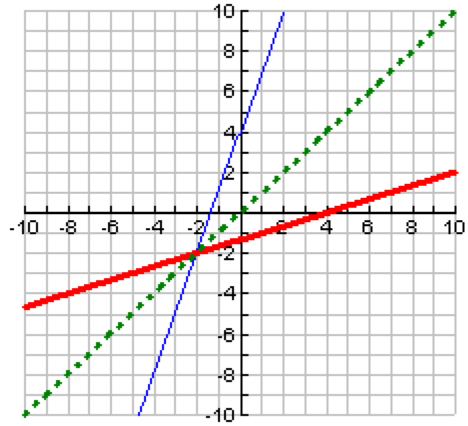
85. Let $f(x) = 3x^2 + 4x + 7$, $g(x) = x - 2$. Then, $h(x) = f(g(x))$.

86. Let $f(x) = \dfrac{x}{1-x}$, $g(x) = \sqrt[3]{x}$. Then, $h(x) = f(g(x))$.

87. Let $f(x)=\frac{1}{\sqrt{x}},\quad g(x)=x^2+7$. Then, $h(x)=f(g(x))$.	**88.** Let $f(x)=\sqrt{x},\quad g(x)=\lvert 3x+4\rvert$. Then, $h(x)=f(g(x))$.
89. The area of a circle with radius $r(t)$ is given by: $$A(t)=\pi\left(r(t)\right)^2=\pi\left(25\sqrt{t+2}\right)^2$$ $$=625\pi(t+2)\text{ in}^2$$	**90.** Since $42=lw,\ \ l=\frac{42}{w}$. So, the perimeter formula becomes: $$36=2l+2w=2\left(\tfrac{42}{w}\right)+2w=\frac{84+2w^2}{w}$$ so that $$2w^2-36w+84=0$$ $$w^2-18w+42=0$$
91. Yes	**92.** Yes
93. Yes	**94.** No, since both (1,1) and $(-1,1)$ satisfy the equation.
95. One-to-one  	**96.** Not one-to-one, since $f(-1)=f(1)=1$, for instance. 
97. Not one-to-one. No function that is a constant value on an entire value can be one-to-one.	**98.** Not one-to-one. For instance, $f(4)=f(-1)=0$.
99. One-to-one	**100.** One-to-one

101.

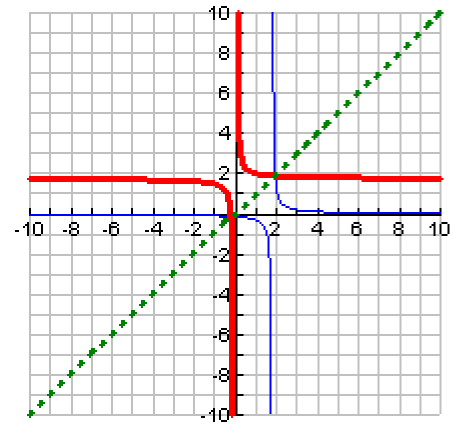
$$f\left(f^{-1}(x)\right) = \cancel{3}\left(\frac{x-4}{\cancel{3}}\right) + 4 = x - 4 + 4 = x$$



102.

$$f\left(f^{-1}(x)\right) = \frac{1}{4\left(\frac{1+7x}{4x}\right)-7} = \frac{1}{\frac{1+7x}{x}-7}$$

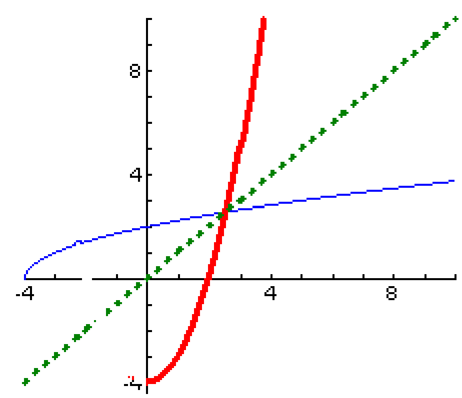
$$= \frac{1}{\frac{1}{x}+7-7} = \frac{1}{\frac{1}{x}} = x$$



