Complete Solutions Manual

Calculus for AP®
A Complete Course

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1.1 The Rule of Four

EXERCISE SOLUTIONS

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1. False — A relation is a function if and only if each element of the *domain* corresponds to exactly one element of the range.
2. True
3. True
4. True
5. False — This function has an exponent (¾) that is not a non-negative integer.
6. False — $f(g(x)) = f(g(x)) \neq f(x)\cdot g(x)$
7. False — $x^{a+b} = x^a \cdot x^b \neq x^a + x^b$
8. True
9. False — $f(1)$ is undefined, but $g(1) = 1$ so $f \neq g$.
10. A function that is described by a table of values is represented <u>numerically.</u>
11. A function is called <u>odd</u>, if for all x in the domain, $f(-x) = -f(x)$.
12. A curve in the coordinate plane is the graph of a function if and only if no <u>vertical line</u> intersects it more than once.
13. The domain of $f(x) = \dfrac{x}{x^2-4}$ is $\{x \in \mathbb{R} \mid x \neq -2, 2\}$.
14. The range of $f(x) = \sqrt{4-x^2}$ is $\{0 \le y \le 2\}$.
15. The function (a) $f(x) = \sqrt{x}$ is increasing for all x in its domain.
16. (a) $f(1) = 3$
    (b) $f(-1) \approx -0.2$
    (c) $f(x) = 1$ when $x = 0$ and $x = 3$.
    (d) $f(x) = 0$ when $x \approx -0.8$.
    (e) The domain of f is $\{-2 \le x \le 4\}$. The range of f is $\{-1 \le y \le 3\}$.
    (f) f is increasing on the interval $\{-2 \le x \le 1\}$.
17. (a) $f(-4) = -2$; $g(3) = 4$
    (b) $f(x) = g(x)$ when $x = -2$ and $x = 2$.
    (c) $f(x) = -1$ when $x \approx -3.4$.
    (d) f is decreasing on the interval $\{0 \le x \le 4\}$.
    (e) The domain of f is $\{-4 \le x \le 4\}$. The range of f is $\{-2 \le y \le 3\}$.
    (f) The domain of g is $\{-4 \le x \le 4\}$. The range of g is $\{0.5 \le y \le 4\}$.
18. (a) $f(2) = 12$ (b) $f(2) = 16$ (c) $f(a) = 3a^2 - a + 2$
    (d) $f(-a) = 3a^2 + a + 2$ (e) $f(a+1) = 3a^2 + 5a + 4$ (f) $2f(x) = 6a^2 - 2a + 4$
    (g) $f(2a) = 12a^2 - 2a + 2$ (h) $f(a^2) = 3a^4 - a^2 + 2$
    (i) $\left[f(a)\right]^2 = \left(3a^2 - a + 2\right)^2 = 9a^4 - 6a^3 + 13a^2 - 4a + 4$

(j) $f(a+h)=3\left(a+h\right)^2-\left(a+h\right)+2=3a^2+3h^2+6ah-a-h+2$

19. $\dfrac{f(3+h)-f(3)}{h}=\dfrac{(4+3(3+h)-(3+h)^2)-4}{h}=\dfrac{9+3h-9-6h-h^2)}{h}=\dfrac{-3h-h^2}{h}=-(3+h)$

20. $\dfrac{f(a+h)-f(a)}{h}=\dfrac{a^3+3a^2h+3ah^2+h^3-a^3}{h}=\dfrac{h\left(3a^2+3ah+h^2\right)}{h}=3a^2+3ah+h^2$

21. $\dfrac{f(x)-f(a)}{x-a}=\dfrac{\frac{1}{x}-\frac{1}{a}}{x-a}=\dfrac{\frac{a}{ax}-\frac{x}{ax}}{x-a}=\dfrac{a-x}{ax(x-a)}=-\dfrac{1}{ax}$

22. $\dfrac{f(x)-f(1)}{x-1}=\dfrac{\frac{x+3}{x+1}-\frac{1+3}{1+1}}{x-1}=\dfrac{\frac{x+3}{x+1}-2}{x-1}=\dfrac{\frac{x+3-2x-2}{x+1}}{x-1}=\dfrac{\frac{-x+1}{x+1}}{x-1}=-\dfrac{\frac{x-1}{x+1}}{x-1}=-\dfrac{1}{x+1}$

23. The domain of $f(x)=\dfrac{x+4}{x^2-9}$ is $\left\{x\in\mathbb{R}\mid x\neq -3,3\right\}$.

24. The domain of $f(x)=\dfrac{2x^3-5}{x^2+x-6}$ is $\left\{x\in\mathbb{R}\mid x\neq -3,2\right\}$.

25. The domain of $f(t)=\sqrt[3]{2t-1}$ is all real numbers.

26. The domain of $g(t)=\sqrt{3-t}-\sqrt{2-t}$ is $\left\{t\leq 2\right\}$.

27. The domain of $h(x)=-\dfrac{1}{\sqrt[4]{x^2-5x}}$ is $\left(-\infty,0\right)\cup\left(5,\infty\right)$.

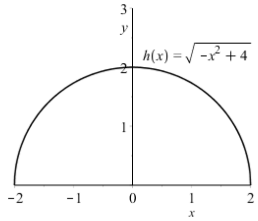
28. The domain of $F(p)=\sqrt{2-\sqrt{p}}$ is $\left\{0\leq p\leq 4\right\}$.

29. The domain of $f(u)=\dfrac{u+1}{1+\dfrac{1}{u+1}}$ is $\left\{u\in\mathbb{R}\mid u\neq -2,-1\right\}$.

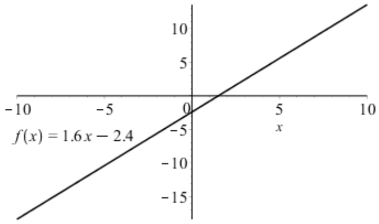
30. (a) This function shifts the graph of $y=|x|$ down two units and to the left one unit.
(b) This function shifts the graph of $y=|x|$ down two units
(c) This function reflects the graph of $y=|x|$ about the x-axis, shifts it up 3 units and then to the left 2 units.
(d) This function reflects the graph of $y=|x|$ about the x-axis and then shifts it up 4 units.
(e) This function reflects the graph of $y=|x|$ about the x-axis, shifts it up 2 units then four units to the left.
(f) This function is a parabola that opens up with vertex at (0, 5). It is not a transformation of $y=|x|$.

31. (a) $g(f(x))=10(x^2+1)$
(b) $f(g(4))=f(10\cdot 4)=40^2+1=1601$
(c) $g(g(-1))=g(10\cdot -1)=10(-10)=-100$
(d) $f(g(f(2)))=f(g(2^2+1))=f(10\cdot 5)=50^2+1=2501$
(e) $\dfrac{1}{f(g(x))}=\dfrac{1}{f(10x)}=\dfrac{1}{100x^2+1}$

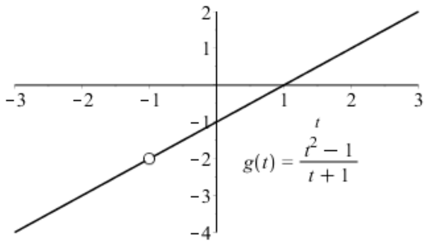
32. The domain of $h(x)=\sqrt{4-x^2}$ is $\{-2 \le x \le 2\}$, and the range is $\{0 \le y \le 2\}$. The graph is the top half of a circle of radius 2 with center at the origin.



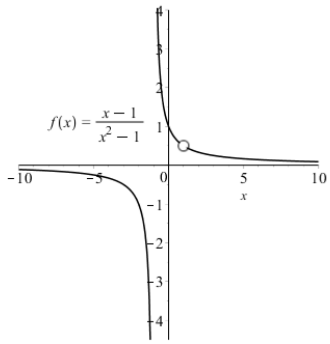
33. The domain of $f(x)=1.6x-2.4$ is all real numbers.



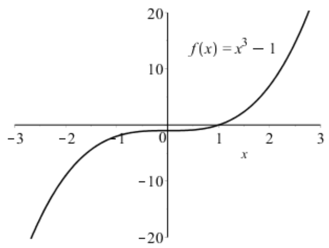
34. The domain of $g(t)=\dfrac{t^2-1}{t+1}$ is $\{t \in \mathbb{R} \mid t \ne -1\}$.



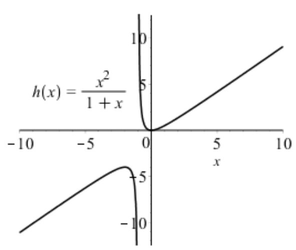
35. The domain of $f(x)=\dfrac{x-1}{x^2-1}$ is $\{x \in \mathbb{R} \mid x \ne -1, 1\}$.



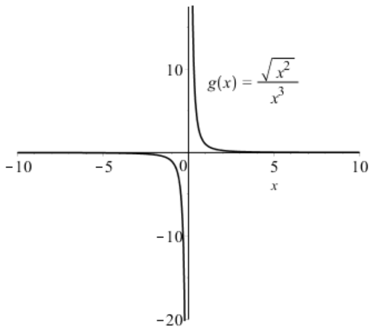
36. The domain of $f(x)=x^3-1$ is all real numbers.



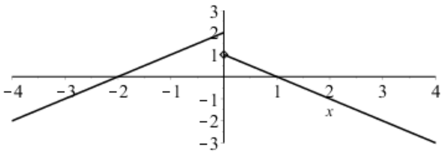
37. The domain of $h(x)=\dfrac{x^2}{x+1}$ is $\{x \in \mathbb{R} \mid x \ne -1\}$.



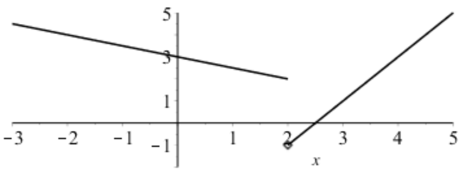
38. The domain of $g(x) = \dfrac{\sqrt{x^2}}{x^3}$ is $\{x \in \mathbb{R} \mid x \neq 0\}$.



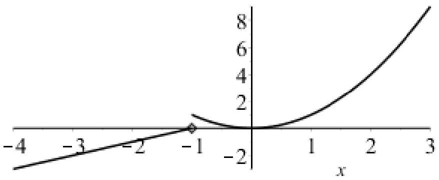
39. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$;
$f(-3) = -1;\ \ f(0) = 1;\ \ f(2) = -1.$



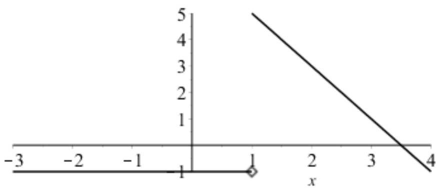
40. $f(x) = \begin{cases} 3-\frac{1}{2}x & \text{if } x < 2 \\ 2x-5 & \text{if } x \geq 2 \end{cases}$;
$f(-3) = \frac{9}{2};\ \ f(0) = 3;\ \ f(2) = -1.$



41. $f(x) = \begin{cases} x+1, & \text{if } x \leq -1 \\ x^2, & \text{if } x > -1 \end{cases}$
$f(-3) = -2;\ \ f(0) = 0;\ \ f(2) = 4.$

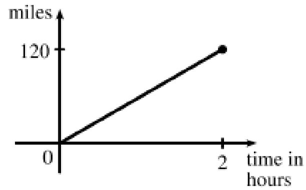


42. $f(x) = \begin{cases} -1 & \text{if } x \leq 1 \\ 7-2x & \text{if } x > 1 \end{cases}$;
$f(-3) = -1;\ \ f(0) = -1;\ \ f(2) = 3.$

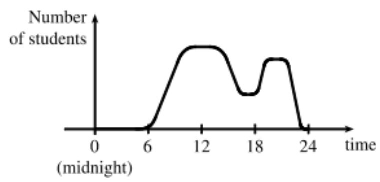


43. Using Figure 1.1, the range appears to be roughly $\{-100 \leq y < 150\}$.

44. *Example 1*: A car is driven at 60 mph for 2 hours. The distance *d* traveled by the car is a function of the time *t*. The domain of the function is $\{t \mid 0 \leq t \leq 2\}$, where *t* is measured in hours. The range of the function is $\{d \mid 0 \leq d \leq 120\}$, where *d* is measured in miles.

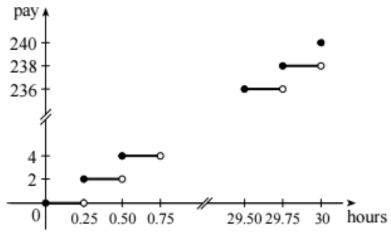


Example 2: At a certain university, the number of students *N* on campus at any time on a particular day is a function of the time *t* after midnight. The domain of the function is $\{t \mid 0 \leq t \leq 24\}$,

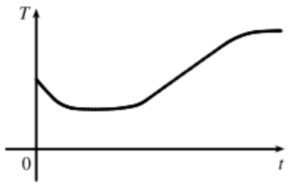


where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at one time.

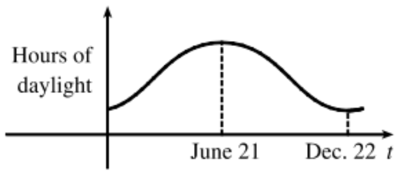
Example 3: A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is [0, 30], and the range of the function is $\{0, 2.00, 4.00, \ldots, 238.00, 240.00\}$.



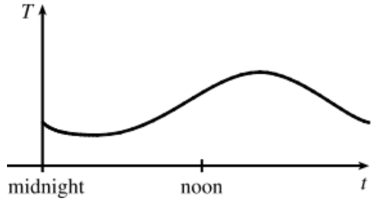
45. This is not the graph of a function because it does not pass the vertical line test.
46. This is the graph of a function. The domain is $\{-2 \le x \le 2\}$ and the range is $\{-1 \le y \le 3\}$.
47. This is the graph of a function. The domain is $\{-3 \le x \le 2\}$ and the range is $\{-3 \le y < -2\}$ $\cup \{-1 \le y \le 3\}$.
48. This is not the graph of a function because it does not pass the vertical line test.
49. (a) When $t = 1950$, $T \approx 13.8$°C, so the global average temperature in 1950 was about 13.8°C.
(b) When $T = 14.2$°C, $t \approx 1990$.
(c) The average global temperature was smallest in 1910 (the year corresponding to the lowest point on the graph) and the largest in year 2005 (the year corresponding to the highest point on the graph).
(d) When $t = 1910$, $T \approx 13.5$°C, and when $t = 2005$, $T \approx 14.5$°C. Thus the range of T is about [13.5, 14.5].
50. (a) The width varies from near 0 mm to about 1.6 mm, so the range of the ring width function is approximately [0, 1.6].
(b) According to the graph, the earth gradually cooled from 1550 to 1700, warmed into the late 1700s, cooled again into the late 1800s, and has been steadily warming since then. In the mid-19th century, there was variation that could have been associated with volcanic eruptions.
51. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



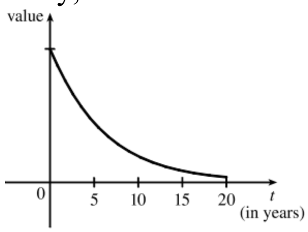
52. The graph indicates that runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. Runner B initially led the race, followed by C, and then A. Runner C passed runner B to lead for a while. Then runner A passed first runner B, then passed runner C to take the lead and finish first. All three runners finished the race.
53. (a) At 6 AM the power consumption was roughly 405 megawatts. At 6 PM the power consumption was roughly 5000 megawatts.
(b) The power consumption was lowest at roughly 3:30 AM. It was highest at about noon. These times do seem reasonable, considering the power consumption schedules of most individuals and businesses.
54. The summer solstice (longest day of the year) is around June 21, and the winter solstice (shortest day) is around December 22 (in the northern hemisphere). Therefore a reasonable graph for the number of hours of daylight vs. time of year is here:



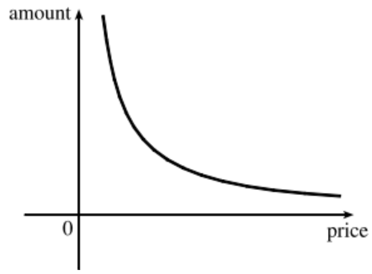
55. The graph will depend upon geographical location, but here is one graph of the outdoor temperature vs. time on a spring day:



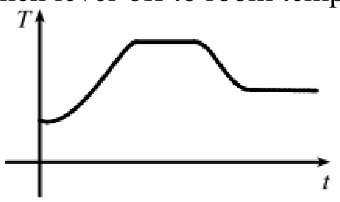
56. The value of the car will decrease rapidly initially, then somewhat less rapidly.



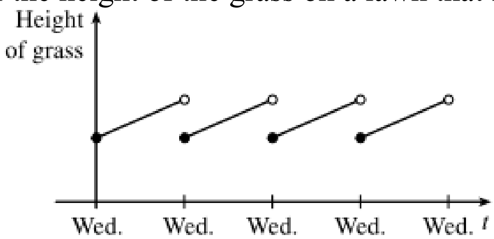
57. As the price increases, the amount of coffee sold will decrease:



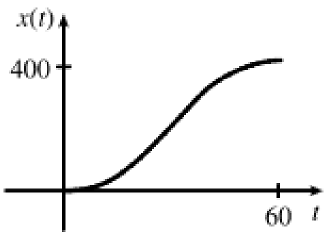
58. The temperature of the pie would increase rapidly, level-off to oven temperature, decrease rapidly when removed from the oven, and then level-off to room temperature:



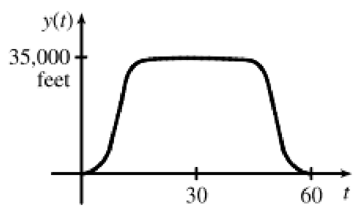
59. Here is a rough graph of the height of the grass on a lawn that is mown every Wednesday:



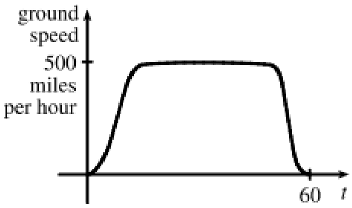
60. (a)



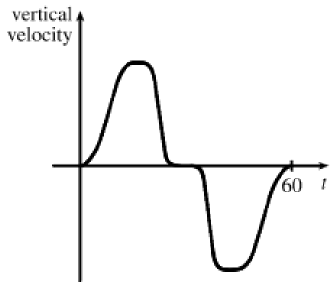
(b)



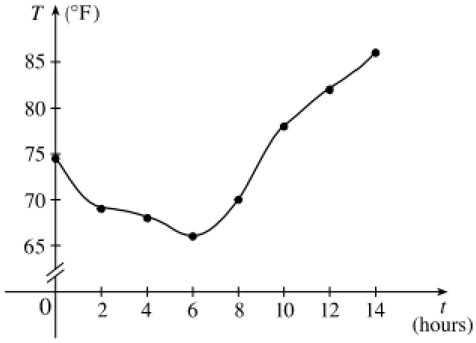
(c)



(d)

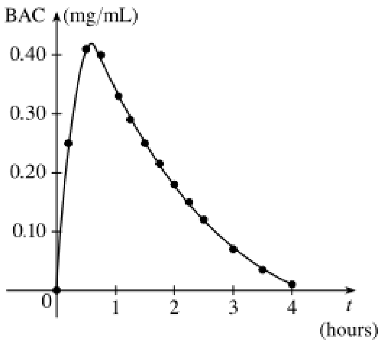


61. (a)



(b) The temperature at 9 AM appears to have been roughly75 °F.

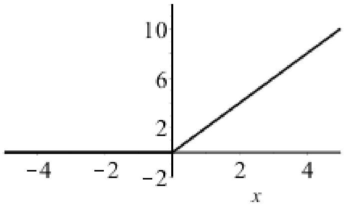
62. (a)



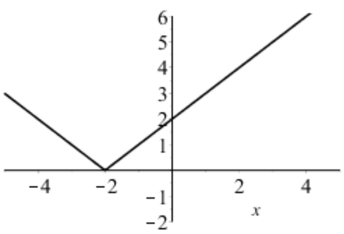
(b) The rate or BAC jumps rapidly from zero to the maximum of 0.41 mg/mL. The concentration then gradually decreases over the next four hours to near zero.

63. With radius r + 1, the balloon has volume $V(r+1)=\frac{4}{3}\pi(r+1)^3=\frac{4}{3}\pi(r^3+3r^2+3r+1)$. We wish to find the amount of air required to inflate the balloon from a radius of r to r+1 inches. Hence we need to find the difference $V(r+1)-V(r)=\frac{4}{3}\pi(3r^3++3r^2+3r+1)-\frac{4}{3}\pi r^3\ =\frac{4}{3}\pi(3r^2+3r+1)$.

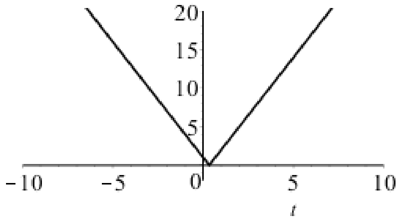
64. A graph of $f(x)=x+|x|$:



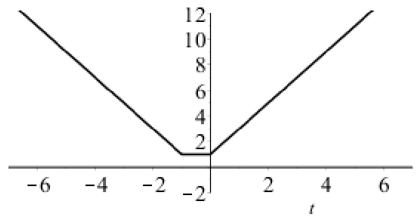
65. A graph of $f(x)=|x+2|$:



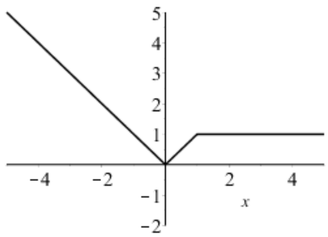
66. A graph of $g(t)=|1-3t|$:



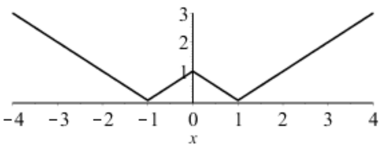
67. A graph of $h(t)=|t|+|t+1|$:



68. A graph of $f(x)=\begin{cases} |x| & \text{if } |x|\le 1 \\ 1 & \text{if } |x|>1 \end{cases}$:



69. A graph of $g(x)=\big||x|-1\big|$:



70. The slope of the line segment joining the points $(1,-3)$ and $(5,7)$ is $\dfrac{7-(-3)}{5-1}=\dfrac{5}{2}$ so a linear equation containing both points is $y-(-3)=\frac{5}{2}(x-1)$. The function is $f(x)=\frac{5}{2}x-\frac{11}{2},\ 1\le x\le 5$.