103.

$$f\left(f^{-1}(x)\right) = \sqrt{\left(x^2-4\right)+4} = \sqrt{x^2} = x,$$

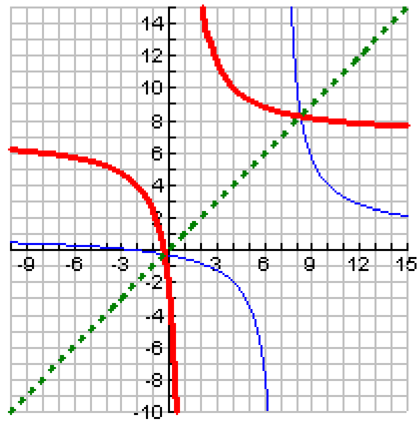
since $x \geq 0$.



104.

$$f\left(f^{-1}(x)\right)=\frac{\frac{7x+2}{x-1}+2}{\frac{7x+2}{x-1}-7}=\frac{\frac{7x+2+2(x-1)}{\cancel{x-1}}}{\frac{7x+2-7(x-1)}{\cancel{x-1}}}$$

$$=\frac{7x+\cancel{2}+2x-\cancel{2}}{\cancel{7x}+2-\cancel{7x}+7}=\frac{9x}{9}=x$$



105. Solve $y=2x+1$ for x:

$$x=\tfrac{1}{2}(y-1)$$

Thus, $f^{-1}(x)=\tfrac{1}{2}(x-1)=\dfrac{x-1}{2}$.

Domains:

$dom(f)=rng(f^{-1})=(-\infty,\infty)$

$rng(f)=dom(f^{-1})=(-\infty,\infty)$

106. Solve $y=x^5+2$ for x:

$$x=\sqrt[5]{y-2}$$

Thus, $f^{-1}(x)=\sqrt[5]{x-2}$.

Domains:

$dom(f)=rng(f^{-1})=(-\infty,\infty)$

$rng(f)=dom(f^{-1})=(-\infty,\infty)$

107. Solve $y=\sqrt{x+4}$ for x:

$$x=y^2-4$$

Thus, $f^{-1}(x)=x^2-4$.

Domains:

$dom(f)=rng(f^{-1})=[-4,\infty)$

$rng(f)=dom(f^{-1})=[0,\infty)$

108. Solve $y=(x+4)^2+3$ for x:

$$\sqrt{y-3}=x+4$$

$$-4+\sqrt{y-3}=x$$

Thus, $f^{-1}(x)=-4+\sqrt{x-3}$.

Domains:

$dom(f)=rng(f^{-1})=[-4,\infty)$

$rng(f)=dom(f^{-1})=[3,\infty)$

109. Solve $y=\frac{x+6}{x+3}$ for x:

$$(x+3)y=x+6$$

$$xy+3y=x+6$$

$$xy-x=6-3y$$

$$x(y-1)=6-3y$$

$$x=\tfrac{6-3y}{y-1}$$

Thus, $f^{-1}(x)=\frac{6-3x}{x-1}$.

Domains:

$dom(f)=rng(f^{-1})=(-\infty,-3)\cup(-3,\infty)$

$rng(f)=dom(f^{-1})=(-\infty,1)\cup(1,\infty)$

110. Solve $y = 2\sqrt[3]{x-5} - 8$ for *x*: $y + 8 = 2\sqrt[3]{x-5}$ $\left(\frac{1}{2}(y+8)\right)^3 = x - 5$ $5 + \left(\frac{1}{2}(y+8)\right)^3 = x$ Thus, $f^{-1}(x) = 5 + \left(\frac{1}{2}(x+8)\right)^3$. <u>Domains</u>: $dom(f) = rng(f^{-1}) = (-\infty, \infty)$ $rng(f) = dom(f^{-1}) = (-\infty, \infty)$
111. Let *x* = total dollars worth of products sold. Then, $S(x) = 22,000 + 0.08x$. Solving $y = 22,000 + 0.08x$ for *x* yields: $x = \frac{1}{0.08}(y - 22,000)$ Thus, $S^{-1}(x) = \frac{x - 22,000}{0.08}$. This inverse function tells you the sales required to earn a desired income.
112. $V(s) = 3s^2$, $s \geq 0$. Solving $y = 3s^2$ for *s* yields: $s = \sqrt{\frac{1}{3}y}$. So, $V^{-1}(s) = \sqrt{\frac{1}{3}s}$. This inverse function tells you the length *s* of a side of a base required to get a desired volume.

Chapter 1 Practice Test Solutions---

1. b (Not one-to-one since both (0,3) and $(-3,3)$ lie on the graph.)	**2. a** (Doesn't pass the vertical line test.)
3. c	**4.** Observe that $f(11) = \sqrt{11-2} = \sqrt{9} = 3$ $g(-1) = (-1)^2 + 11 = 12$ So, $f(11) - 2g(-1) = 3 - 2(12) = -21$.
5. $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{x^2+11}$ <u>Domain</u>: $[2, \infty)$	**6.** $\left(\frac{g}{f}\right)(x) = \frac{x^2+11}{\sqrt{x-2}}$ <u>Domain</u>: $(2, \infty)$
7. $g(f(x)) = \left(\sqrt{x-2}\right)^2 + 11 = x - 2 + 11 = x + 9$ Domain: $[2, \infty)$	**8.** $(f+g)(6) = f(6) + g(6)$ $= \sqrt{6-2} + \left(6^2 + 11\right) = 2 + 47 = \boxed{49}$
9. $f\left(g\left(\sqrt{7}\right)\right) = f\left(\left(\sqrt{7}\right)^2 + 11\right) = f(18)$ $= \sqrt{18-2} = \boxed{4}$	**10.** $f(-x) = \lvert -x \rvert - (-x)^2 = \lvert x \rvert - x^2 = f(x)$ So, $\boxed{\text{even}}$. Therefore, *f* cannot be odd.

11.

$$f(-x) = 9(-x)^3 + 5(-x) - 3$$
$$= -\left[9x^3 + 5x + 3\right] \neq f(x)$$

So, not even.

$$-f(-x) = -\left(-\left[9x^3 + 5x + 3\right]\right)$$
$$= 9x^3 + 5x + 3 \neq f(x)$$

So, not odd. Thus, neither.

12. $f(-x) = \frac{2}{-x} = -\frac{2}{x} = -f(x)$

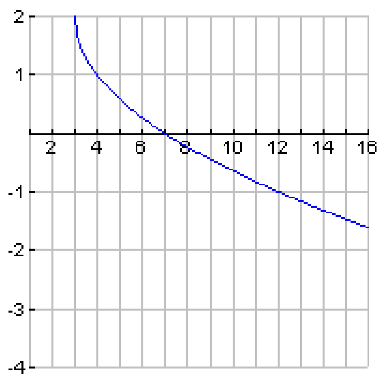
So, odd.

Therefore, f cannot be even.

13. $f(x) = -\sqrt{x-3} + 2$

Reflect the graph of $\sqrt{x}$ over the x-axis, then shift right 3 units, and then up 2 units.

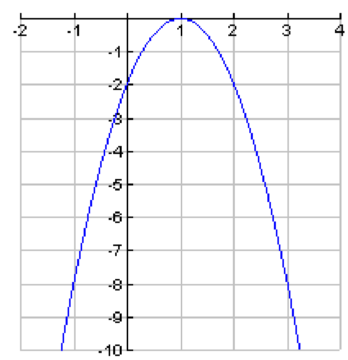
Domain: $[3, \infty)$ Range: $(-\infty, 2]$



14. $f(x) = -2(x-1)^2$

Reflect the graph of x^2 over the x-axis, then expand vertically by a factor of 2, and then shift right 1 unit.