71. The slope of the line segment joining the points $(-5,10)$ and $(7,-10)$ is $-\dfrac{5}{3}$ so a linear equation containing both points is $y-10=-\frac{5}{3}(x-(-5))$. The function is $f(x)=-\frac{5}{3}x+\frac{5}{3},\ -5\le x\le 7$.

72. We need to solve the given equation for y: $x+\left(y-1\right)^2=0 \Leftrightarrow y-1=\pm\sqrt{-x} \Leftrightarrow y=1\pm\sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Therefore the function is $f(x)=1-\sqrt{-x}$, and the domain is $x\le 0$.

73. $x^2+\left(y-2\right)^2=4 \Leftrightarrow y-2=\pm\sqrt{4-x^2} \Leftrightarrow y=2\pm\sqrt{4-x^2}$. The top half of the circle is given by the function $f(x)=2+\sqrt{4-x^2}$, $-2\le x\le 2$.

74. For $0\le x\le 3$ the graph is a line with slope –1 and y-intercept 3. For $3<x\le 5$ the graph is a line with slope 2 that passes through the point (3, 0).

Thus the function is $f(x)=\begin{cases}-x+3, & \text{if } 0\le x\le 3\\ 2x-6, & \text{if } 3<x\le 5\end{cases}$.

75. For $-4\le x\le -2$ the graph is a line with slope –3/2 passing through the point (–2, 0). For $-2<x<2$ the graph is the top half of the circle with center (0, 0) and radius 2. For $2\le x\le 4$ the graph is the line with slope 3/2 passing through the point (2, 0). Therefore the function is

$$f(x)=\begin{cases}-\frac{3}{2}x-3, & \text{if } -4\le x\le -2\\ \sqrt{4-x^2}, & \text{if } -2<x<2\\ \frac{3}{2}x-3, & \text{if } 2\le x\le 4\end{cases}.$$

76. Let the length and width of the rectangle be l and w. Then the perimeter is $2l+2w=20$ and the area is $A=lw$. Solving the first equation for $w$in terms of l gives $w=\dfrac{20-2l}{2}=10-l$. Thus $A(l)=l(10-l)=10l-l^2$. Since the length must be positive, the domain of A is $0<l<10$. If we further restrict l to be larger than w, the $5<l<10$ would be the domain.

77. Let the length and width of the rectangle be l and w. Then the area is $lw=16$ so that $w=16/l$. The perimenter is $P=2l+2w$, so $P(l)=2l+2(16/l)=2l+32/l$, and the domain of P is $l>0$ since the lengths must be positive. If we further require l to be larger than w, then the domain would be $l>4$.

78. Let the length of a side of the triangle be x. Then by the Pythagorean Theorem, the height y of the triangle satistfies $y^2=\left(\frac{1}{2}x\right)^2$, so that $y^2=x^2-\frac{1}{4}x^2=\frac{3}{4}x^2$ and $y=\frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle, A = ½(base)(height), we find $A(x)=\frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right)=\frac{\sqrt{3}}{4}x^2$, with domain $x>0$.

79. Let the length, width, and height of the closed rectangular box be denoted by l, w, and h respectively. The length is twice the width, so $l=2w$. The volume V of the box is $V=lwh$. Since $V=8$, we have $8=(2w)wh\Rightarrow 8=2w^2h\Rightarrow h=\dfrac{8}{2w^2}=\dfrac{4}{w^2}$, so $h=f(w)=\dfrac{4}{w^2}$.

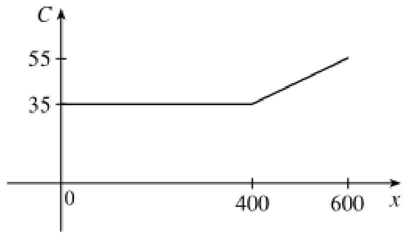
80. Let each side of the base of the box have length x, and let the height of the box be h. Since the volume is 2, we know that $2=hx^2$ so that $h=2/x^2$, and the surface area is $S=x^2+4xh$. Thus, $S(x)=x^2+4x\left(2/x^2\right)=x^2+8/x$, with domain $x>0$.

81. The area of the window is $A=xh+\frac{1}{2}\pi\left(\frac{1}{2}x\right)^2=xh+\dfrac{\pi x^2}{8}$, where h is the height of the rectangular portion of the window. The perimeter is $P=2h+x+\frac{1}{2}\pi x=30\iff 2h=30-x-\frac{1}{2}\pi x$ $\iff h=\frac{1}{2}(60-2x-\pi x)$. Thus, $A(x)=x\left(\dfrac{60-2x-\pi x}{4}\right)+\dfrac{\pi x^2}{8}=15x-\frac{1}{2}x^2-\frac{\pi}{4}x^2+\frac{\pi}{8}x^2$ $=15x-\frac{4}{8}x^2-\frac{\pi}{8}x^2=15x-x^2\left(\dfrac{\pi+4}{8}\right)$. Since the lengths x and h must be positive, we have $x>0$ and $h>0$. For $h>0$, we have $2h>0\iff 30-x-\frac{1}{2}\pi x>0\iff 60>2x+\pi x\iff x<\dfrac{60}{2+\pi}$. Therefore the domain of A is $0<x<\dfrac{60}{2+\pi}$.

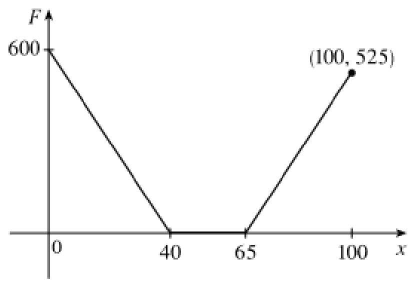
82. The height of the box is x and the length and width are $l = 20-2x,\ w = 12-2x.$ Then $V = lwx$ so $V(x) = (20-2x)(12-2x)x = 4(10-x)(6-x)x = 4x(60-16x+x^2) = 4x^3-64x^2+240x.$ Because the sides must have positive lengths, $l > 0 \Leftrightarrow 20-2x < 0 \Leftrightarrow x < 10;\ w > 0 \Leftrightarrow x < 6;$ and $x > 0$. Combining these restrictions indicates the domain is $0 < x < 6$.

83. We can model the monthly cost with a piecewise function:

$$C(x) = \begin{cases} \$35 & \text{if } 0 \le x \le 400 \\ \$35 + 0.1(x-400) & \text{if } x > 400 \end{cases}.$$

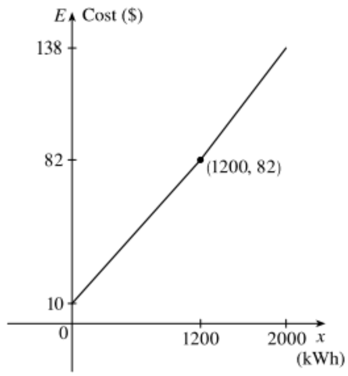


84. We can model the amount of the fine with a piecewise function: $F(x) = \begin{cases} 15(40-x), & \text{if } 0 \le x < 40 \\ 0, & \text{if } 40 \le x \le 65 \\ 15(x-65), & \text{if } 65 < x \le 100 \end{cases}.$

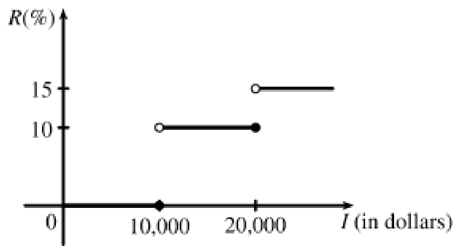


85. $E(x) = \begin{cases} \$10 + 0.06x, & \text{if } 0 \le x \le 1200 \\ \$10 + 0.06(1200) + 0.07(x-1200), & \text{if } x > 400 \end{cases}$ or

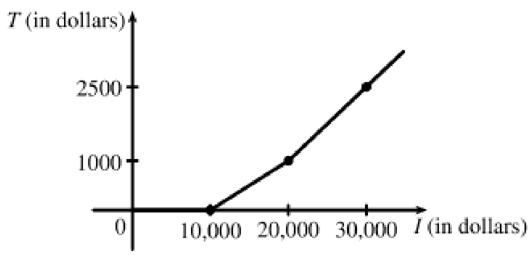
equivalently, $E(x) = \begin{cases} \$10 + 0.06x, & \text{if } 0 \le x \le 1200 \\ \$82 + 0.07(x-1200), & \text{if } x > 400 \end{cases}$



86. (a) Here is the graph:

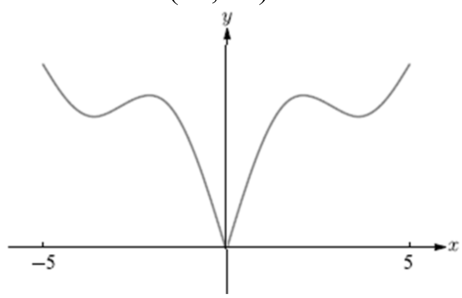


(b) On \$14,000 tax is assessed on \$4000, and 10% of \$4000 is \$400. On \$26,000, tax is assessed on \$16,000 and 10%(\$10,000) + 15%(\$6000) = \$1000 + \$900 = \$1900.
(c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of T is a line segment from (10,000, 0) to (20,000, 1000). The tax on \$30,000 is \$2500, so the graph of T for $x > 20{,}000$ is the ray with the

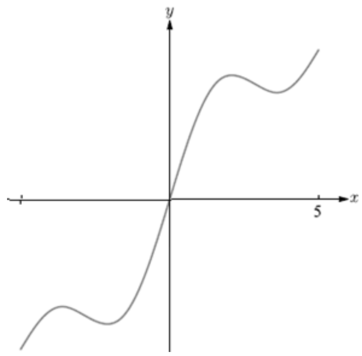


initial point (20,000, 1000) that passes through (30,000, 2500).

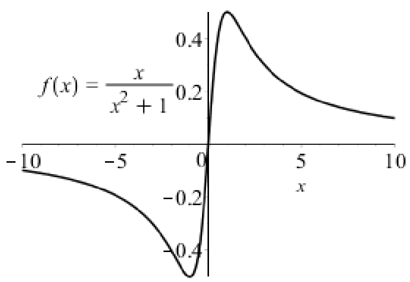
87. One example is the amount paid for cable or appliance repair in the home, usually measured to the nearest quarter hour. Another example is the amount paid by a student in college tuition fees, if the fees vary according to the number of credits for which the student registers.
88. The function *f* is odd because its graph is symmetric about the point (0, 0). The function *g* is even because its graph is symmetric across the *y*-axis.
89. The function *f* is even because its graph is symmetric across the *y*-axis. The function *g* is neither even nor odd because it is not symmetric about the *y*-axis or the origin.
90. (a) If the point (5, 3) is on the graph of an even function then (–5, 3) must also be on the graph.
(b) If the point (5, 3) is on the graph of an odd function then (–5, –3) must also be on the graph.
91. (a) Here is the complete graph if *f* is even:



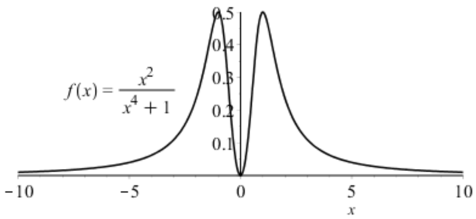
(b) Here is the complete graph if *f* is odd:



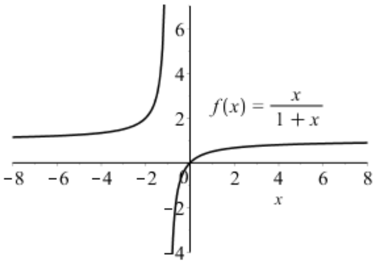
92. $f(x)=\dfrac{x}{x^2+1}$ is an *odd* function because $f(-x)=\dfrac{-x}{(-x)^2+1}=-f(x)$.



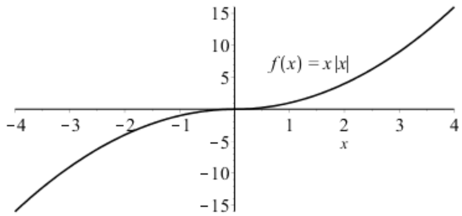
93. $f(x)=\dfrac{x^2}{x^4+1}=f(-x)$ is an even function.



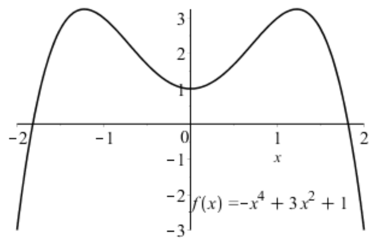
94. $f(x) = \dfrac{x}{x+1}$ is neither even nor odd.



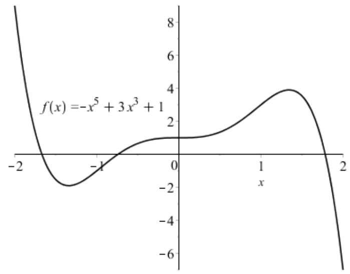
95. $f(x) = x|x|$ is an odd function because
$f(-x) = -x|-x| = -f(x)$.



96. $f(x) = 1 + 3x^2 - x^4 = 1 + 3(-x)^2 - (-x)^4 = f(-x)$ is an even function.



97. $f(x) = 1 + 3x^3 - x^5$ is neither even nor odd.



98. If f and g are both even functions then $f(-x) = f(x)$ and $g(-x) = g(x)$ so
$(f+g)(-x) = f(x) + g(x) = (f+g)(x)$ and therefore the sum $f + g$ is also *even*.
If f and g are both odd functions then $f(-x) = -f(x)$ and $g(-x) = -g(x)$ so
$(f+g)(-x) = -f(x) + -g(x) = -(f+g)(x)$ so the sum is also an *odd* function.
If f is even and g is odd then $f(-x) = f(x)$ and $g(-x) = -g(x)$ so the sum is
$(f+g)(-x) = f(x) + -g(x) = (f-g)(x)$. Therefore the sum is *neither* even nor odd.
(Exception: if f is the zero function, then $f + g$ will be *odd*. If g is the zero function, then $f + g$ will be *even*.

99. If f and g are both even functions then $f(-x) = f(x)$ and $g(-x) = g(x)$ so $fg(-x) = f(x)g(x) = fg(x)$ and the product fg is also *even*.
If f and g are both odd functions then $f(-x) = -f(x)$ and $g(-x) = -g(x)$ so

$fg(-x) = -f(x) \cdot -g(x) = fg(x)$. Therefore the product *fg* is an *even* function.

If *f* is even and *g* is odd then $f(-x) = f(x)$ and $g(-x) = -g(x)$ so the product is

$fg(-x) = f(x) \cdot -g(x) = -fg(x)$. Therefore the product is an *odd* function.

100. If *f* and *g* are both even then $f(-x) = f(x)$ and $g(-x) = g(x)$, so $f(g(-x)) = f(g(x))$ and $f \circ g$ is an *even* function.

If *f* and *g* are both odd then $f(-x) = -f(x)$ and $g(-x) = -g(x)$ so $f(g(-x)) = f(-g(x)) = -f(g(x))$ and $f \circ g$ is an *odd* function.

If *f* is even and *g* is odd then $f(-x) = f(x)$ and $g(-x) = -g(x)$ so

$f(g(-x)) = f(-g(x)) = f(g(x))$ so $f \circ g$ is an *even* function.

1.2 Mathematical Models: A Catalog of Essential Functions

EXERCISE SOLUTIONS

1. True
2. True
3. False
4. False
5. True
6. (a) $f(x)=\sqrt[4]{x}$ is a root function.

   (b) $g(x)=\dfrac{2x^3}{1-x^2}$ is a rational function.

   (c) $u(t)=1-2t+3t^2$ is a polynomial of degree 2.

   (d) $v(t)=t^7$ is a power function as well as a polynomial of degree 7.

   (e) $h(x)=x^5-7x^2+9$ is a polynomial of degree 5.

   (f) $k(x)=(x-3)^4(x+2)^2$ is a polynomial of degree 6.
7. (a) $y=x^\pi$ is an algebraic function.

   (b) $y=x^2\left(2-x^3\right)$ is a polynomial of degree 5.

   (c) $y=\dfrac{s}{1+s}$ is a rational function.

   (d) $y=\dfrac{\sqrt{x^3-1}}{1+\sqrt[3]{x}}$ is an algebraic function.

   (e) $y=x^{3/5}+7$ is an algebraic function.

   (f) $y=\sqrt{\dfrac{x^2}{x^2+1}}$ is an algebraic function.
8. (a) $y=x^2$ is the blue curve (function h) because it is a parabola.

   (b) $y=x^5$ is the red curve (function f) because it is an odd power function.

   (c) $y=x^8$ is the green curve (function g) because it is an even power function whose degree is more than 2.
9. (a) $y=3x$ is the blue curve (function f) because it is a linear function.

   (b) $y=x^3$ is the red curve (function g) because it is an odd power function.

   (c) $y=\sqrt[3]{x}$ is the green curve (function h) because it is the cube-root function.
10. The linear function that contains the points (1, 3) and (2, –1) (c) $y=-4x+7$.
11. The quadratic polynomial that contains the points $(1,3)$, $(2,4)$ and $(3,11)$ is (**A**)

    $y=2x^2-3x+2$.
12. (C) $h(x)=\dfrac{\sqrt{x}}{x^2-1}$ is not a rational function.

13. Reciprocal (**D**) does not describe the function $f(x) = x^6$ but rational, power and algebraic do.
14. The domain and range of $f(x) = 2x - 3$ are all real numbers.
15. The domain of $f(x) = x^2 + 4$ is all real numbers. The range is $\{y \in \mathbb{R} \mid y \ge 4\}$.
16. The domain of $g(x) = x^3 + 1$ is all real numbers. The range is also all real numbers.
17. The domain of $g(x) = \dfrac{x-2}{x^2-4}$ is $\{x \in \mathbb{R} \mid x \ne -2, 2\}$. The range is $\{y \in \mathbb{R} \mid y \ne 0\}$.
18. The domain of $h(x) = x^3 + 2x + 2$ is all real numbers. The range is also all real numbers.
19. The domain of $h(x) = \sqrt{x} + 2$ is $\{x \in \mathbb{R} \mid x \ge 0\}$. The range is $\{y \in \mathbb{R} \mid y \ge 2\}$.
20. The domain of $f(x) = \sqrt{x^2 + 4}$ is all real numbers. The range is $\{y \in \mathbb{R} \mid y \ge 2\}$.
21. The domain of $f(x) = \dfrac{1}{|x+2|}$ is $\{x \in \mathbb{R} \mid x \ne -2\}$. The range is $\{y \in \mathbb{R} \mid y > 0\}$.
22. The graph of $f(x) = \dfrac{x}{x+4}$ has a vertical asymptote at $x = -4$ and a horizontal asymptote at $y = 1$.
23. The graph of $f(x) = \dfrac{x^2}{x+3}$ has a vertical asymptote at $x = -3$ and no horizontal asymptotes.
24. The graph of $g(x) = \dfrac{x-2}{(x+2)(x-2)}$ has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 0$.
25. The graph of $g(x) = \dfrac{x}{x^3-1}$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 0$.
26. The graph of $h(x) = \dfrac{|x+2|}{x-2}$ has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 0$.
27. The graph of $h(x) = \dfrac{x+1}{x^3-1}$ has a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 0$.
28. By the Vertical Line Test the given graph is a function.
29. By the Vertical Line Test the given graph is a function.
30. By the Vertical Line Test the given graph is not a function.
31. By the Vertical Line Test the given graph is not a function.
32. The linear function such that $f(3) = 11$ and $f(7) = 9$ must have slope = $\dfrac{19-11}{7-3} = 2$. An equation for this line is $y = 2(x-3) + 11 = 2x + 5$.
33. If $f(x) = 3x + 5$ then $\dfrac{f(b) - f(a)}{b-a} = \dfrac{(3b+5) - (3a+5)}{b-a} = \dfrac{3b - 3a}{b-a} = 3\left(\dfrac{b-a}{b-a}\right) = 3$. This will always be the value because the rate of change (i.e. "slope") of a linear function is constant.
34. If $f(x) = 5x + 2$ then $\dfrac{f(x+h) - f(x)}{h} = \dfrac{5(x+h) + 2 - (5x+2)}{h} = \dfrac{5x + 5h + 2 - 5x - 2}{h} = \dfrac{5h}{h} = 5$.
35. The domain of $f(x) = 2(x-3)^2 + 5$ is all real numbers. The range is $\{y \in \mathbb{R} \mid y \ge 5\}$.
36. A quadratic function with range $\{y \mid y \le 6\}$ such that $f(4+d) = f(4-d)$ is $f(x) = 6 - (x-4)^2$ or

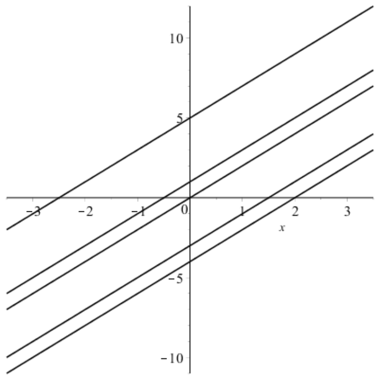
$f(x) = -x^2 + 8x - 10$. This function has a maximum of 6 that occurs when $x = 4$ and for all d, $f(4+d) = f(4-d) = 6 - d^2$.

37. A root function for which $g(64) = 4$ is $g(x) = \sqrt[3]{x}$.

38. A rational function with a linear denominator which has an x-intercept at 4 and a vertical asymptote at $x = 3$ is $f(x) = \dfrac{x-4}{x-3}$.

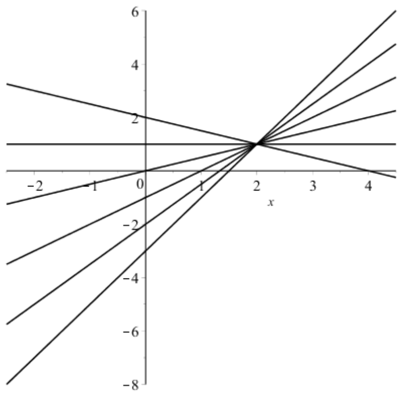
39. A rational function that is undefined at $x = 2$ but does not have a vertical asymptote at $x = 2$ is $f(x) = \dfrac{x^2-4}{x-2}$.

40. (a) An equation for the family of linear functions with slope 2 is $y = 2x + b$. To the right is a graph of several members of this family.

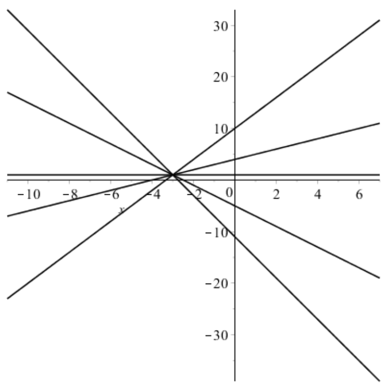


(b) An equation for the family of linear functions such that $f(2) = 1$ is $y = \frac{1-b}{2}x + b$. To the right is a graph of several members of this family.

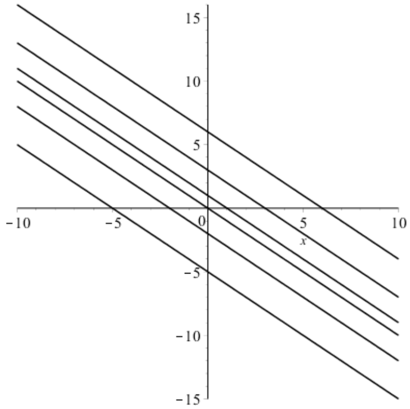
(c) The function $y = 2x - 3$ belongs to both families in (a) and (b).



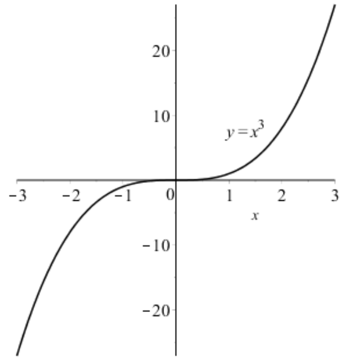
41. All members of the family of linear functions $f(x) = 1 + m(x+3)$ go through the point $(-3, 1)$. Here is a graph of several members of this family.



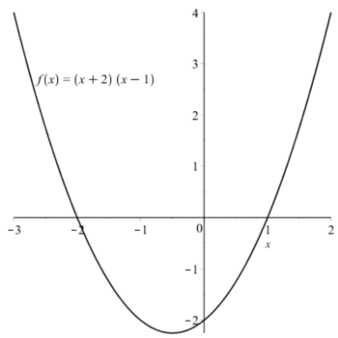
42. All members of the family of linear functions $f(x)=c-x.$ have a common slope of -1. Each has a y-intercept of (0, c). Below is a graph of several members of this family.



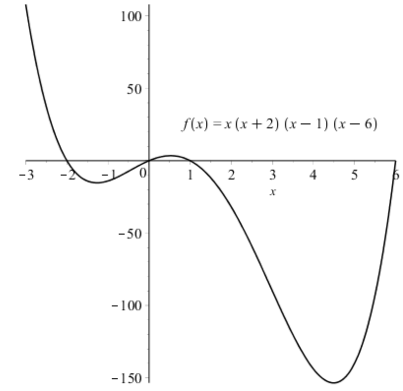
43. (a) A quadratic function whose graph has vertex $(0,3)$ and goes through the point $(4,2)$ is $f(x)=2(x-3)^2$.
(b) A quadratic function whose graph goes through the points $(-2,2)$, $(0,1)$, and $(1,-2.5)$, is $f(x)=-x^2-2.5x+1$.
44. One expression for a cubic function f such that $f(1)=6$, and $f(-1)=f(0)=f(2)=0$ is $f(x)=-3x(x-2)(x+1)$.
45. One cubic function that is increasing on the interval $(-\infty, \infty)$ is $f(x)=x^3$:



46. One quadratic function that intersects the x-axis at -2 is $f(x)=(x+2)(x-1)$:



47. One fourth degree polynomial that has zeros –2, 0, 1 and 6 is $f(x) = x(x+2)(x-1)(x-6)$.



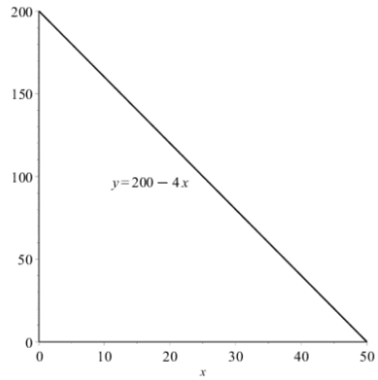
48. (a) In the equation $T = 0.02t + 8.50$, the slope of 0.02 indicates that for each additional year after 1900 the temperature is predicted to rise 0.02 °C. This is the rate of change in temperature with respect to time. The T-intercept of 8.5 °C represents the average temperature of the earth's surface in 1900.
(b) The predicted average global surface temperature in 2100 is $0.02(200) + 8.50 = 12.5$ °C.

49. (a) If $c = 0.0417 \cdot 200 \cdot (a+1)$ then the slope of the graph of c is 8.34 mg/year . The slope is the increase in the mg of dose for each additional year in the age of a child and represents the rate of change in a child's dose with respect to age.
(b) The dosage for a newborn child would be $c = 0.0417 \cdot 200 \cdot (0+1) = 8.34$ mg.

50. If $d = kv^2$ and $28 = k \cdot 20^2$ then $k = 0.07$ so $d = 0.07v^2$. Then a car traveling 40 mph requires $d(40) = 0.07 \cdot 40^2 = 112$ ft. to stop.

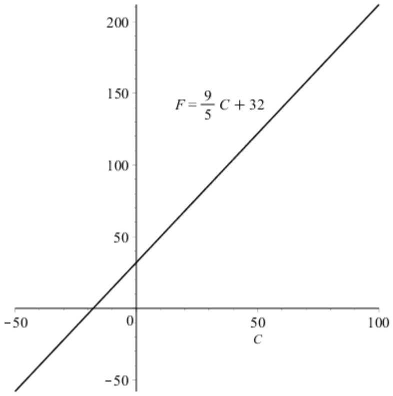
51. The slope for this linear model would be $\dfrac{\$7.25 - \$3.35}{2015 - 1981} = \dfrac{\$3.9}{34}$. A linear model for these data would be $y = \dfrac{\$3.9}{34}\left(x - 1981\right) + \$3.35$. The estimated minimum wage in 1996 would be $\dfrac{\$3.9}{34}\left(1996 - 1981\right) + \$3.35 = \$5.07$. This is \$0.32 more than the actual minimum wage in 1996.

52. (a) Here is a graph of the model $y = 200 - 4x$:

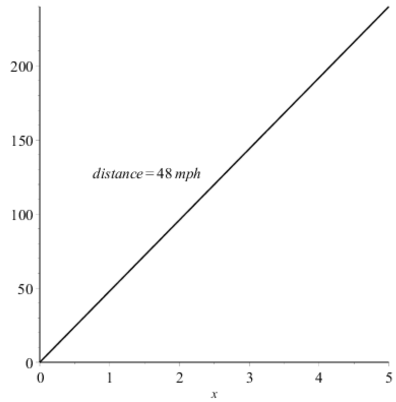


(b) The slope of this line (–1) indicates that each additional dollar charged for rent is predicted to result in a decrease of four spaces rented. This is the rate of change in the number of spaces with respect to the rent charged. The y-intercept of (0, 200) indicates that if he charges nothing, he can rent 200 total spaces. The x-intercept of (50, 0) indicates that if he charges \$50 per space, he will be able to rent no spaces. Both of these intercepts are unrealistic parts of the model.

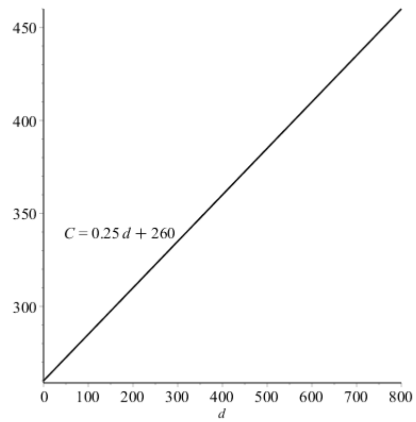
53. (a) Here is a graph of the function:
(b) The slope of this graph is 9/5. The slope indicates that each increase of one degree in the Celsius temperature will result in a 9/5 degree increase in the Fahrenheit temperature. The slope represents the rate of change in the Fahrenheit temperature with respect to the Celsius temperature. The *F*-intercept is (0,32) and it tells us that a temperature of 0 °C is equivalent to 32 °F.



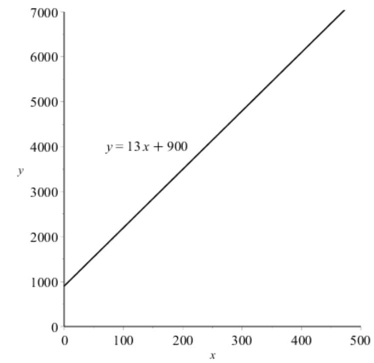
54. (a) Jason is driving at a constant rate of 40/50 = 4/5 miles per minute = 48 miles per hour.
(b) A graph of this equation shown.
(c) The slope of this line is 48 miles per hour. At this rate, each hour Jason will travel 48 miles.



55. (a) A linear equation for this model is
$$T = \left(\frac{80-70}{173-113}\right)(N-113)+70 = \tfrac{1}{6}(N-113)+70.$$
(b) The slope is ⅙ °F per chirps per minute. Each additional chirp per minute is predicted to occur at an increased temperature of ⅙ °F. The slope is the rate of change in temperature with respect to the number of chirps per hour.
(c) If the crickets are chirping at 150 chirps per minute, then the temperature is estimated to be $\frac{1}{6}(150-113)+70 = \frac{457}{6} = 76.1\overline{6}$ °F.



56. (a) Let y = cost of producing a chair and x = number of chairs. A linear function relating the cost and number of chairs produced is
$y = \$13(x - 100) + \$2200 = \$13x + \$900.$
(b) The slope is 13 and it indicates that the cost of producing each additional chair is \$13. The slope is the rate in change of the cost with respect to the number of chairs produced.
(c) The y-intercept is (0, \$900). This indicates that the cost of producing no chairs is \$900.



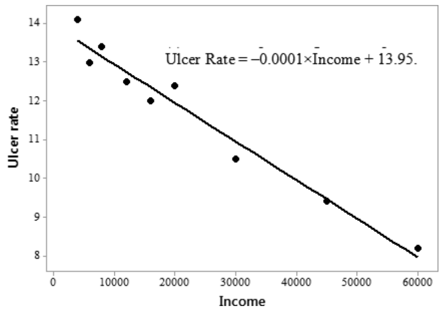
57. (a) $p = 0.434d + 15,$ where p = pressure and d = depth below the surface.
(b) The pressure is 100 lb/in^2 at a depth of 195.85 ft.

58. (a) $C = 0.25(d - 480) + 380 = 0.25d + 260.$
(b) The cost of driving 1500 miles per month is predicted to be \$635.
(c) The slope of \$0.25/mile indicates that each additional mile driven is predicted to cost an additional \$0.25 each month. The slope is the rate of change in the cost per month with respect to the number of miles driven.
(d) The C-intercept is (0, \$260) and it indicates that it costs Lynn \$260 each month to own her car without driving any miles.
(e) A linear function is a suitable model in this case because there are maintenance costs (\$260) each month regardless of the number of miles she drives, and the cost of per mile of driving is constant (\$0.25/mile).

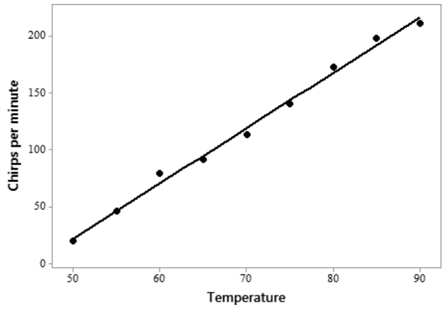
59. (a) A cosine function would provide the best model for this graph because the graph is oscillating and $y(0)$ is positive.
(b) A linear function with negative slope would provide the best model for this scatterplot because the data are roughly linear.

60. (a) A quadratic function would provide the best model for this graph because the graph is in the shape of a parabola.
(b) A power function with $n < 0$ would provide the best model for this scatterplot because the data are decreasing at a decreasing rate.

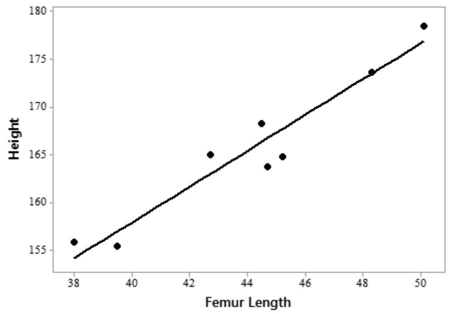
61. (a) The scatterplot indicates that a linear model would be appropriate.
(b) Using the first and last points we would compute a slope of
$$\frac{8.2 - 14.1}{60,000 - 4000} \approx -0.0001.$$
A linear equation for this model would be
Ulcer Rate = –0.0001(Income –4000) + 14.1.
(c) The least squares regression equation is
Ulcer Rate = –0.0001×Income + 13.95.
(d) The ulcer rate for an income of \$25,000 is predicted to be 11.45%.
(e) A person with an income of \$80,000 is predicted to have a 5.95% chance of peptic ulcers.



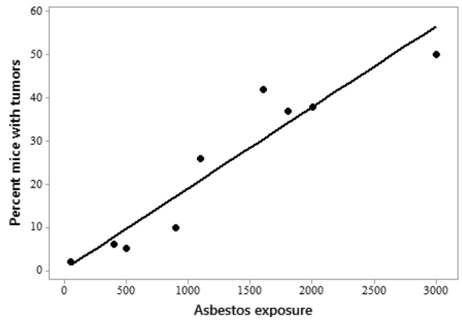
(f) An income of \$200,000 is quite far from the given income values so it would not be appropriate to apply the model in this case. If the model were applied, the resulting rate would be negative which makes no sense.



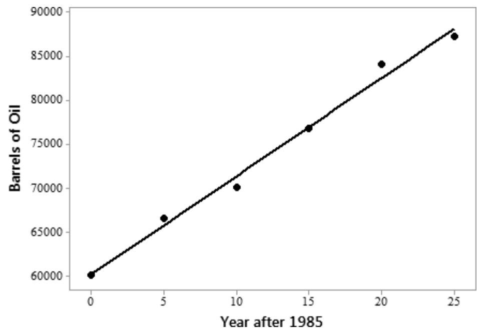
62. (a) Graph
(b) The regression equation is
Chirp rate = 4.857×Temperature – 221.
(c) The predicted chirping rate for an outdoor temperature of 100 °F is 48.57 – 221 = 264.7 chirps per minute.



63. (a) Graph
(b) The regression equation is
Height = 1.881 × Femur Length + 82.65.
(c) A person with a femur 53 cm long is predicted to have been 182.343 cm tall.

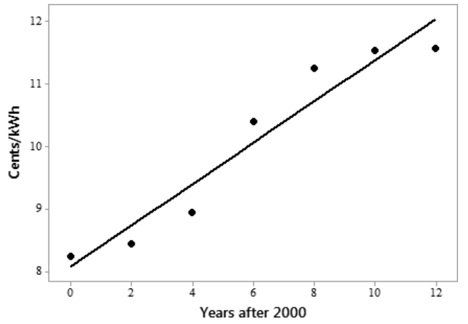


64. (a) The regression equation is
Percent Tumors = 0.01879×Asbestos Exposure + 0.305.
(b) The regression line does appear to be a suitable model for the data as the data is fairly linear.
(c) The y-intercept, (0, 0.305), indicates that if there is no asbestos exposure then 0.305% of the mice are predicted to develop lung tumors.



65. (a) A linear model is definitely appropriate as shown in the scatterplot in part (b).
(b) The regression equation is
Oil = 1117×(Year after 1985) + 60,188.
(c) The predicted oil consumption in 2002 is
$1117 \cdot 17 + 60{,}188 = 79{,}177$ thousand barrels.
The predicted oil consumption in 2012 is
$1117 \cdot 27 + 60{,}188 = 90{,}347$ thousand barrels.

66. (a) The scatterplot indicates a linear model is appropriate.
(b) The regression equation is
Cents/kWh $= 0.3309 \times$ (Year after 2000) +8.073.
(c) The predicted average retail price of electricity in 2005 is
$0.3309 \cdot 5 + 8.073 = 9.7275$.
The predicted average retail price of electricity in 2013 is
$0.3309 \cdot 13 + 8.073 = 12.3747$.



67. The light would be $\left(\frac{1}{2}\right)^{-2} = 4$ times brighter.

68. (a) $S(60) = 0.7 \cdot 60^{0.3} = 2.39$ so you would expect to find 2 species of bats living in that cave.

(b) $4 = S(A) = 0.7 \cdot A^{0.3} \Rightarrow 4/0.7 = A^{0.3}$. Then $\dfrac{\ln\left(\frac{4}{0.7}\right)}{0.3} = \ln A \Rightarrow e^{\frac{\ln\left(\frac{4}{0.7}\right)}{0.3}} = A \approx 333.585\ \text{m}^2$.

69. (a) One power model for these data is $T = d^{3/2}$.
(b) This model does agree with Kepler's Third Law because an equivalent equation is $T^2 = d^3$.

70. The function $f(x) = |x| + |x+3| + 2$ can also be written as $f(x) = \begin{cases} 2x+5, & x \geq 0 \\ 5, & -3 < x < 0 \\ -1-2x, & x \leq -3 \end{cases}$.

1.3 Trigonometric Functions

EXERCISE SOLUTIONS

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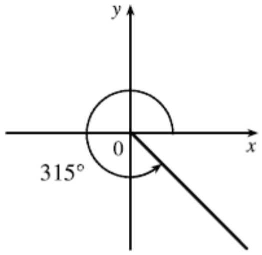
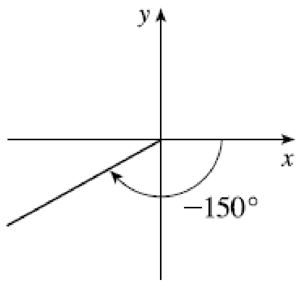
1. False $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ $V(t)=\begin{cases}0, & t<0\\ 2t, & 0\le t\le 60\end{cases}$ so $V(t)=2tH(t), t\le 60.$
2. True $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$
3. False $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
4. False The range of $y=\sin x$ is $[-1,1]$.
5. If $\sin\theta<0$ and $\tan\theta>0$, then θ lies in Quadrant III.
6. $210° = 210°\left(\dfrac{\pi}{180°}\right) = \dfrac{7\pi}{6}$ rad
7. $300° = 300°\left(\dfrac{\pi}{180°}\right) = \dfrac{5\pi}{3}$ rad
8. $9° = 9°\left(\dfrac{\pi}{180°}\right) = \dfrac{\pi}{20}$ rad
9. $-315° = -315°\left(\dfrac{\pi}{180°}\right) = -\dfrac{7\pi}{4}$
10. $900° = 900°\left(\dfrac{\pi}{180°}\right) = 5\pi$ rad
11. $36° = 36°\left(\dfrac{\pi}{180°}\right) = \dfrac{\pi}{5}$ rad
12. $4\pi \text{ rad} = 4\pi\left(\dfrac{180°}{\pi}\right) = 720°$
13. $-\dfrac{7\pi}{2} \text{ rad} = -\dfrac{7\pi}{2}\left(\dfrac{180°}{\pi}\right) = -630°$
14. $\dfrac{5\pi}{12} \text{ rad} = \dfrac{5\pi}{12}\left(\dfrac{180°}{\pi}\right) = 75^{\circ}$
15. $\dfrac{8\pi}{3} \text{ rad} = \dfrac{8\pi}{3}\left(\dfrac{180°}{\pi}\right) = 480°$
16. $-\dfrac{3\pi}{8} = -\dfrac{3\pi}{8}\left(\dfrac{180°}{\pi}\right) = -67.5°$
17. $5 \text{ rad} = 5\left(\dfrac{180°}{\pi}\right) = \left(\dfrac{900}{\pi}\right)^{\circ} \approx 286.48°$
18. $a = r\theta = 36\cdot\frac{\pi}{12} = 3\pi$ cm

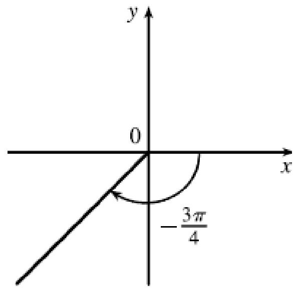
19. $a = r\theta = 10 \cdot 72°\left(\frac{\pi}{180°}\right) = 4\pi$ cm

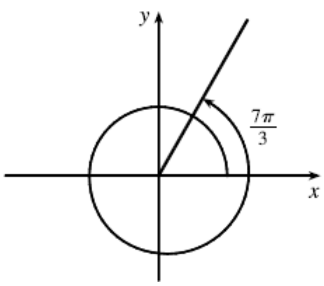
20. $\theta = a / r = \frac{1}{1.5} = \frac{2}{3}$ rad $= \frac{2}{3}\left(\frac{180°}{\pi}\right) = \left(\frac{120}{\pi}\right)^{\circ} \approx 38.2°$

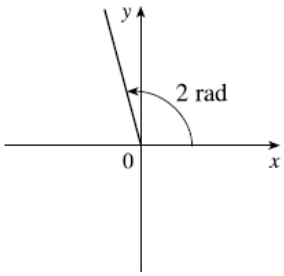
21. $a = r\theta \Rightarrow r = \frac{a}{\theta} = \frac{6}{3\pi/4} = \dfrac{8}{\pi}$ cm

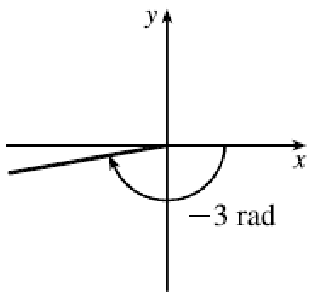
22. 