Domain: $(-\infty, \infty)$ Range: $(-\infty, 0]$

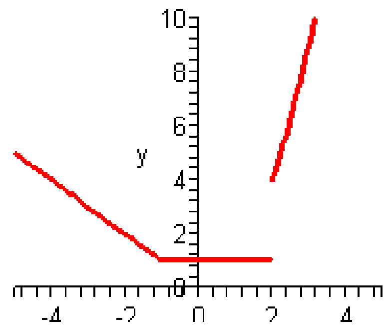


15. $f(x) = \begin{cases} -x, & x < -1 \\ 1, & -1 < x < 2 \\ x^2, & x \geq 2 \end{cases}$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $[1, \infty)$

Open holes at (-1,1) and (2,1); closed hole at (2,4)



16. **(a)** 3 **(b)** 1 **(c)** 6 **(d)** when $x = -1.5$, 2

(e) when $x = 3.5$

17.

(a) -2

(b) 4

(c) -3

(d) when $x = -3, 2$

18.

(a) -3

(b) never

(c) -1

(d) 1

19.

$$\frac{\left(3(x+h)^2-4(x+h)+1\right)-\left(3x^2-4x+1\right)}{h}=\frac{3x^2+6xh+3h^2-4x-4h+1-3x^2+4x-1}{h}$$

$$=\frac{h\left(6x+3h-4\right)}{h}=\boxed{6x+3h-4}$$

20.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[\left(x+h\right)^3-\frac{1}{\sqrt{x+h}}\right]-\left[x^3-\frac{1}{\sqrt{x}}\right]}{h}$$

$$=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}-\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$$

$$=\frac{h\left(3x^2+3xh+h^2\right)}{h}-\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$=\left(3x^2+3xh+h^2\right)-\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$

$$=\left(3x^2+3xh+h^2\right)-\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$=\boxed{\left(3x^2+3xh+h^2\right)+\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}}$$

21.

$$\frac{\left(64-16(2)^2\right)-\left(64-16(0)^2\right)}{2}=\frac{0-64}{2}=\boxed{-32}$$

22.

$$\frac{\sqrt{10-1}-\sqrt{2-1}}{10-2}=\frac{3-1}{8}=\boxed{\frac{1}{4}}$$

23. Solve $y=\sqrt{x-5}$ for x:

$$y^2=x-5$$

$$y^2+5=x$$

Thus, $f^{-1}(x)=x^2+5$.

Domains:

$dom\left(f\right)=rng\left(f^{-1}\right)=[5,\infty)$

$rng\left(f\right)=dom\left(f^{-1}\right)=[0,\infty)$

24. Solve $y=x^2+5$ for x:

$$y=x^2+5$$

$$\sqrt{y-5}=x,\ \text{since } x\geq 0.$$

Thus, $f^{-1}(x)=\sqrt{x-5}$.

Domains:

$dom\left(f\right)=rng\left(f^{-1}\right)=[0,\infty)$

$rng\left(f\right)=dom\left(f^{-1}\right)=[5,\infty)$

25. Solve $y = \frac{2x+1}{5-x}$ for x:

$$(5-x)y = 2x+1$$
$$5y - xy = 2x+1$$
$$5y - 1 = x(y+2)$$
$$x = \frac{5y-1}{y+2}$$

Thus, $f^{-1}(x) = \frac{5x-1}{x+2}$.

Domains:

$dom(f) = rng(f^{-1}) = (-\infty, 5) \cup (5, \infty)$

$rng(f) = dom(f^{-1}) = (-\infty, -2) \cup (-2, \infty)$

26. We compute the inverse of f piecewise:

For $x \le 0$: Solve $y = -x$ for x: $x = -y$. So, $f^{-1}(x) = -x$ on $(-\infty, 0]$.