23. 

24. 

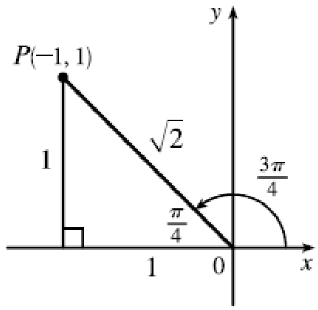
25. 

26. 

27. 

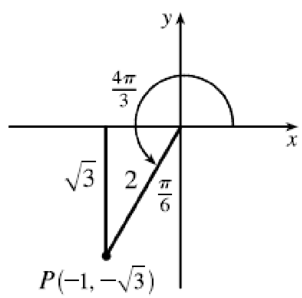
28. From the diagram we see that a point on the terminal side is $P(-1,1)$. Therefore, taking $x = -1, y = 1, r = \sqrt{2}$ in the definitions of the trigonometric ratios, we have

$$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}, \quad \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}, \quad \tan\frac{3\pi}{4} = -1,$$
$$\csc\frac{3\pi}{4} = \sqrt{2}, \quad \sec\frac{3\pi}{4} = -\sqrt{2}, \quad \text{and} \quad \cot\frac{3\pi}{4} = -1.$$



29. From the diagram, we see that appoint on the terminal side is $P(-1,-\sqrt{3})$. Therefore, taking $x = -1, y = -\sqrt{3}, r = 2$ in the definitions of the trig ratios, we have

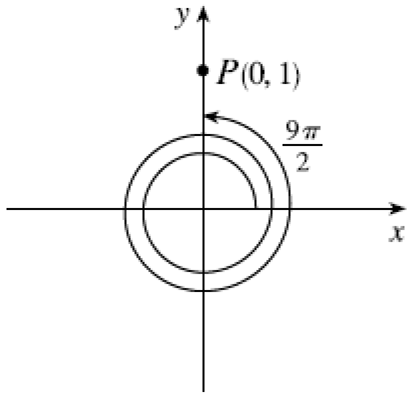
$$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}, \quad \cos\frac{4\pi}{3} = -\frac{1}{2}, \quad \tan\frac{4\pi}{3} = \sqrt{3},$$
$$\csc\frac{4\pi}{3} = -\frac{2}{\sqrt{3}}, \quad \sec\frac{4\pi}{3} = -2, \text{ and } \cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}}.$$



30. From the diagram, we see that appoint on the terminal side is $P(-1,0)$. Therefore, taking $x=-1, y=0, r=1$ in the definitions of the trig ratios, we have

$\sin\frac{9\pi}{2}=1,\quad \cos\frac{9\pi}{2}=0,\quad \tan\frac{9\pi}{2}=\text{undefined},$

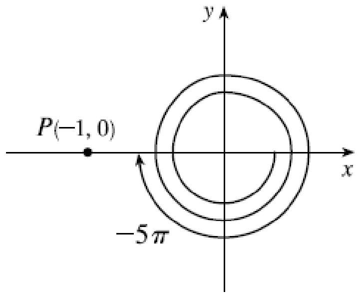
$\csc\frac{9\pi}{2}=1,\quad \sec\frac{9\pi}{2}=\text{undefined, and } \cot\frac{9\pi}{2}=0.$



31. From the diagram, we see that appoint on the terminal side is $P(0,1)$. Therefore, taking $x=0, y=2, r=1$ in the definitions of the trig ratios, we have

$\sin(-3\pi)=0,\quad \cos(-3\pi)=-1,\quad \tan(-3\pi)=\ 0,$

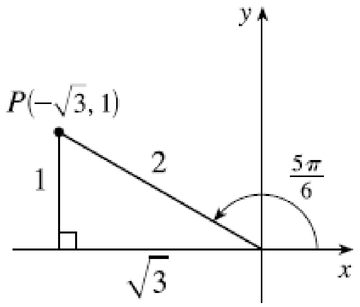
$\csc(-3\pi)=\text{undefined},\quad \sec(-3\pi)=-1, \text{and } \cot(-3\pi)=\text{undefined}.$



32. From the diagram, we see that appoint on the terminal side is $P\left(-\sqrt{3},1\right)$. Therefore, taking $x=-\sqrt{3}, y=1, r=2$ in the definitions of the trig ratios, we have

$\sin\frac{5\pi}{6}=\frac{1}{2},\quad \cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2},\quad \tan\frac{5\pi}{6}=\ -\frac{1}{\sqrt{3}},$

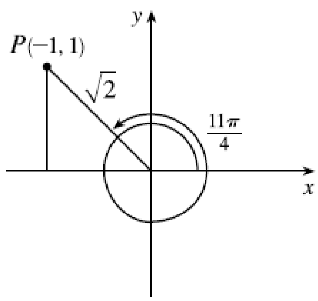
$\csc\frac{5\pi}{6}=2,\quad \sec\frac{5\pi}{6}=-\frac{2}{\sqrt{3}},\ \text{and}\ \cot\frac{5\pi}{6}=-\sqrt{3}.$



33. From the diagram, we see that appoint on the terminal side is $P(-1,1)$. Therefore, taking $x=-1, y=1, r=\sqrt{2}$ in the definitions of the trig ratios, we have

$\sin\frac{11\pi}{4}=\frac{\sqrt{2}}{2},\quad \cos\frac{11\pi}{4}=-\frac{\sqrt{2}}{2},\quad \tan\frac{11\pi}{4}=-1,$

$\csc\frac{11\pi}{4}=\sqrt{2},\ \sec\frac{11\pi}{4}=-\sqrt{2},\ \text{and}\ \cot\frac{11\pi}{4}=-1.$



34. (a) $\sin\theta=\frac{1}{2}$ when $\theta=\frac{\pi}{6},\frac{5\pi}{6}$

(b) $\cos\theta=1$ when $\theta=0,2\pi$

(c) $\sin\theta=1.2$ for no values of θ

(d) $\tan\theta=1$ when $\theta=\frac{\pi}{4},\frac{5\pi}{4}$

(e) $\cos\theta=-\frac{3}{4}$ when $\theta=2.491, 3.864$

(f) $\sin\theta=0.2$ when $\theta=0.201, 2.940$

(g) $\tan\theta=3$ when $\theta=1.294, 4.391$

(h) $\tan\theta=-2$ when $\theta=2.034, 5.176$

35. (**D**) $\frac{13\pi}{6}$ is not a solution of $\sin\theta = -\frac{1}{2}$.

36. (**C**) $\frac{3\pi}{8}$ is not a solution of $\cos 2\theta = \frac{\sqrt{2}}{2}$.

37. $\sin\theta = y/r = \frac{3}{5} \Rightarrow y = 3, r = 5,$ and $x = \sqrt{r^2 - y^2} = 4$ $\left(\text{since } 0 < \theta < \frac{\pi}{2}\right)$. Therefore taking

$x = 4, y = 3, r = 5$ in the definitions of the trigonometric ratios, we have $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$,

$\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, and $\cot\theta = \frac{4}{3}$.

38. Because α is in the first quadrant where x and y are both positive. Therefore

$\tan\alpha = y/x = 2 \Rightarrow y = 2, x = 1,$ and $r = \sqrt{x^2 + y^2} = \sqrt{5}$. Then $\sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = \frac{1}{\sqrt{5}}$

$\csc\alpha = \frac{\sqrt{5}}{2}$, $\sec\alpha = \sqrt{5}$, and $\cot\alpha = \frac{1}{2}$.

39. $\frac{\pi}{2} < \phi < \pi \Rightarrow \phi$ is in the second quadrant where x is negative and y is positive. Therefore

$\sec\phi = r/x = -1.5 = -\frac{3}{2} \Rightarrow r = 3, x = 2,$ and $y = \sqrt{r^2 - x^2} = \sqrt{5}$. Taking $x = -2, y = \sqrt{5},$ and $r = 3$ in

the definitions of the trigonometric ratios, we have $\sec\phi = -1.5$, $\sin\phi = \frac{\sqrt{5}}{3}$, $\cos\phi = -\frac{2}{3}$,

$\tan\phi = -\frac{\sqrt{5}}{2}$, $\csc\phi = \frac{3}{\sqrt{5}}$, and $\cot\phi = -\frac{2}{\sqrt{5}}$.

40. Since $\pi < x < \frac{3\pi}{2}$, x is in the third quadrant where x and y are both negative. Therefore

$\cos x = x/r = -\frac{1}{3} \Rightarrow x = -1, r = 3$ and $y = -\sqrt{r^2 - x^2} = -\sqrt{8} = 2\sqrt{2}$. Taking

$x = -1, y = -2\sqrt{2},$ and $r = 3$ in the definitions of the trigonometric ratios, we have $\cos\theta = -\frac{1}{3}$,

$\sin\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = 2\sqrt{2}$, $\csc\theta = -\frac{3}{2\sqrt{2}}$, $\sec\theta = -3$, and $\cot\theta = \frac{1}{2\sqrt{2}}$.

41. $\pi < \beta < 2\pi$ means that β is in the third or fourth quadrant where y is negative. Also since

$\cot\beta = x/y = 3$ which is positive, x must also be negative. Therefore $\cot\beta = x/y = \frac{3}{1} \Rightarrow$

$\Rightarrow x = -3, y = -1,$ and $r = \sqrt{x^2 + y^2} = \sqrt{10}$. Taking $x = -3, y = -1,$ and $r = \sqrt{10}$ in the definitions of

the trigonometric ratios, we have $\sin\beta = -\frac{1}{\sqrt{10}}$, $\cos\beta = -\frac{3}{\sqrt{10}}$, $\tan\beta = \frac{1}{3}$, $\csc\beta = -\sqrt{10}$, and $\sec\beta = -\dfrac{\sqrt{10}}{3}$.

42. Since $\dfrac{3\pi}{2} < \theta < 2\pi$, θ is in the fourth quadrant where x is positive and y is negative. Therefore $\csc\theta = r/y = -\dfrac{4}{3} \Rightarrow r = 4, y = -3,$ and $x = \sqrt{r^2 - y^2} = \sqrt{7}$. Taking $x = \sqrt{7}, y = -3,$ and $r = 4$ in the definitions of the trigonometric ratios, we have $\sin\theta = -\dfrac{3}{4}$, $\cos\theta = \dfrac{\sqrt{7}}{4}$, $\tan\theta = -\dfrac{3}{\sqrt{7}}$, $\sec\theta = \dfrac{4}{\sqrt{7}}$, and $\cot\theta = -\dfrac{\sqrt{7}}{3}$.

43. $\sin 35° = \dfrac{x}{10} \Rightarrow x = 10 \cdot \sin 35° \approx 5.736$

44. $\cos 40° = \dfrac{x}{25} \Rightarrow x = 25 \cdot \cos 40° \approx 19.151$

45. $\tan\dfrac{2\pi}{5} = \dfrac{x}{8} \Rightarrow x = 8 \cdot \tan\dfrac{2\pi}{5} \approx 24.621$

46. $\cos\dfrac{3\pi}{8} = \dfrac{22}{x} \Rightarrow x = \dfrac{22}{\cos\frac{3\pi}{8}} \approx 57.489$

47. (a) $f(x) = x\tan x$ is *even*.

(b) $g(x) = \dfrac{\sin x}{x}$ is *even*.

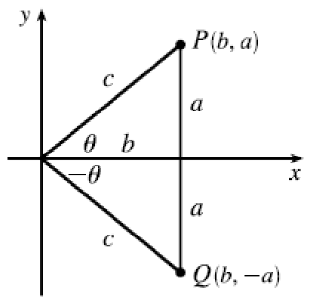
(c) $h(x) = 5\cos x - \sin^2 x$ is *even*.

48. (a) For $f(x) = \sin x$, $0 \le x \le \dfrac{\pi}{6}$ the average rate of change is $\dfrac{\sin\frac{\pi}{6} - \sin 0}{\frac{\pi}{6} - 0} = \dfrac{0.5}{\frac{\pi}{6}} = \dfrac{3}{\pi} \approx 0.955$.

(b) For $g(x) = \cos 2x$, $\dfrac{\pi}{6} \le x \le \dfrac{\pi}{4}$ the average rate of change is

$$\frac{\cos\left(\frac{2\pi}{4}\right) - \cos\left(\frac{2\pi}{6}\right)}{\frac{\pi}{4} - \frac{\pi}{6}} = \frac{0 - 0.5}{\frac{\pi}{12}} = -\frac{6}{\pi} \approx -1.910.$$

49. (a) From the diagram we see that $\sin\theta = \dfrac{y}{r} = \dfrac{a}{c}$, and $\sin(-\theta) = \dfrac{-a}{c} = -\dfrac{a}{c} = -\sin\theta$.



(b) Again from the diagram we see that $\cos\theta = \dfrac{x}{r} = \dfrac{b}{c} = \cos(-\theta)$.

50. (a) Using the sine and cosine addition formulas from Equations 15a and 15b, we have:

$$\tan(x+y)=\frac{\sin(x+y)}{\cos(x+y)}=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}=\frac{\dfrac{\sin x\cos y}{\cos x\cos y}+\dfrac{\cos x\sin y}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y}-\dfrac{\sin x\sin y}{\cos x\cos y}}=\frac{\tan x+\tan y}{1-\tan x\tan y}.$$

(b) Since $\tan(-\theta)=-\tan\theta$,

$$\tan(x-y)=\tan(x+(-y))=\frac{\tan x+\tan(-y)}{1-\tan x\tan(-y)}=\frac{\tan x-\tan y}{1+\tan x\tan y}.$$

51. (a) Working from the right side of Equation 21a, we have:

$$\frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right]=\frac{1}{2}\left[\sin x\cos y+\cos x\sin y+\sin x\cos y-\cos x\sin y\right]$$
$$=\frac{1}{2}\left[2\sin x\cos y\right]=\sin x\cos y$$

(b) Working from the right side of Equation 21b, we have

$$\frac{1}{2}\left[\cos(x+y)+\cos(x-y)\right]=\frac{1}{2}\left[\cos x\cos y-\sin x\sin y+\cos x\cos y-\sin x\sin y\right]$$
$$=\frac{1}{2}\left[2\cos x\cos y\right]=\cos x\cos y$$

(c) Working from the right side of Equation 21c, we have

$$\frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]=\frac{1}{2}\left[\cos x\cos y+\sin x\sin y-\cos x\cos y+\sin x\sin y\right]$$
$$=\frac{1}{2}\left[2\sin x\sin y\right]=\sin x\sin y$$

52. $\cos\left(\frac{\pi}{2}-x\right)=\cos\left(\frac{\pi}{2}\right)\cos x+\sin\left(\frac{\pi}{2}\right)\sin x=0+1\cdot\sin x=\sin x$

53. $\sin\left(\frac{\pi}{2}+x\right)=\sin\left(\frac{\pi}{2}\right)\cos x+\cos\left(\frac{\pi}{2}\right)\sin x=\cos x+0=\cos x$

54. $\sin(\pi-x)=\sin x\cos x-\cos\pi\sin x=0-(-1)\cdot\sin x=\sin x$

55. $\sin\theta\cot\theta=\sin\theta\cdot\dfrac{\cos\theta}{\sin\theta}=\cos\theta$

56. $(\sin x+\cos x)^2=\sin^2 x+2\sin x\cos x+\cos^2 x=1+2\sin x\cos x=1+2\sin x$

57. $\sec y-\cos y=\dfrac{1}{\cos y}-\cos y=\dfrac{1-\cos^2 y}{\cos y}=\dfrac{\sin^2 y}{\cos y}=\dfrac{\sin y}{\cos y}\cdot\sin y=\tan y\sin y$

58. $\tan^2\alpha-\sin^2\alpha=\dfrac{\sin^2\alpha}{\cos^2\alpha}-\sin^2\alpha\dfrac{\cos^2\alpha}{\cos^2\alpha}=\dfrac{\sin^2\alpha\left(1-\cos^2\alpha\right)}{\cos^2\alpha}=\dfrac{\sin^2\alpha\cdot\sin^2\alpha}{\cos^2\alpha}=\tan^2\alpha\sin^2\alpha$

59. $$\cot^2\theta+\sec^2\theta=\frac{\cos^2\theta}{\sin^2\theta}+\frac{1}{\cos^2\theta}=\frac{\cos^2\theta\cos^2\theta+\sin^2\theta}{\sin^2\theta\cos^2\theta}=\frac{\left(1-\sin^2\theta\right)\left(1-\sin^2\theta\right)+\sin^2\theta}{\sin^2\theta\cos^2\theta}$$
$$=\frac{1-\sin^2\theta+\sin^4\theta}{\sin^2\theta\cos^2\theta}=\frac{\cos^2\theta+\cos^4\theta}{\sin^2\theta\cos^2\theta}=\frac{1}{\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\csc^2\theta+\tan^2\theta.$$

60. $2\csc 2t = \dfrac{2}{\sin 2t} = \dfrac{2}{2\sin t\cos t} = \dfrac{1}{\sin t\cos t} = \sec t\csc t$

61. $\tan 2\theta = \tan(\theta+\theta) = \dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

62. $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = \dfrac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} = \dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$

63. $\sin x\sin 2x + x\cos\cos 2x = \sin x(2\sin x\cos x) + \cos(2\cos^2 x - 1) = 2\sin^2 x\cos x + 2\cos^3 x - \cos x$

$= 2(1-\cos^2 x)\cos x + 2\cos^3 x - \cos x = 2\cos x - 2\cos^3 x - \cos x = \cos x$ or

$\sin x\sin 2x + x\cos\cos 2x = \cos(2x - x) = \cos x$

64. $\sin(x+y)\sin(x-y) = (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$= \sin^2 x\cos^2 y - \sin x\cos y\cos x\sin y + \cos x\sin y\sin x\cos y - \cos^2 x\sin^2 y$

$= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y$

$= \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x\sin^2 y = \sin^2 x - \sin^2 y$

65. $\dfrac{\sin\phi}{1-\cos\phi} = \dfrac{\sin\phi}{1-\cos\phi}\cdot\dfrac{1+\cos\phi}{1+\cos\phi} = \dfrac{\sin\phi(1+\cos\phi)}{1-\cos^2\phi} = \dfrac{\sin\phi(1+\cos\phi)}{\sin^2\phi}$

$= \dfrac{1+\cos\phi}{\sin\phi} = \dfrac{1}{\sin\phi} + \dfrac{\cos\phi}{\sin\phi} = \csc\phi + \cot\phi$

66. $\tan x + \tan y = \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y} = \dfrac{\sin x\cos y + \cos\sin y}{\cos x\cos y} = \dfrac{\sin(x+y)}{\cos x\cos y}$

67. $\sin 3\theta + \sin\theta = \sin(2\theta+\theta) + \sin\theta = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta + \sin\theta$

$= \sin 2\theta\cos\theta + (2\cos^2\theta - 1)\sin\theta + \sin\theta$

$= \sin 2\theta\cos\theta + 2\cos^2\theta - \sin\theta + \sin\theta = \sin 2\theta\cos\theta + \sin 2\theta\cos\theta$

$= \sin 2\theta\cos\theta$

68. Let $x = 2\theta, y = \theta$. Then

$\cos 3\theta = \cos(2\theta+\theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2(1-\cos^2\theta)\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta = 4\cos^3\theta - 3\cos\theta$

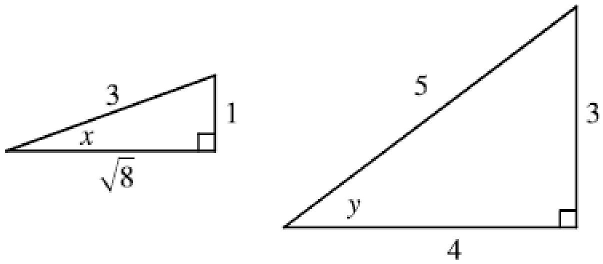
69. $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta = 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta = 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

70. $\sin(-\theta) = \sin(0-\theta) = \sin 0\cos\theta - \cos 0\sin\theta = 0 - 1\cdot\sin\theta = -\sin\theta$

71. Because $\sin x = \frac{1}{3}$ we can label the opposite side as having length 1, the hypotenuse as having length 3 and use the Pythagorean Theorem to get that the adjacent side has length $\sqrt{8}$. Then, from the diagram, $\cos x = \frac{\sqrt{8}}{3}$. Similarly, we have that $\sin y = \frac{3}{5}$. Finally $\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{1}{3}\cdot\frac{4}{5} + \frac{\sqrt{8}}{3}\cdot\frac{3}{5} =$ $\frac{4}{15} + \frac{3+\sqrt{8}}{15} = \frac{4+6\sqrt{2}}{15}$



72. $\cos(x+y) = \cos x \cos y - \sin x \sin y = \frac{\sqrt{8}}{3}\cdot\frac{4}{5} - \frac{1}{3}\cdot\frac{3}{5} = \frac{8\sqrt{2}-3}{15}$

73. $\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{\sqrt{8}}{3}\cdot\frac{4}{5} + \frac{1}{3}\cdot\frac{3}{5} = \frac{8\sqrt{2}+3}{15}$

74. $\sin(x-y) = \sin x \cos y - \cos x \sin y = \frac{1}{3}\cdot\frac{4}{5} - \frac{\sqrt{8}}{3}\cdot\frac{3}{5} = \frac{4-6\sqrt{2}}{15}$

75. $\sin 2y = 2\sin y \cos y = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

76. $\cos 2y = 2\cos^2 y - 1 = 2\cdot\left(\frac{4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \frac{7}{25}$

77. $2\cos x - 1 = 0 \Rightarrow 2\cos x = 1 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

78. $3\cot^2 x = 1 \Rightarrow \cot^2 x = \frac{1}{3} \Rightarrow \cot x = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

79. $2\sin^2 x = 1 \Rightarrow \sin^2 x = 2 \Rightarrow \sin x = \pm\frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

80. $|\tan x| = 1 \Rightarrow \tan x = \pm 1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$, or $x = \frac{\pi}{4}, \frac{5\pi}{4}$,

81. $\sin 2x = \cos x \Rightarrow 2\sin x \cos x = \cos x \Rightarrow \cos x(2\sin x - 1) = 0 \Rightarrow \cos x = 0 \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

or $2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

82. $2\cos x + \sin 2x = 0 \Rightarrow 2\cos x - \sin 2x = 0 \Rightarrow 2\cos x - 2\sin x \cos = 0 \Rightarrow 2\cos x(1 - \sin x) = 0$

$\Rightarrow 1 - \sin x = 0 \Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2}$ or $2\cos x = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

83. $\sin x = \tan x \Rightarrow \sin x - \frac{\sin x}{\cos x} = 0 \Rightarrow \sin x\left(1 - \frac{1}{\cos x}\right) = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

or $1 - \frac{1}{\cos x} = 0 \Rightarrow \cos x = 1 \Rightarrow x = 0, 2\pi$. So the solutions are $x = 0, \pi, 2\pi$.

84. $2+\cos 2x = 3\cos x \;\Rightarrow\; 2\cos^2 x - 1 + 2 = 3\cos x \;\Rightarrow\; 2\cos^2 x - 3\cos x + 1 = 0 \;\Rightarrow$
$(2\cos x - 1)(\cos x - 1) = 0 \;\Rightarrow\; \cos x = \dfrac{1}{2}$ or $\cos x = 1 \;\Rightarrow\; x = 0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3} 2\pi$

85. $\cos 3x = -\sin 3x \;\Rightarrow\; \dfrac{\cos 3x}{\sin 3x} = -1 \;\Rightarrow\; \cot 3x = -1 \;\Rightarrow\; x = \dfrac{\cot^{-1}(-1)}{3} = \dfrac{\pi}{4}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}, \dfrac{5\pi}{4}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}$

86. $\sin 2x = \cos 2x \;\Rightarrow\; \tan 2x = 1 \Rightarrow 2x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4} \;\Rightarrow\; x = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$

87. $\tan x = -\sec x \;\Rightarrow\; \dfrac{\sin x}{\cos x} = -\dfrac{1}{\cos x} \;\Rightarrow\; \sin x = -1 \;\Rightarrow\; \sin x = \dfrac{3\pi}{2}$

88. $\cos 2x = \cos x \;\Rightarrow\; 2\cos^2 x - 1 = \cos x \;\Rightarrow\; 2\cos^2 x - \cos x - 1 = 0 \;\Rightarrow\; \big(2\cos x + 1)(\cos x - 1\big) = 0$
$\Rightarrow\; \cos x = -\dfrac{1}{2} \cos x = 1 \;\Rightarrow\; x = 0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 2\pi$

89. We know that $\sin x = \dfrac{1}{2}$ when $x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$, and from a graph we see that $\sin x \le \dfrac{1}{2} \;\Rightarrow 0 \le x \le \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6} \le x \le 2\pi$ for $x \in [0, 2\pi]$.

90. $2\cos x + 1 > 0 \;\Rightarrow\; 2\cos > -1 \;\Rightarrow \cos > -\dfrac{1}{2}$. Now $\cos x = -\dfrac{1}{2}$ when $x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ and from a graph we see that $\cos x > -\dfrac{1}{2}$ when $0 \le x < \dfrac{2\pi}{3}$, and when $\dfrac{4\pi}{3} < x \le 2\pi$.

91. $\tan x = -1$ when $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$ and $\tan x = 1$ when $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$. Using a graph we see that
$-1 < \tan x < 1 \;\Rightarrow\; 0 \le x < \dfrac{\pi}{4}$, $\dfrac{3\pi}{4} < x \dfrac{5\pi}{4}$, and $\dfrac{7\pi}{4} < x \le 2\pi$.

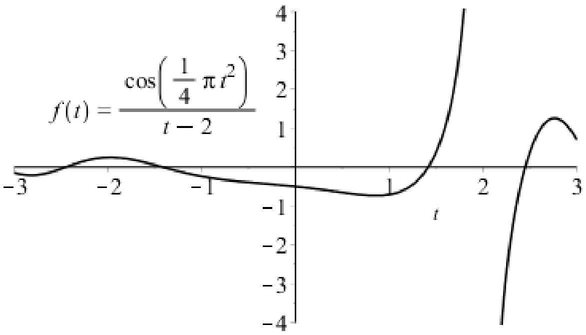
92. We know that $\sin x = \cos x$ when $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$, and from a graph we find that $\sin x > \cos x$ when $\dfrac{\pi}{4} < x < \dfrac{5\pi}{4}$.

93. Using technology, we find that $\sin 2x + 2 \ge 2\cos 2x$ for $0 \le x \le 2.678$ and $\pi \le x \le 5.820$ or when $x = 2\pi$.

94. $\cos^2 x - 3\cos x \le \sin^2 x - 1 \;\Rightarrow\; \cos^2 x - 3\cos x \le 1 - \cos^2 x - 1 \;\Rightarrow 2\cos^2 x - 3x \le 0$
$\Rightarrow \cos x(2\cos x - 3) \le 0$. Because $2\cos x - 3 < 0$ for all x, we need to know when $\cos x \ge 0$. Thus $0 \le x \le \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2} \le x \le 2\pi$.

95. We start with a graph of the function on this interval as shown below.
From the graph we see that there are 4 solutions. Two obvious roots are $t = \pm\sqrt{2}$ since
$\cos\left(\dfrac{\pi\left(\pm\sqrt{2}\right)^2}{4}\right) = \cos\left(\dfrac{\pi}{2}\right) = 0$.

To find the remaining solutions, we resort to technology which tells us the other roots are roughly ± 2.449489. Thus $\dfrac{\cos\left(\frac{\pi t^2}{4}\right)}{t-2} \le 0$ when $-3 \le t \le -2.449489$ or $-\sqrt{2} \le t \le \sqrt{2}$ or $2 \le t \le 2.449489$.



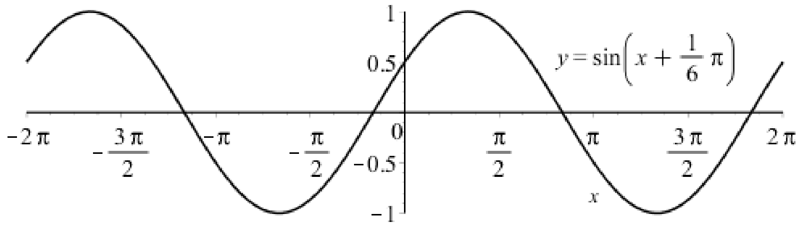
96. (a) $y = 2\sin x$: Domain: $\mathbb{R}$, Range: [–2, 2], Period: 2π, Amplitude: 2.

(b) $y = \dfrac{1}{2}\cos 2x$: Domain: $\mathbb{R}$, Range: [–½ , ½], Period: π, Amplitude: ½.

(c) $y = \tan 2x - 2$: Domain: all reals except odd multiples of $\pi/4$. Range: $\mathbb{R}$, Period: $\pi/2$, Amplitude: N/A.

(d) $y = 2(\sin 3x + 1)$: Domain: $\mathbb{R}$, Range: [0, 4], Period: $2\pi/3$, Amplitude: 2.

97.



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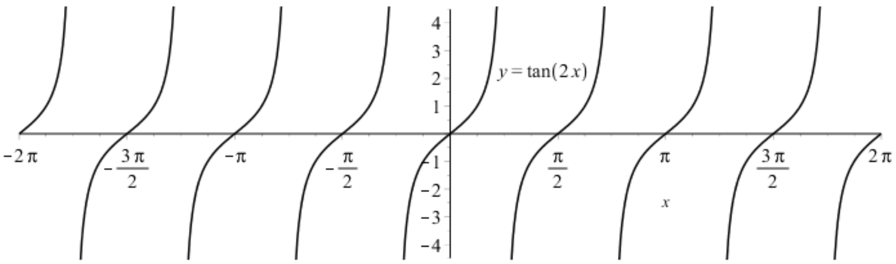

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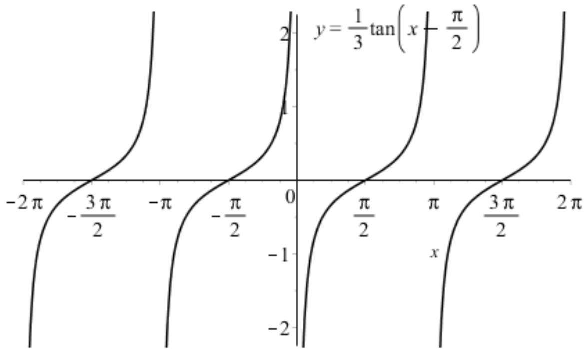
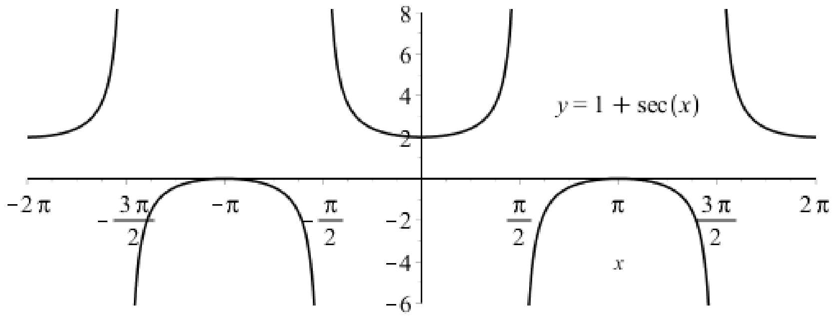


100.



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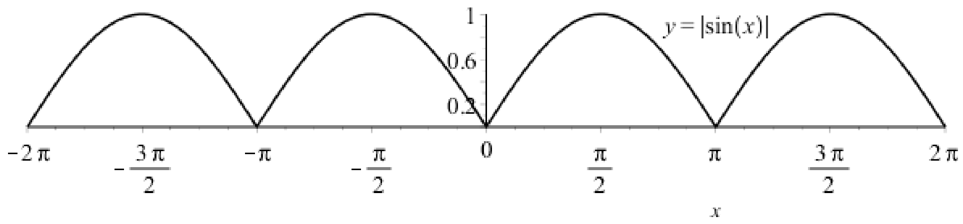
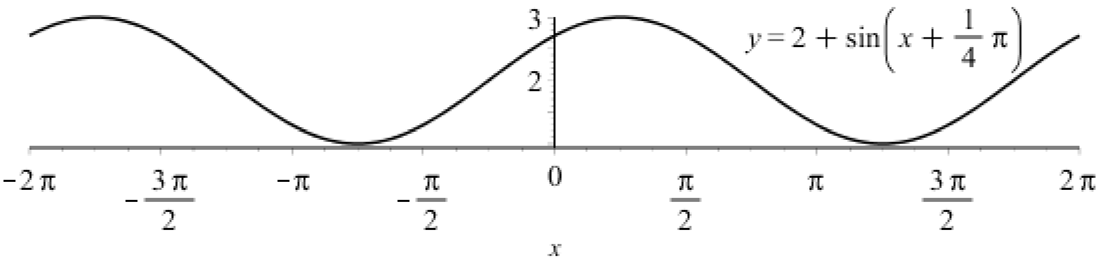
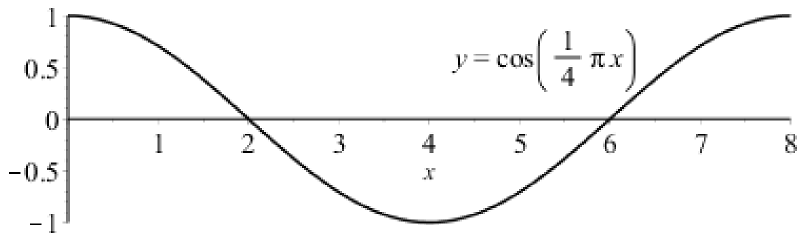


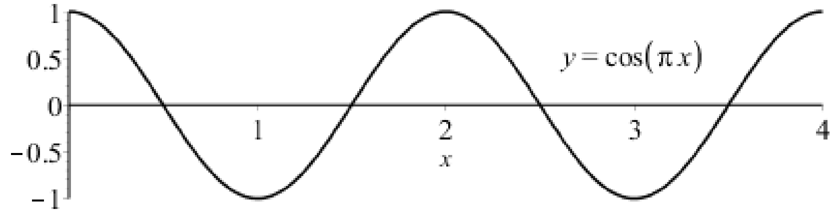
102.



103. Using the graph it appears that $y = \cos\left(\dfrac{\pi}{4}x\right)$ is increasing on $[4,8]$.



104. Using the graph we find that $y = \cos \pi x$ appears to be decreasing on $[-1,1]$, and $[2,3]$.



105. $\dfrac{\sin(x+h)-\sin x}{h}=\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}=\dfrac{\sin x\left(\cos h-1\right)+\sin h\cos x}{}$

$$=\sin x\left(\frac{\cos h-1}{h}\right)-\frac{\sin h}{h}\sin x$$

106. $\dfrac{\cos(x+h)-\cos x}{h}=\dfrac{\cos x\cos h-\sin x\sin h}{h}=\dfrac{\cos x\left(\cos h-1\right)-\sin x\sin h}{h}$

$$=\frac{\cos x\left(\cos h-1\right)}{h}-\frac{\sin h}{h}\sin x$$

107. $\dfrac{\sin(x+h)-\sin(x-h)}{2h}=\dfrac{\left(\sin x\cos h+\cos x\sin h\right)-\left(\sin x\cos h-\cos x\sin h\right)}{2h}$

$$=\frac{2\cos\sin h}{2h}=\frac{\sin h}{h}\cos x$$

108. From the figure in the text, we see that $x=b\cos\theta, y=b\sin\theta,$ and from the distance formula we have that the distance c from (x,y) to $(a,0)$ is $c=\sqrt{(x-a)^2+\left(y-0\right)^2}\ \Rightarrow$

$$c^2=\left(b\cos\theta-a\right)^2+\left(b\sin\theta\right)^2=b^2\cos^2\theta-2ab\cos\theta+a^2+b^2\sin^2\theta$$
$$=a^2+b^2\left(\cos^2\theta+\sin^2\theta\right)-2ab\cos\theta=a^2+b^2-2ab\cos\theta$$

109. The law of cosines is $c^2=a^2+b^2-2ab\cos\theta.$ If θ is in a right triangle, then $\cos\theta=\cos\dfrac{\pi}{2}=0\ \Rightarrow\ c^2=a^2+b^2-2ab(0)=a^2+b^2.$

110. $\left|AB\right|^2=\left|AB\right|^2+\left|BC\right|^2-2\left|AC\right|\left|BC\right|\cos\angle C=820^2+910^2-2(820)(910)\cos 103°\approx 1{,}836{,}217\ \Rightarrow$ $\left|AB\right|\approx 1355$ m.

111. Using the Law of Cosines, we have $c^2=1^2+1^2-2(1)(1)\cos\left(\alpha-\beta\right)=2\left[1-\cos\left(\alpha-\beta\right)\right].$ Now, using the distance formula, $c^2=\left|AB\right|^2=\left(\cos\alpha-\cos\beta\right)^2+\left(\sin\alpha-\sin\beta\right)^2$. Equating these two expressions for c^2, we get

$2\left[1-\cos\left(\alpha-\beta\right)\right]=\cos^2\alpha+\sin^2\alpha+\cos^2\beta+\sin^2\beta-2\cos\alpha\cos\beta-2\sin\alpha\sin\beta\ \Rightarrow$

$1-\cos\left(\alpha-\beta\right)=1-\cos\alpha\cos\beta-\sin\alpha\sin\beta\ \Rightarrow\ \cos\left(\alpha-\beta\right)=\cos\alpha\cos\beta+\sin\alpha\sin\beta.$

112. $\cos\left(x+y\right)=\cos(x-(-y))=\cos x\cos(-y)+\sin x\sin(-y)\qquad=\cos x\cos y-\sin x\sin y$

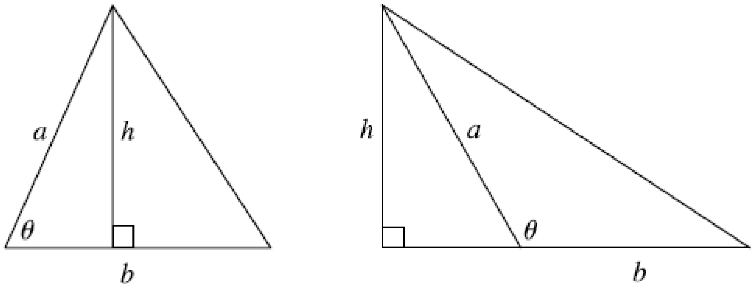
113. Using the addition formula for cosine with $x=\frac{\pi}{2}-\alpha, y=\beta,$ we get

$\cos\left[\left(\frac{\pi}{2}-\alpha\right)+\beta\right]=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta-\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta\ \Rightarrow\cos\left[\frac{\pi}{2}-\left(\alpha-\beta\right)\right]$

$=\cos\left(\frac{\pi}{2}-\alpha\right)\cos\beta-\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta.$ Now we using the identities given in the problem, $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ and $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta,$ to get $\sin\left(\alpha-\beta\right)=\sin\alpha\cos\beta-\cos\alpha\sin\beta.$

114. If $0<\theta<\frac{\pi}{2}$, we have the case depicted in the first diagram. In this case, we see that the height of the triangle is $h=a\sin\theta.$ If $\frac{\pi}{2}\le\theta<\pi,$ we have the case depicted in the second diagram. In

this case, the height of the triangle is $h = a\sin(\pi - \theta) = a\sin\theta$. So in either case, the area of the triangle is $\frac{1}{2}bh = \frac{1}{2}ab\sin\theta$.



115. Using the previous exercise, the area of the triangle is $\frac{1}{2}(10)(3)\sin 107° \approx 14.34457\ \text{cm}^2$.

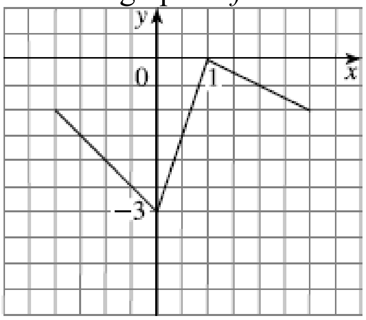
1.4 New Functions from Old Functions

EXERCISE SOLUTIONS

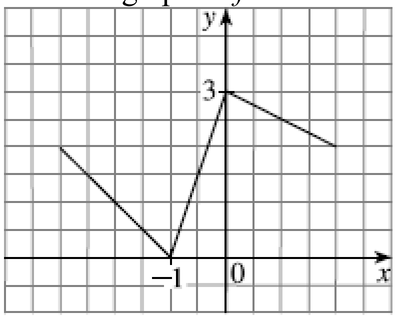
1. False; the graph of $y=-f(x)$ is a reflection of the graph of $y=f(x)$ about the x-axis.
2. True
3. True; because $|2| = 2$, these functions have the same graph.
4. False; the graph of $y=2\sin x$ has a period of 2π.
5. $f\circ g$ denotes the composition of f and g.
6. The domain of $f(g(x))$ consists of all x in the domain of g such that $g(x)$ is in the domain of f.
7. False
8. If $c > 0$, then the graph of $y=f(x+c)$ is a shift of the graph $y=f(x)$ c units to the left.
9. If $f(g(x))=\sqrt{k-\sqrt{x}}$ has domain [0, 9], then $k = 3$.
10. If $f(g(x))=\sqrt{\dfrac{5-2x}{x}}$, then the domain of $f(g(x))$ is $\{x \mid 0<x\le 5/2\}$.
11. If $f(x)=x^2$, $g(x)=\sqrt{x+3}$, and $h(x)=\dfrac{1}{x}$, then the composition that results in $\sqrt{3+\dfrac{1}{x^2}}$ is (D) $g\circ f\circ h$.
12. To obtain the graph of $y=-3\sin(3x+2)$ from the graph of $y=\sin x$ first shrink the graph horizontally by a factor of 3, then shift ⅔ units to the right, then flip across the x-axis, stretch by 3 in the vertical direction, and finally move down 4 units.
13. (a) To shift the graph of f up 3 units, graph $y=f(x)+3$
    (b) To shift the graph of f down 3 units, graph $y=f(x)-3$.
    (c) To shift the graph of f 3 units to the right, graph $y=f(x-3)$.
    (d) To shift the graph of f 3 units to the left, graph $y=f(x+3)$.
    (e) To reflect the graph of f about the x-axis, graph $y=-f(x)$.
    (f) To reflect the graph of f about the y-axis, graph $y=f(-x)$.
    (g) To stretch the graph of f vertically by a factor of 3, graph $y=3f(x)$.
    (h) To shrink the graph of f vertically by a factor of 3, graph $y=\frac{1}{3}f(x)$.
14. (a) To obtain the graph of $y=f(x)+8$ from the graph of $y=f(x)$, shift the graph up 8 units.
    (b) To obtain the graph of $y=f(x+8)$ from the graph of $y=f(x)$, shift the graph 8 units to the left.
    (c) To obtain the graph of $y=8f(x)$ from the graph of $y=f(x)$, stretch the graph vertically by a factor of 8.
    (d) To obtain the graph of $y=f(8x)$ from the graph of $y=f(x)$, shrink the graph horizontally by a factor of 8.
    (e) To obtain the graph of $y=-f(x)-1$ from the graph of $y=f(x)$, first reflect the graph about the x-axis, then shift it down 1 unit.
    (f) To obtain the graph of $y=8f\left(\frac{1}{8}x\right)$ from the graph of $y=f(x)$, stretch the graph horizontally and vertically by a factor of 8.