For $x > 0$: Solve $y = -x^2$ (≤ 0) for x: $x = -\sqrt{-y}$. So, $f^{-1}(x) = -\sqrt{-x}$ on $(0, \infty)$.

Thus, the inverse function is given by

$$f^{-1}(x) = \begin{cases} -x, & x \ge 0 \\ -\sqrt{-x}, & x < 0 \end{cases}$$

27. Can restrict to $[0, \infty)$ so that *f* will have an inverse. Also, one could restrict to any interval of the form $[a, \infty)$ or $(-\infty, -a]$, where *a* is a positive real number, to ensure *f* is one-to-one.

28. The point $(5, -2)$ (switch *x* and *y* coordinates to get a point on the inverse.)

29. $\frac{\Delta P}{\Delta d} = \frac{28-10}{100-0} = 0.18 \ {}^{\text{psi}}\!/_{\text{ft.}}$ $\qquad \frac{\Delta d}{\Delta t} = \frac{5}{1} = 5 \ {}^{\text{ft}}\!/_{\text{sec.}}$

So, $\frac{\Delta P}{\Delta t} = \frac{\Delta P}{\Delta d} \cdot \frac{\Delta d}{\Delta t} = (0.18)(5) \ {}^{\text{psi}}\!/_{\text{sec.}} = 0.9 \ {}^{\text{psi}}\!/_{\text{sec.}}$ is the slope of the line.

Using (0, 10) as a point on the line, we see that the equation is $\boxed{P(t) = \frac{9}{10}t + 10}$.

30. Recall that $V = \frac{4}{3}\pi R^3$ **(1)** and $S = 4\pi R^2$ **(2)**

Solve **(2)** for R: $R = \sqrt{\frac{S}{4\pi}}$ Then, substitute this into **(1)**:

$$V = \frac{4}{3}\pi\left(\sqrt{\frac{S}{4\pi}}\right)^3 = \frac{S}{3}\sqrt{\frac{S}{4\pi}} = \frac{S}{6}\sqrt{\frac{S}{\pi}}$$

31. Consider $f(x) = -\sqrt{1-x^2}$, $-1 \le x \le 0$. (The graph of *f* is the quarter unit circle in the third quadrant.) To find its inverse, solve $y = -\sqrt{1-x^2}$ for x:

$y = -\sqrt{1-x^2} \ \Rightarrow \ (-y)^2 = 1-x^2 \ \Rightarrow \ x^2 = 1-y^2 \ \Rightarrow \ x = -\sqrt{1-y^2}$ since $-1 \le x \le 0$

So, $f^{-1}(x) = -\sqrt{1-x^2}$ (The graph looks identical to that of *f*.)

32. Solve $r(t)=15$ (At this point, the puddle just touches the sidelines.) $10\sqrt{t}=15$ $\sqrt{t}=1.5$ $t=(1.5)^2=2.25$ So, after 2.25 hours, the puddle will reach the sidelines.	**33.** Let x = number of minutes. Then, $C(x)=\begin{cases} 15, & 0\le x\le 30 \\ 15+\underbrace{1(x-30)}_{\text{Amount for minutes beyond the initial 30.}}, & x>30 \end{cases}$ $=\begin{cases} 15, & 0\le x\le 30 \\ x-15, & x>30 \end{cases}$

34. slope = $\dfrac{\Delta T}{\Delta CO_2}=\dfrac{46.23-45.86}{379.7-369.4}=\dfrac{0.37}{10.3}\approx 0.036$

Using the point (369.4, 45.86), we find that the equation of the line is

$$T(x)=0.036(x-369.4)+45.86\,.$$

As such, $T(375)=0.036(375-369.4)+45.86=46.1^{\circ}F$.