15. (a) $y = f(x-4)$ is curve (3) because the graph of f has been shifted 4 units to the right.
(b) $y = f(x)+3$ is curve (1) because the graph of f has been shifted up 3 units.
(c) $y = \frac{1}{3}f(x)$ is curve (4) because the graph of f has been shrunk vertically by a factor of 3.
(d) $y = -f(x+4)$ is curve (5) because the graph of f has been shifted 4 units to the left and reflected about then x-axis.
(e) $y = 2f(x+6)$ is curve (2) because the graph of f has been shifted 6 units to the left and stretched vertically by a factor of 2.

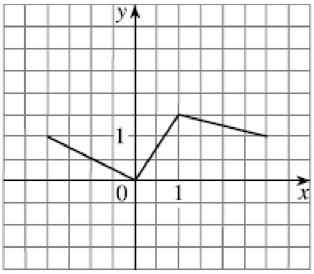
16. (a) $y = f(x)-3$:
Shift the graph of f down 3 units:



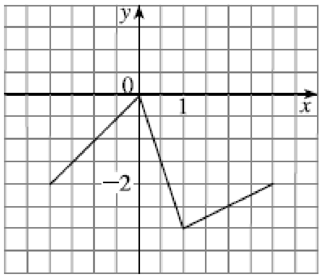
(b) $y = f(x+1)$:
Shift the graph of f 1 unit to the left:



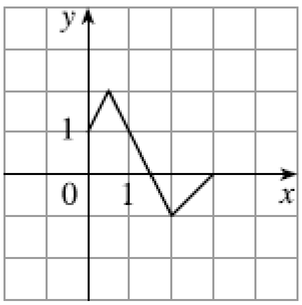
(c) $y = \frac{1}{2}f(x)$:
Shrink the graph of f vertically by a factor of 2:



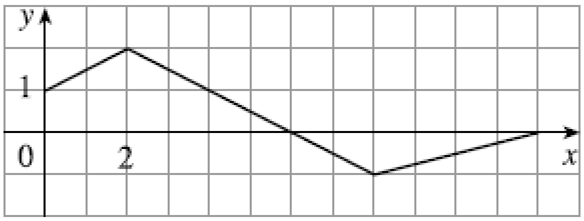
(d) $y = -f(x)$:
Reflect the graph of f about the x-axis.



17. (a) $y = f(2x)$:
Shrink the graph of f horizontally by a factor of 2:

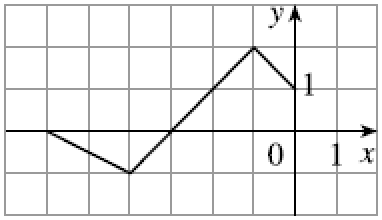


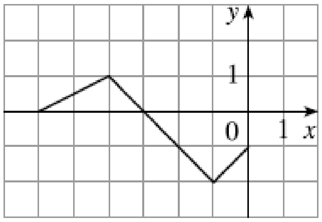
(b) $y = f\left(\frac{1}{2}x\right)$:
Stretch the graph of f horizontally by a factor of 2:



(c) $y = f(-x)$:
Reflect the graph of f about the y-axis:

(d) $y = -f(-x)$:
Reflect the graph of f about the y-axis and then about the x-axis.





18. The graph of. $y = f(x) = \sqrt{3x - x^2}$. has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus a function describing the graph is
$y = 2f(x-2) = 2\sqrt{3(x-2)-(x-2)^2} = 2\sqrt{-x^2+7x-10}$.

19. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x-axis, and shifted down 1 unit Thus a function describing the graph is
$y = -f(x+4) - 1 = -\sqrt{3(x+4)-(x+4)^2} - 1 = -\sqrt{-x^2-5x-4} - 1$

20. If (–6, –3) is on the graph of $y = f(x)$ then (**A**) (–12, 6) is on the graph of $y = -2f\left(\frac{1}{2}x\right)$.

21. If (6, –2) is on the graph of $y = f(x)$ then (**A**) (–18, –2) is on the graph of $y = 2f\left(-\frac{1}{3}x\right)$.

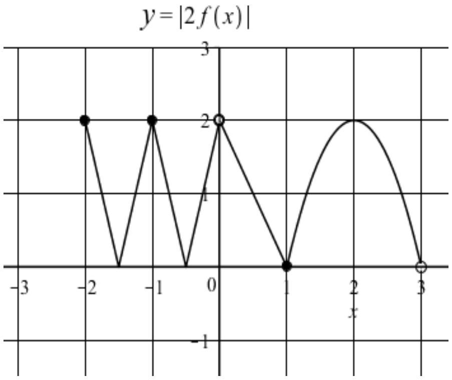
22. If an x-intercept of the graph of $y = f(x)$ is –4, then (**D**), an x-intercept of the graph of $f\left(|x|\right)$ is –7.

23. If an x-intercept of the graph of $y = f(x)$ is –6, then (**C**), an x-intercept of the graph $f\left(\frac{1}{2}x\right)$ of is –12.

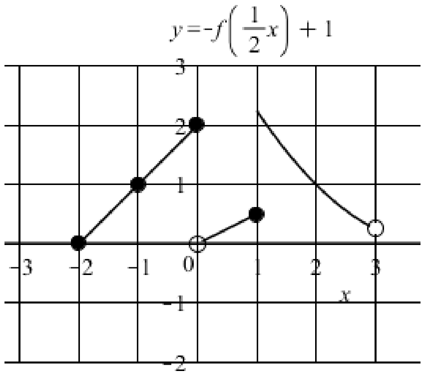
24. If $f(x) = \{(1,4),(0,2),(-4,3),(2,9)\}$ and $g(x) = \frac{1}{x}$ then (**D**), there are 4 ordered pairs in $f \circ g$. These pairs are $= \{(1,\frac{1}{4}),(0,\frac{1}{2}),(-4,\frac{1}{3}),(2,\frac{1}{9})\}$.

25. (a) The range of $y = f\left(|x|\right)$ shown is $\{0,1,2,3\}$
(b) The range of the function $y = f(-x)$ is $\{-1,0,1,2,3\}$
(c) The range of the function $y = f(f(x))$ is $\{-1,0,1,2,\}$

26. (a)



(b)



27. (a) The domain of $y = g(x)$ is $(0,\infty)$
(b) The domain of $y = (f+g)(x)$ is the domain of f which is $[-2,3)$.
(c) The domain of $f(g(x))$ is roughly $[-2,2.25]$.
(d) The domain of $y = f\left(|x|\right)$ is the domain of f which is $[-2,3)$.

(e) The domain of $y = f(2x)$ is $[-1, \frac{3}{2})$.

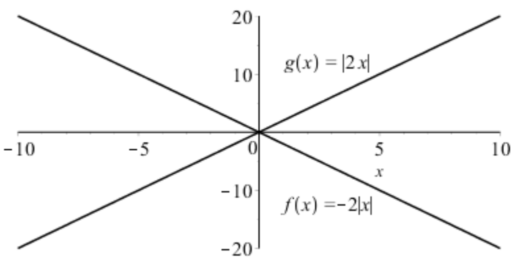
28. (a) The domain of $y = \left(\dfrac{f}{g}\right)(x)$ is $[-1,1)\cup(1,3]$.

(b) The domain of $y = (g \circ f)(x)$ is $[-7,-6)\cup[-3,3)$.

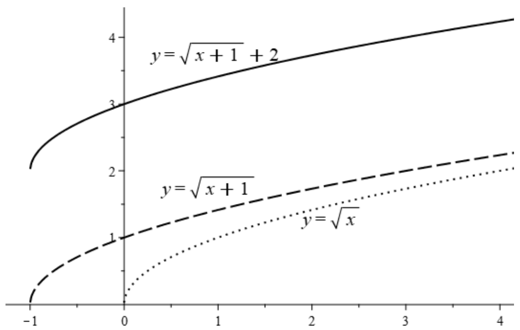
(c) The domain of $y = f(|x|)$ is $[-8,-6)\cup(-6,6)\cup(6,8)$.

(d) The domain of $y = f(2x)$ is $[-4,3)\cup(3,4)$.

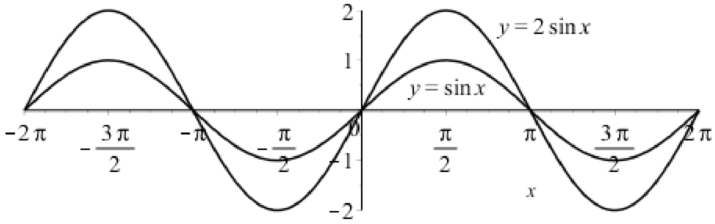
29. The graphs of $f(x) = -2|x|$ and $g(x) = |2x|$ are reflections through the x-axis.



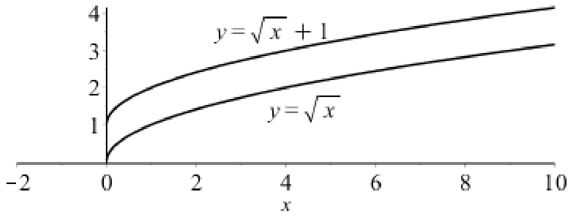
30. First graph the function $g_1(x) = \sqrt{x}$, then shift to the left one unit $\left(g_2(x) = \sqrt{x+1}\right)$. Finally, shift up 2 units to plot $f(x) = \sqrt{x+1} + 2$.



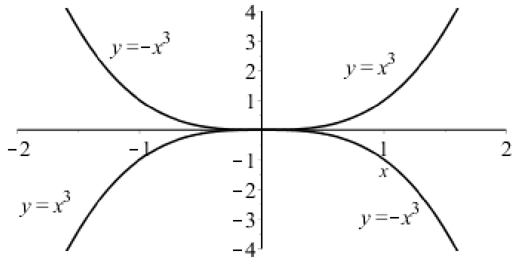
31. (a) The graph of $y = 2\sin x$ is the graph of $y = \sin x$ stretched vertically by a factor of 2.



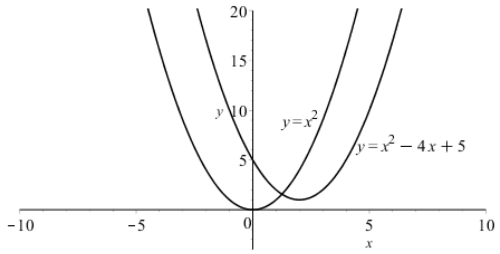
(b) The graph of $y = 1 + \sqrt{x}$ is the graph of $y = \sqrt{x}$ shifted up 1 unit.



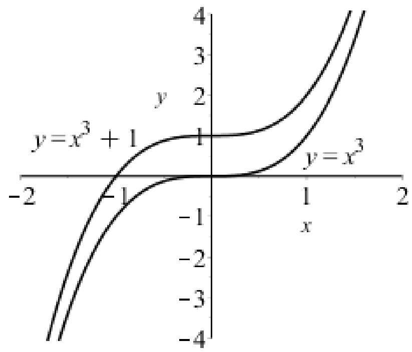
32. Start with the graph of $y = x^3$ and reflect about the x-axis.



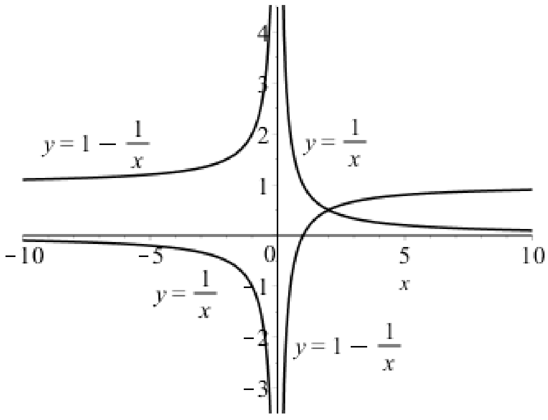
33. Start with the graph of $y = x^2$ and shift 3 units to the right.



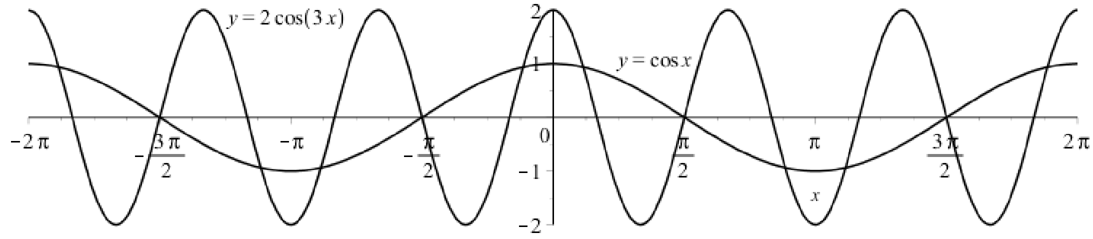
34. Start with the graph of $y = x^3 + 1$ and shift up 1 unit:



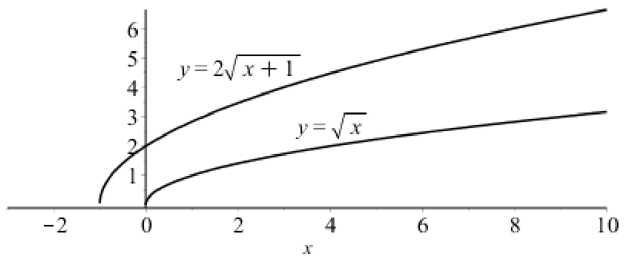
35. Start with the graph of $y = \frac{1}{x}$ reflect about the x-axis, and then shift up 1 unit.



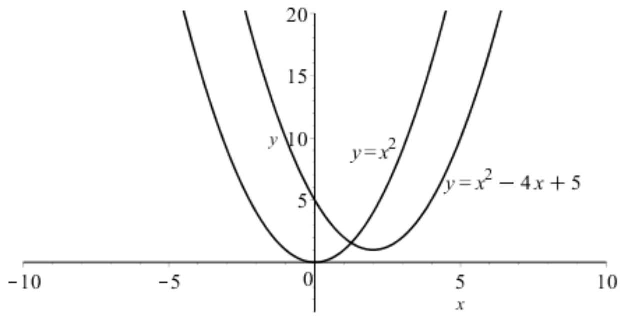
36. Start with the graph of $y = \cos x$, compress horizontally by a factor of 3, then stretch vertically by a factor of 2:



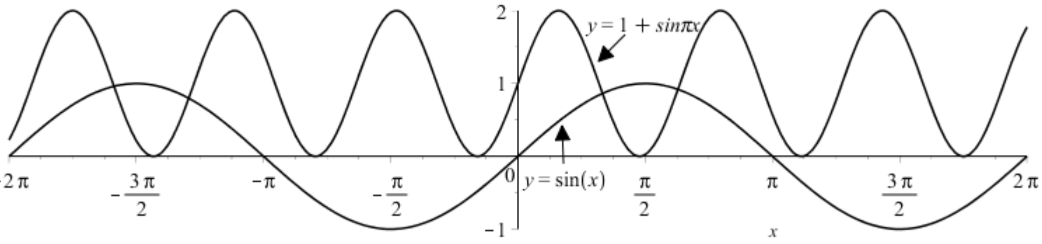
37. Start with the graph of $y = \sqrt{x}$, shift 1 unit to the left, and then stretch vertically by a factor of 2:



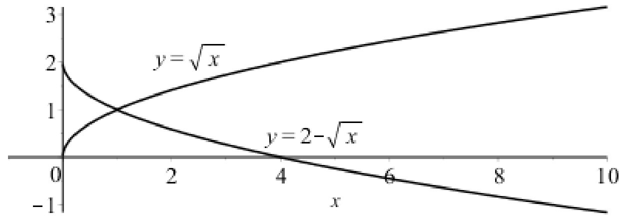
38. First note that $y = x^2 - 4x + 5 = (x-2)^2 + 1$. Now start with the graph of $y = x^2$, shift 2 units to the right, then up 1 unit:



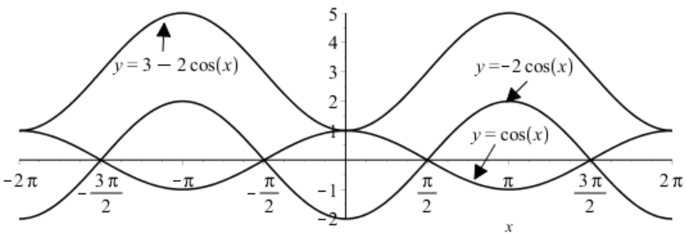
39. Start with the graph of $y = \sin x$, compress horizontally by a factor of π, then shift up 1 unit.



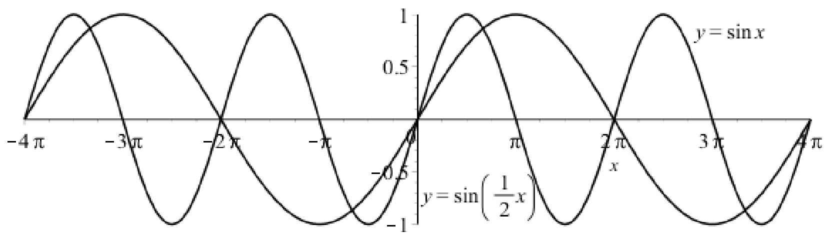
40. Start with the graph of $y = \sqrt{x}$, reflect about the x-axis, and then shift up 2 units:



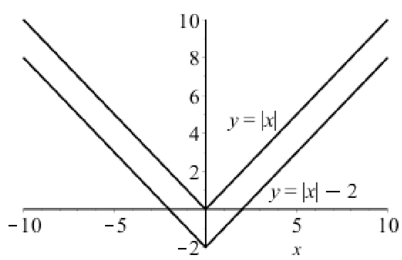
41. Start with the graph of $y = \cos x$, stretch vertically by a factor of 2, reflect about the x-axis, and then shift up 3 units:



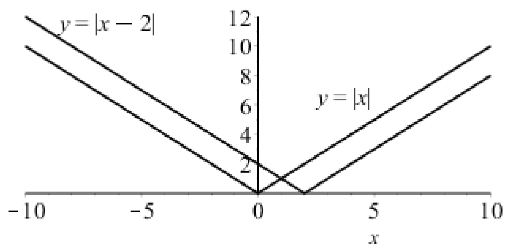
42. Start with the graph of $y = \sin x$ and stretch horizontally by a factor of 2:



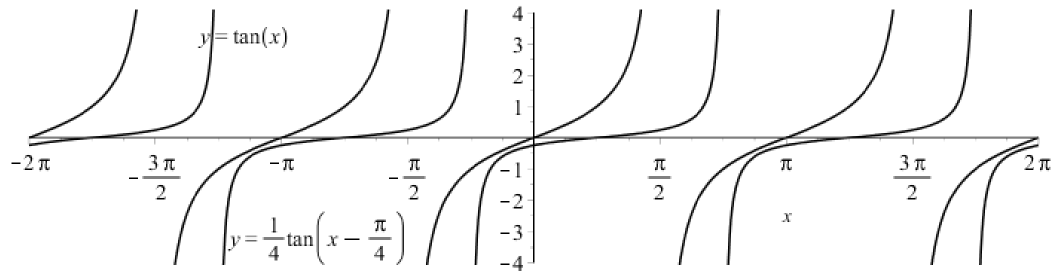
43. Start with the graph of $y = |x|$ and shift 2 units down:



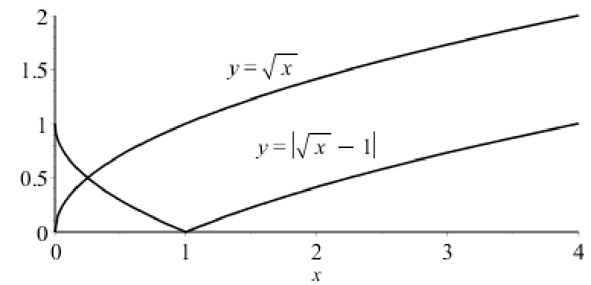
44. Start with the graph of $y = |x|$ and shift 2 units to the right:



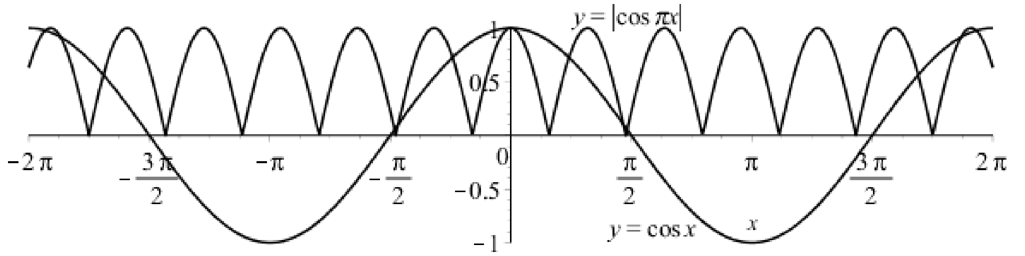
45. Start with the graph of $y = \tan x$, shift $\pi/4$ units to the right, then compress vertically by a factor of 4.



46. Start with the graph of $y = \sqrt{x}$, shift down 1 unit, and then reflect the portion from $x = 0$ to $x = 1$ across the x–axis to obtain the graph of $y = \left|\sqrt{x} - 1\right|$.



47. Start with the graph of $y = \cos x$, shrink it horizontally by a factor of π, then reflect all the parts of the graph below the x-axis about the x-axis.



48. The amplitude of the curve is 14 – 12 = 2. So the function is $L(t) = 12 + 2\sin\left(\frac{2\pi}{365}(t - 80)\right)$. March 31 is the 90th day of the year, so the model gives $L(90) \approx 12.34$ hours. The daylight time (5:51 AM to 6:18 PM) is 12 hours and 27 minutes or 12.45 h. The model value differs from the actual value by $\frac{12.45-12.34}{12.45} \approx 0.009$, less than 1%.

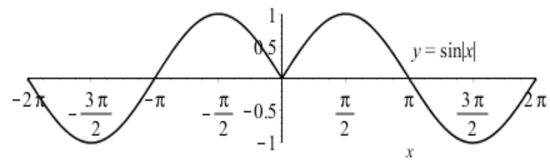
49. Using a sine function to model the brightness of Delta Cephei as a function of time, we take its period to be 5.4 days, its amplitude to be 0.35 (on the scale of magnitude), and its average magnitude to be 4.0. If we let $t = 0$ at the time of average brightness, then the magnitude (brightness) as a function of time t in days can be modeled by $M(t) = 4.0 + 0.34\sin\left(\frac{2\pi}{5.4}t\right)$.

50. The water depth $D(t)$ can be modeled by a cosine function with amplitude (12 – 2)/2 = 5 m, average magnitude (12 + 2)/2 = 7 m, and period 12 hours. High tide occurred at 6:45 AM (t = 6.75 h), so the curve begins a cycle at time t = 6.75 h (we need to shift 6.75 units to the right). Thus $D(t) = 5\cos\left(\frac{2\pi}{12}(t-6.75)\right)+7 = 5\cos\left(\frac{\pi}{6}(t-6.75)\right)+7,$ where D is measured in meters and t is the number of hours after midnight.

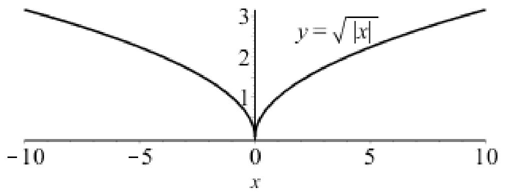
51. The total volume of air $V(t)$ in the lungs can be modeled by a sine function with amplitude (2500 – 200)/2 = 250 mL, average volume = (2500 + 2000)/2 = 2250 mL, and period 4 seconds. Thus $V(t) = 250\sin\frac{2\pi}{4}t + 2250 = 250\sin\frac{\pi}{2}t + 2250,$ where V is in mL and t is in seconds.

52. (a) To obtain the graph of $y = f\left(|x|\right)$, the portion of the graph of $y = f(x)$ to the right of the y-axis is reflected about the y-axis.

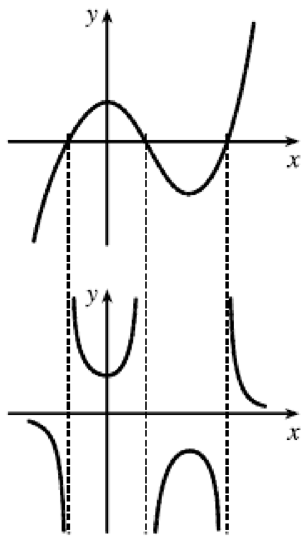
(b)



(c)



53. The most important features of the given graph are the x-incepts and the maximum and minimum points. The graph of $y = 1/f(x)$ has vertical asymptotes at the x-values where the x-intercepts are on the graph of $y = f(x)$. The maximum of 1 on the graph of $y = f(x)$ corresponds to a minimum of 1/1 = 1 on Similarly, the minimum on the graph of $y = f(x)$ corresponds to a maximum on the graph of $y = 1/f(x)$. As the values of y get large (positively or negatively) on the graph of $y = f(x)$, the values of y get close to zero on the graph of $y = 1/f(x)$.



54. (a) $(f+g)(x) = x^3 + 5x^2 - 1$. Its domain is all real numbers.

(b) $(f-g)(x) = x^3 - x^2 + 1$. Its domain is all real numbers.

(c) $(fg)(x) = 3x^5 + 6x^4 - x^3 - 2x^2$. Its domain is all real numbers.

(d) $(f/g)(x) = \dfrac{x^3+2x^2}{3x^2-1}$. Its domain is $\left\{x \in \mathbb{R} \mid x \neq \pm\dfrac{\sqrt{3}}{3}\right\}$.

55. (a) $(f+g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$. Its domain is $(-\infty,-1]\cup[1,3]$.

(b) $(f-g)(x) = \sqrt{3-x} - \sqrt{x^2-1}$. Its domain is $(-\infty,-1]\cup[1,3]$.

(c) $(fg)(x) = \sqrt{3-x}\cdot\sqrt{x^2-1}$. Its domain is $(-\infty,-1]\cup[1,3]$.

(d) $(f/g)(x) = \dfrac{\sqrt{3-x}}{\sqrt{x^2-1}}$. Its domain is $(-\infty,-1)\cup(1,3]$.

56. (a) $(f \circ g)(x) = 3x^2 + 3x + 5$. Its domain is all real numbers.

(b) $(g \circ f)(x) = (3x+5)^2 + 3x + 5 = 9x^2 + 32x + 30$. Its domain is all real numbers.

(c) $(f \circ f)(x) = 3(3x+5) + 5 = 9x = 20$. Its domain is all real numbers.

(d) $(g \circ g)(x) = (x^2 + x)^2 + x^2 + x = x^4 + 2x^3 + 2x^2 + x$. Its domain is all real numbers.

57. (a) $(f \circ g)(x) = (1-4x)^3 - 2 = -64x^3 + 48x^2 - 12x - 1$. Its domain is all real numbers.

(b) $(g \circ f)(x) = 1 - (x^3 - 2)^4$. Its domain is all real numbers.

(c) $(f \circ f)(x) = (x^3 - 2)^3 - 2 = x^9 - 6x^6 + 12x^3 - 10$. Its domain is all real numbers.

(d) $(g \circ g)(x) = 1 - 4(1-4x) = 16x - 3$. Its domain is all real numbers.

58. (a) $(f \circ g)(x) = \sqrt{4x-2}$. Its domain is $\{x \in \mathbb{R} \mid x \geq -\frac{1}{2}\}$.

(b) $(g \circ f)(x) = 4\sqrt{x+1} - 3$. Its domain is $\{x \in \mathbb{R} \mid x \geq -1\}$.

(c) $(f \circ f)(x) = \sqrt{\sqrt{x}+1+1}$. Its domain is $\{x \in \mathbb{R} \mid x \geq -1\}$.

(d) $(g \circ g)(x) = 4(4x-3) - 3 = 16x - 15$. Its domain is all real numbers.

59. (a) $(f \circ g)(x) = \sin(x^2 + 1)$. Its domain is all real numbers.

(b) $(g \circ f)(x) = \sin^2 x + 1$. Its domain is all real numbers.

(c) $(f \circ f)(x) = \sin(\sin x)$. Its domain is all real numbers.

(d) $(g \circ g)(x) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$. Its domain is all real numbers.

60. (a) $(f \circ g)(x) = \dfrac{x+1}{x+2} + \dfrac{x+2}{x+1} = \dfrac{2x^2 + 6x + 5}{(x+1)(x+2)}$. Its domain is $\{x \in \mathbb{R} \mid x \neq -2, -1\}$.

(b) $(g \circ f)(x) = \dfrac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} = \dfrac{x^2 + x + 1}{x^2 + 2x + 1}$. Its domain is $\{x \in \mathbb{R} \mid x \neq -1, 0\}$.

(c) $(f \circ f)(x) = x + \dfrac{1}{x} + \dfrac{1}{x + \frac{1}{x}} = \dfrac{x^4 + 3x^2 + 1}{x(x^2+1)}$. Its domain is $\{x \in \mathbb{R} \mid x \neq 0\}$.

(d) $(g \circ g)(x) = \dfrac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2} = \dfrac{2x+3}{3x+5}$. Its domain is $\{x \in \mathbb{R} \mid x \neq -2, -\frac{5}{3}\}$.

61. (a) $(f \circ g)(x) = \dfrac{\sin 2x}{1 + \sin 2x}$. Its domain is $\{x \in \mathbb{R} \mid x \neq \left(\frac{3}{4} + n\right)\pi\},\ n \in \mathbb{Z}$.

(b) $(g \circ f)(x) = \sin\left(\dfrac{x}{1+x}\right)$. Its domain is $\{x \in \mathbb{R} \mid x \neq -1\}$.

(c) $\left(f\circ f\right)(x)=\dfrac{\dfrac{x}{1+x}}{1+\dfrac{x}{1+x}}=\dfrac{\left(\dfrac{x}{1+x}\right)\cdot(1+x)}{\left(1+\dfrac{x}{1+x}\right)\cdot(1+x)}=\dfrac{x}{2x+1}$ Its domain is $\left\{x\in\mathbb{R}\mid x\neq -1,-\tfrac{1}{2}\right\}$.

(d) $\left(g\circ g\right)(x)=\sin(2\sin 2x)$. Its domain is all real numbers.

62. $\left(f\circ g\circ h\right)(x)=f\left(g\left(x^2\right)\right)=f\left(\sin(x^2\right)=3\sin x^2-2$

63. $\left(f\circ g\circ h\right)(x)=\left|\left(\sqrt{x}\right)^2-4\right|=\left||x|-4\right|$

64. $\left(f\circ g\circ h\right)(x)=f\left(g\left(x^3+2\right)\right)=f\left(\left(x^3+2\right)^2\right)=\sqrt{\left(x^3+2\right)^2-3}=\sqrt{x^6+4x^3+1}$

65. $\left(f\circ g\circ h\right)(x)=f\left(g\left(\sqrt[3]{x}\right)\right)=f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

66. If $f(x)=x^4,\ g(x)=2x+x^2,$ then $\left(f\circ g\right)(x)=\left(2x+x^2\right)^4=F(x)$.

67. If $f(x)=x^2,\ g(x)=\cos x.$ then $\left(f\circ g\right)(x)=\left(\cos x\right)^2=\cos^2 x=F(x)$.

68. If $f(x)=\dfrac{x}{1+x},\ g(x)=\sqrt[3]{x},$ then $\left(f\circ g\right)(x)=\dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}=F(x)$.

69. If $f(x)=\sqrt[3]{x},\ g(x)=\dfrac{x}{1+x},$ then $\left(f\circ g\right)(x)=\sqrt[3]{\dfrac{x}{1+x}}=F(x)$.

70. If $f(t)=\sec t\tan t,\ g(t)=t^2,$ then $\left(f\circ g\right)(t)=f\left(t^2\right)=\sec\left(t^2\right)\tan\left(t^2\right)=F(t)$.

71. If $f(t)=\dfrac{t}{1+t},\ g(t)=\tan t,$ then $\left(f\circ g\right)(t)=f\left(\tan t\right)=\dfrac{\tan t}{1+\tan t}=F(t)$.

72. Let $f(x)=\sqrt{x},\ g(x)=x-1,\ h(x)=\sqrt{x}$. Then
$\left(f\circ g\circ h\right)(x)=f\left(g\left(\sqrt{x}\right)\right)=f\left(\sqrt{x}-1\right)=\sqrt{\sqrt{x}-1}=F(x)$.

73. Let $f(x)=\sqrt[8]{x},\ g(x)=2+x,\ h(x)=|x|$. Then
$\left(f\circ g\circ h\right)(x)=f\left(g\left(|x|\right)\right)=f\left(|x|+2\right)=\sqrt[8]{|x|+2}=F(x)$.

74. Let $f(t)=t^2,\ g(t)=\sin t,\ h(t)=\cos t$. Then
$\left(f\circ g\circ h\right)(t)=f\left(g\left(\cos t\right)\right)=f\left(\sin(\cos t)\right)=(\sin(\cos t))^2=\sin^2(\cos t)=F(t)$.

75. Let $f(t)=\tan t,\ g(t)=\left|\dfrac{1}{t}\right|,\ h(t)=t^3$. Then $\left(f\circ g\circ h\right)(t)=f\left(g\left(t^3\right)\right)=f\left(\left|\dfrac{1}{t^3}\right|\right)=\tan\left|\dfrac{1}{t^3}\right|=F(t)$.

76. (a) $f(g(1))=f(6)=5$ (b) $g(f(1))=g(3)=2$
(c) $f(f(1))=f(3)=4$ (d) $g(g(1))=g(6)=3$
(e) $\left(g\circ f\right)(3)=g(f(3))=g(4)=1$ (f) $\left(f\circ g\right)(6)=f(g(6))=f(3)=4$

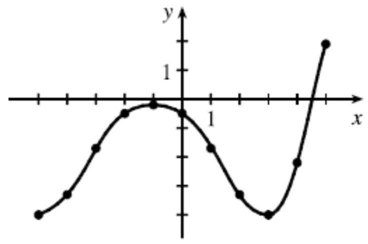
77. (a) $f(g(2))=f(5)=4$ (b) $g(f(0))=g(0)=3$
(c) $\left(f\circ g\right)(0)=f(g(0))=f(3)=0$ (d) $\left(g\circ f\right)(6)=g(f(6))=g(6)=$ undefined

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$ (f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

78. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus $f(g(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table are obtained in a similar fashion.

x	**g(x)**	**f(g(x))**
–5	–0.2	–4
–4	1.2	–3.3
–3	2.2	–1.7
–2	2.8	–0.5
–1	3	–0.2

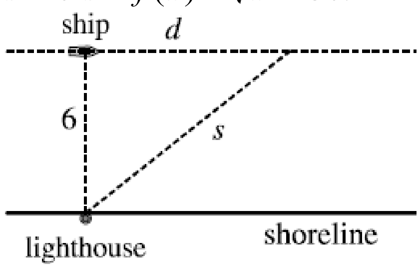
x	**g(x)**	**f(g(x))**
0	2.8	–0.5
1	2.2	–1.7
2	1.2	–3.3
3	–0.2	–4
4	–1.9	–2.2
5	–4.1	1.9



79. (a) Using the relationship *distance* = *rate* · *time* with the radius as the distance, we have $r = 60t$.
(b) $A = \pi r^2 \Rightarrow (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. The area of the circle is increasing at a rate of 3600π cm/s^2.

80. (a) The radius of the balloon is increasing at a rate of 2 cm/s, so $r(t) = 2t$ (in cm)
(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$. This tells us the volume of the balloon (in cm^3) as a function of time in seconds.

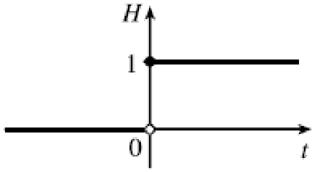
81. From the figure, we have a right triangle with legs of length 6 and *d*, and hypotenuse *s*. By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \Rightarrow s = f(d) = \sqrt{d^2 + 36}$.



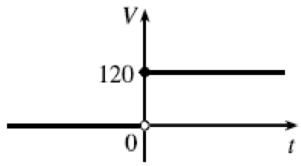
(b) Using $d = rt$, we find d = (30 km)×(t hours) = $30t$ (in km). Thus $d = g(t) = 30t$.
(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{900t^2 + 36}$. This represents the distance between the ship and the lighthouse as a function of the time elapsed since noon.

82. (a) $d = rt \Rightarrow d(t) = 350t$.
(b) There is a Pythagorean relationship involving the legs with length *d* and 1 and the hypotenuse with length *s*: $d^2 + 1^2 = s^2$. Thus, $s(d) = \sqrt{d^2 + 1}$.
(c) $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$

83. (a) Heaviside function: $H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

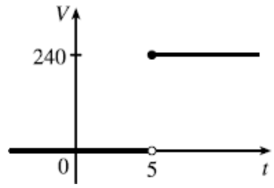


(b)

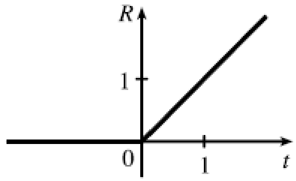


$V(t)=\begin{cases}0, & t<0\\ 120, & t\geq 0\end{cases}$ so $V(t)=120H(t)$.

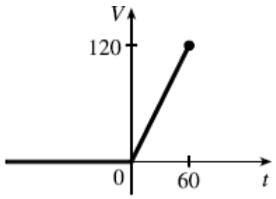
(c) Starting with the equation in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of $t=0,$ we replace t with $t-5$.
Thus the formula is $V(t)=240H(t-5)$.



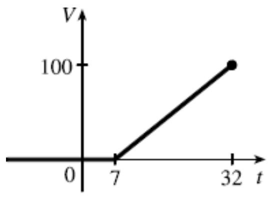
84. (a) $R(t)=tH(t)=\begin{cases}0, & t<0\\ t, & t\geq 0\end{cases}$



(b)



(c) $V(t)=\begin{cases}0, & t<7\\ 4(t-7), & 7\leq t\leq 32\end{cases}$ so $V(t)=4(t-7)H(t-7), t\leq 32.$



85. If $f(x)=m_1x+b_1$ and $g(x)=m_2x+b_2$ then $(f\circ g)(x)=m_1(m_2x+b_2)+b_1=m_1m_2x+(m_1b_2+b_1)$ which is also a linear function. The slope of this function is m_1m_2.

86. If $A(x)=1.04^2x,$ then $(A\circ A)(x)=1.04^2x$, and $(A\circ A\circ A)(x)=1.04^3x$, and $(A\circ A\circ A\circ A)(x)=1.04^4x.$ These compositions represent the investment amounts after 2, 3 and 4 years respectively. After n compositions, the investment amount would be $1.04^n x.$

87. (a) If $g(x)=2x+1$ and $f(x)=x^2+6$ then
$f\circ g=f(2x+1)=(2x+1)^2+6=4x^2+4x+7=h(x).$

(b) If $f(x)=x+4$ and $g(x)=x^2+x-1$ then
$f\circ g=f(x^2+x-1)=3(x^2+x-1)+5=3x^2+3x-3+5=3x^2+3x+2=h(x).$

88. If $f(x)=x+4$ and $h(x)=4x-1$ then $h=g\circ f$ if $g(x)=4x-17$.

89. Suppose g is an even function and $h=f\circ g$. Then $h(-x)=(f\circ g)(-x)=f(g(-x))=f(g(x)=h(x)$ so $h=f\circ g$ is an *even* function.

90. Suppose g is an odd function and $h = f \circ g$. First suppose f is an odd function. Then
$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -h(x)$ so $h = f \circ g$ is an *odd* function.
Now assume f is an even function. Then $h(-x) = (f \circ g)(-x)$
$= f(g(-x)) = f(-g(x)) = f(g(x)) = h(x)$ so $h = f \circ g$ is an *even* function.
If f is neither either nor odd, $h = f \circ g$ will be neither even nor odd.

1.5 Exponential Functions

EXERCISE SOLUTIONS

1. False An exponential function has the form $y = b^x$ where b is any positive constant.
2. False $b^{x+y} = b^x \cdot b^y$
3. True The graph $f(x) = b^x$, $0 < b < 1$ of decreases for all x.
4. True The graph of $f(x) = b^x$, $0 < b < 1$ has a horizontal asymptote at $y = 0$.
5. True The value of 2^{π} is a unique real number.
6. False $\sqrt{16} = 4$
7. The expression (**D**) $y = \dfrac{1}{4^x}$ is equivalent to $y = 16^{-x/2}$.
8. The expression (**B**) $y = 2 - \dfrac{e}{e^x}$ is equivalent to $y = 2 - e^{1-x}$.
9. The graph of $y = \left(\dfrac{1}{2}\right)^{x+3}$ has (**D**) a horizontal asymptote at $y = 0$.
10. To reflect the graph of $y = 2^x$ across the y-axis, we would replace x with $-x$. To shift the graph down 2 units, we would subtract 2 from the original function. The resulting graph would have equation $y = 2^{-x} - 2$.
11. (a) $\dfrac{4^{-3}}{2^{-8}} = \dfrac{2^8}{4^3} = \dfrac{2^8}{\left(2^2\right)^3} = \dfrac{2^8}{2^6} = 2^{8-6} = 2^2 = 4$ (b) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$
12. (a) $8^{4/3} = 2^4 = 16$ (b) $x\left(3x^2\right)^3 = x3^3x^{2^3} = 27x \cdot x^6 = 27x^7$
13. (a) $b^8\left(2b\right)^4 = b^8 \cdot 2^4 \cdot b^4 = 16 \cdot b^{12}$ (b) $\dfrac{\left(6y^3\right)^4}{2y^5} = \dfrac{1296y^{3^4}}{2y^5} = \dfrac{648y^{12}}{y^5} = 648y^7$
14. (a) $5^{3-2\sqrt{7}} \cdot 25^{-1+\sqrt{7}} = \dfrac{5^3}{5^{2\sqrt{7}}} \cdot \dfrac{25^{\sqrt{7}}}{25} = \dfrac{5^3}{5^{2\sqrt{7}}} \cdot \dfrac{5^{2\sqrt{7}}}{5^2} = 5$

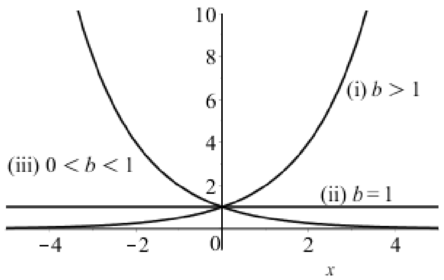
    (b) $\dfrac{4^{n-2} \cdot 8^{2-n}}{16^{2-n}} = \dfrac{4^n \cdot 8^2 \cdot 16^n}{4^2 \cdot 8^n \cdot 16^2} = \dfrac{2^{2n} \cdot 2^6 \cdot 2^{4n}}{2^4 \cdot 2^{3n} \cdot 2^8} = \dfrac{2^{6n} \cdot 2^6}{2^{3n} \cdot 2^{12}} = \dfrac{2^{3n}}{2^6} = 2^{3n-6}$
15. (a) $\left(20\right)^{\frac{1}{2}} + 5^{\frac{3}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} + 5^{\frac{3}{2}} = 2 \cdot 5^{\frac{1}{2}} + 5 \cdot 5^{\frac{1}{2}} = 7 \cdot 5^{\frac{1}{2}} = 7\sqrt{5}$

    (b) $8^{\frac{1}{2}} + 2^{\frac{3}{2}} = 2^{\frac{3}{2}} + 2^{\frac{3}{2}} = 2 \cdot 2^{\frac{3}{2}} = 2^{\frac{5}{2}}$
16. (a) $\left(3^{x+1}\right)\left(9^{2x}\right) = 3^{x+1} \cdot \left(3^2\right)^{2x} = 3^{x+1} \cdot 3^{4x} = 3^{5x+1}$ (b) $\dfrac{3^{\frac{1}{3}}}{3^{-\frac{2}{3}}} = 3^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} = 3$
17. (a) $\dfrac{x^{2n} \cdot x^{3n-1}}{x^{n+2}} = x^{2n+3n-1-(n+2)} = x^{4n-3}$

(b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{a^{\frac{1}{2}}\left(b^{\frac{1}{2}}\right)^{\frac{1}{2}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = \dfrac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}} = a^{\frac{1}{2}-\frac{1}{3}} \cdot b^{\frac{1}{4}-\frac{1}{3}} = a^{\frac{1}{6}} \cdot b^{-\frac{1}{12}}$

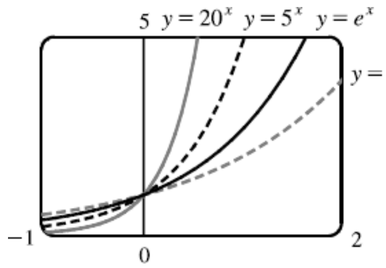
18. (a) $2^{\sqrt{5}}$ is in the interval [4, 5]
(b) $3^{\sqrt{2}}$ is in the interval [4, 5]
(c) $4^{1.6}$ is in the interval [9, 10]
(d) $\sqrt{2}^{\sqrt{2}}$ is in the interval [1, 2]

19. (a) $f(x) = b^x$, $b > 0$
(b) The domain is $\mathbb{R}$.
(c) The range (for $b \neq 1$) is $(0, \infty)$.
(d) Here are graphs:

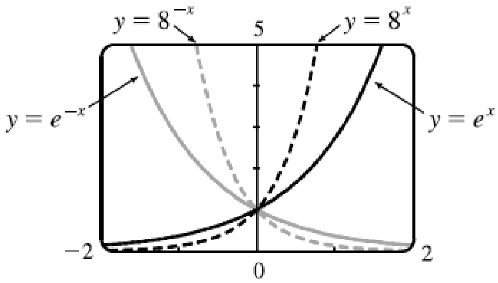


20. (a) The number e is the value of a such that the slop of the tangent line at $x = 0$ on the graph of $y = a^x$ is exactly 1.
(b) $e \approx 2.71828$
(c) $f(x) = e^x$

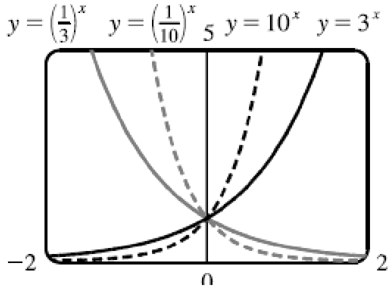
21. All of these graphs approach 0 as $x \to -\infty$, all of them pass through the point (0, 1), and all of them are increasing and approach ∞ as $x \to \infty$. The larger the base, the faster the function increases for $x > 0$, and the faster it approaches 0 as $x \to -\infty$.



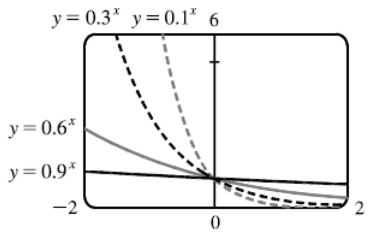
22. The graph of e^{-x} is the reflection of the graph of e^x about the y-axis, and the graph of is the reflection of that of 8^x about the y-axis. The graph of 8^x increases more quickly than that of e^x for $x > 0$, and approaches 0 faster as $x \to -\infty$.



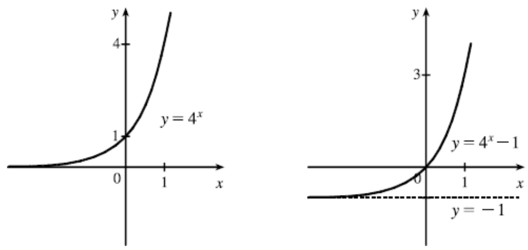
23. The functions with bases greater than 1 (3^x and 10^x) are increasing, whereas those with bases less than 1 $\left(\left(\frac{1}{3}\right)^x \text{ and } \left(\frac{1}{10}\right)^x\right)$ are decreasing. The graph of $\left(\frac{1}{3}\right)^x$ is the reflection of 3^x about the y-axis, and the graph of $\left(\frac{1}{10}\right)^x$ is the reflection of that of 10^x about the y-axis. The graph of 10^x increases more quickly than that of 3^x for $x > 0$, and approaches 0 faster as $x \to -\infty$.



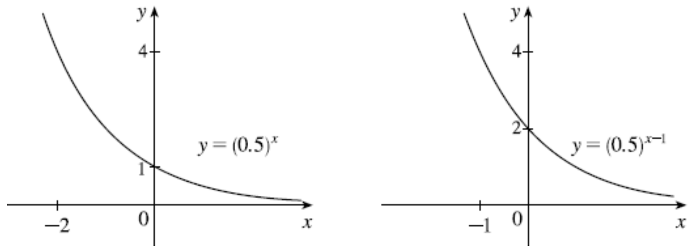
24. Each of these graphs approaches ∞ as $x \to -\infty$, and each approaches 0 as $x \to \infty$. The smaller the base, the faster the function grows as $x \to -\infty$, and the faster it approaches 0 as $x \to \infty$.



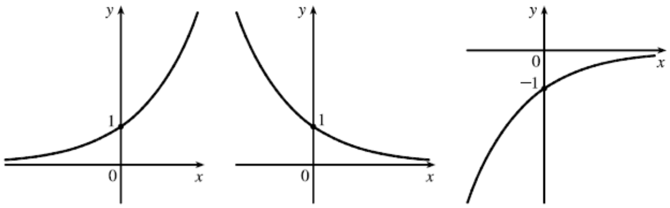
25. If we start with the graph of $y = 4^x$ and shift it 1 unit down to obtain the graph of $y = 4^x - 1$.



26. If we start with the graph of $y = (0.5)^x$ and shift it 1 unit to the right we obtain the graph of $y = (0.5)^{x-1}$.

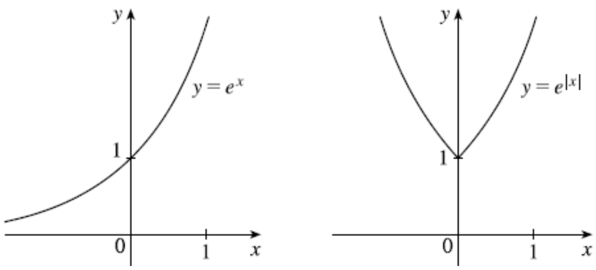


27. If we start with the graph of $y = 2^x$ and reflect it about the y-axis, then about the x-axis, we obtain the graph of $y = -2^{-x}$. In each case, $y = 0$ is the horizontal asymptote.



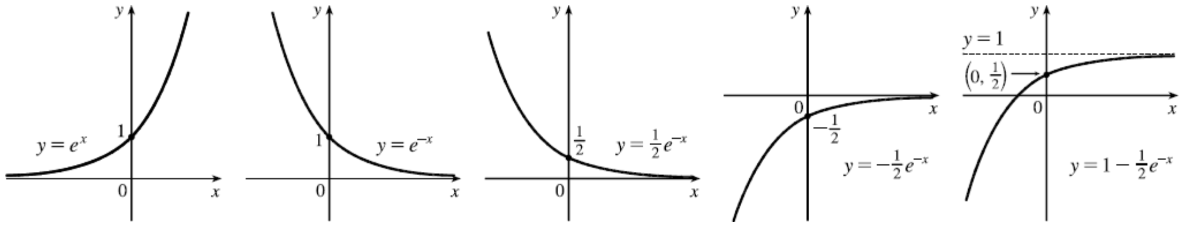
$y = 2^x$ $y = 2^{-x}$ $y = -2^{-x}$

28. If we start with the graph of $y = e^x$ and reflect the portion of the graph in the first quadrant about the y-axis, we obtain the graph of $y = e^{|x|}$.

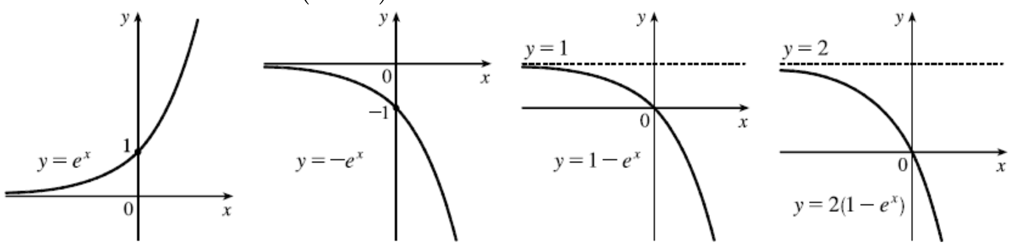


29. We start with the graph of $y = e^x$ and reflect about the y-axis to get the graph of $y = e^{-x}$. Then we compress the graph vertically by a factor of 2 to obtain the graph of $y = \frac{1}{2}e^x$. and then reflect about

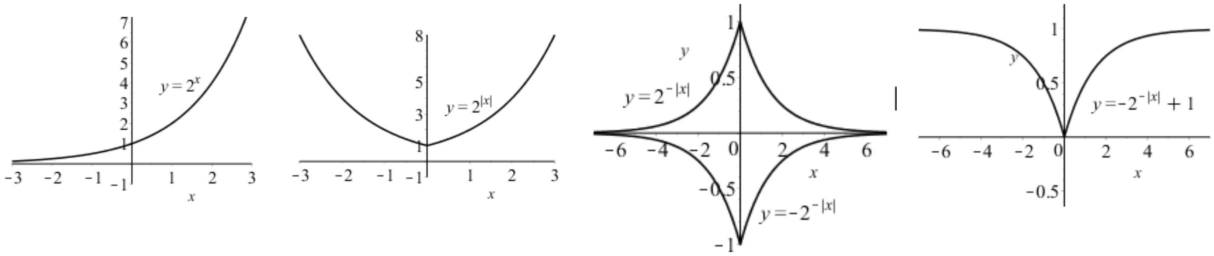
the x-axis to get the graph of $y=-\frac{1}{2}e^{-x}$. Finally we shift the graph up one unit to obtain the graph of $y=1-\frac{1}{2}e^{-x}$.



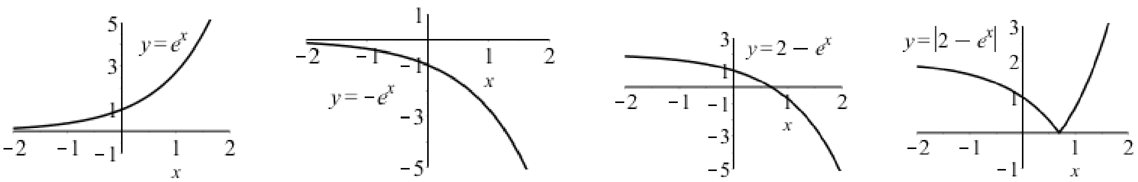
30. We start with the graph of $y=e^x$ and reflect about the x-axis to get the graph of $y=-e^x$. Then shift the graph up one unit to get the graph of $y=1-e^x$. Finally, we stretch the graph vertically by a factor of 2 to obtain the graph of $y=2\left(1-e^x\right)$.



31. We start with the graph of $y=2^x$ and then reflect the portion of the graph where $x > 0$ across the y-axis and add this to the original graph. Then we flip the coordinates of every point on the graph of $y=2^{|x|}$ to get the corresponding values of $y=2^{-|x|}$. Finally, shift the graph up one unit to obtain the graph of $y=2^{-|x|}+1$.



32. We start with the graph of $y=e^x$ and flip across the x-axis to obtain the graph of $y=-e^x$. Then we shift the graph up 2 units to graph $y=-e^x+2$. Finally, we flip the part to the right of $x > \ln(2)$ across the x-axis to obtain the graph $y=\left|-e^x+2\right|$.



33. (a) To find the equation of the graph that results from shifting the graph of $y = e^x$ 2 units downward, we subtract 2 from the original function to get $y=e^x-2$.
(b) To find the equation of the graph that results from shifting the graph of $y = e^x$ two units to the right, we replace x with $x - 2$ in the original function to get $y=e^{(x-2)}$.

(c) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the x-axis, we multiply the original function by –1 to get $y = -e^x$.
(d) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the y-axis, we replace x with $-x$ in the original function to get $y = e^{-x}$.
(e) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the x-axis and then about the y-axis, we first multiply the original function by –1 (to get $y = -e^x$) and then replace x with $-x$ in this equation to get $y = -e^{-x}$.

34. (a) This reflection consists of first reflecting the graph about the x-axis and then shifting this graph by 2·4 = 8 units up. So the equation is $y = -e^x + 8$.
(b) This reflection consists of first reflecting the graph about the y-axis and then shift this graph 2·2 = 4 units to the right. So the equation is $y = e^{-(x-4)}$.

35. The denominator is zero when $1 - e^{1-x^2} = 0 \Leftrightarrow e^{1-x^2} = 1 \Leftrightarrow 1 - x^2 = 0 \Leftrightarrow x = \pm 1$. Thus, the function has domain $\{x \mid x \neq \pm 1\}$.

36. The denominator is never equal to zero, so the function $f(x) = \dfrac{1+x}{e^{\cos x}}$ has domain $\mathbb{R}$.

37. The function $g(t) = \sqrt{10^t - 100}$ has domain $\{t \mid 10^t - 100 \geq 0\} = \{t \mid 10^t \geq 10^2\} = \{t \mid t \geq 2\}$.

38. The sine and exponential functions have domain $\mathbb{R}$, so $g(t) = \sin\left(e^t - 1\right)$ also has domain $\mathbb{R}$.

39. $9^x = \left(\dfrac{1}{27}\right)^{x-2} \Leftrightarrow \left(3^2\right)^x = \left(3^{-\frac{1}{3}}\right)^{x-2} \Leftrightarrow 3^{2x} = 3^{-3(x-2)} \Leftrightarrow 2x = -3x + 6 \Leftrightarrow 5x = 6 \Leftrightarrow x = \dfrac{6}{5}$

40. $\left(\sqrt{2}\right)^{2x-1} = \left(\dfrac{1}{4}\right)^{x+5} \Leftrightarrow 2^{\frac{1}{2}(2x-1)} = 2^{-2(x+5)} \Leftrightarrow \frac{1}{2}(2x-1) = -2(x+5) \Leftrightarrow x - \frac{1}{2} = -2x - 10$
$\Leftrightarrow 3x = -\dfrac{19}{2} \Leftrightarrow x = -\dfrac{19}{6}$

41. $8^x = \left(\sqrt{2}\right)^{2x^2+4} \Leftrightarrow 8^x = 2^{\frac{1}{2}\left(2x^2+4\right)} \Leftrightarrow \left(2^3\right)^x = 2^{x^2+2} \Leftrightarrow 2^{3x} = 2^{x^2+2} \Leftrightarrow 3x = x^2 + 2 \Leftrightarrow x^2 - 3x + 2 = 0$
$\Leftrightarrow (x-1)(x-2) = 0 \Leftrightarrow x = 1$ or $x = 2$

42. $\left(5^{3x}\right)^2 = \left(5^x\right)^3 \cdot 5^{x+6} \Leftrightarrow 5^{6x} = 5^{3x} \cdot 5^{x+6} \Leftrightarrow 5^{6x} = 5^{3x+x+6} \Leftrightarrow 6x = 4x + 6 \Leftrightarrow 2x = 6 \Leftrightarrow x = 3$

43. $4 \cdot 2^x - 6 \cdot 4^x + 8 = 0 \Leftrightarrow -3 \cdot \left(2^x\right)^2 + 2 \cdot 2^x + 4 = 0 \Leftrightarrow 3 \cdot \left(2^x\right)^2 - 2 \cdot 2^x - 4 = 0$ Let $u = 2^x$, and the equation becomes $3 \cdot u^2 - 2 \cdot u - 4 = 0$. Using the quadratic formula, we find that $u = \dfrac{1 \pm \sqrt{13}}{3}$, so
$2x = \dfrac{1 \pm \sqrt{13}}{3} \Leftrightarrow x = \dfrac{1 \pm \sqrt{13}}{6}$.

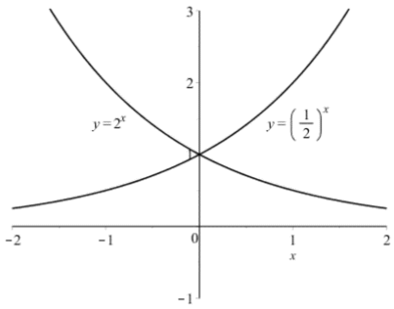
44. $4^{x+4} - 4^{x+3} = 96 \Leftrightarrow 4^x 4^4 - 4^x 4^3 = 96 \Leftrightarrow 4^x 4^3 \left(4 - 1\right) = 96 \Leftrightarrow 4^x = \dfrac{96}{192} \Leftrightarrow 2x = -1 \Leftrightarrow x = -\dfrac{1}{2}$

45. Using the points (1, 6) and (3, 24), we have $6 = Cb^1 \Rightarrow C = \frac{6}{b}$ and $24 = Cb^3 \Rightarrow 24 = \left(\frac{6}{b}\right)b^3 \Rightarrow 4 = b^2$. Then since $b > 0$, $b = 2$ and $C = 6/2 = 3$. The function is $f(x) = 3 \cdot 2^x$.

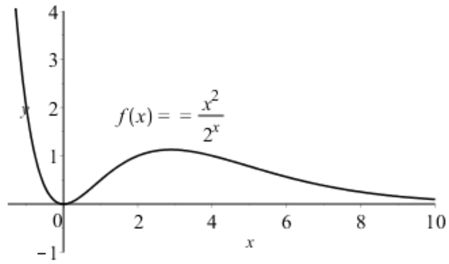
46. Using the points (–1, 3) and (1, 4/3), we have $3 = Cb^{-1} \Rightarrow C = 3b$ and $\frac{4}{3} = Cb^1 \Rightarrow$ $\frac{4}{3} = (3b)b \Rightarrow \frac{4}{9} = b^2$. Then since $b > 0, b = \frac{2}{3}$ and $C = 3 \cdot \frac{2}{3} = 2$. The function is $f(x) = 2 \cdot \left(\frac{2}{3}\right)^x$.

47. This function is $y = e^x$ reflected through both the x- and y-axes, then shifted up 2 units, and then stretched vertically 3 units, because it goes through the point (0, 0) and has a horizontal asymptote at $y = 3$. The equation of this graph is $y = 3\left(1 - e^{-x}\right)$.

48. These graphs are reflections of each other through the y-axis because $y = \left(\frac{1}{2}\right)^x = 2^{-x}$.



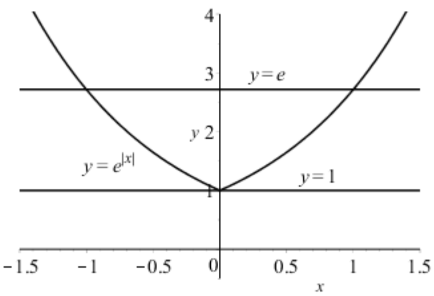
49. From the graph of $f(x) = \dfrac{x^2}{2^x}$ we see that as x increases, the function approaches zero.



50. If $f(x) = 2^x$, the average rate of change of f for $0 \le x \le 4$ is $\dfrac{f(4) - f(0)}{4 - 0} = \dfrac{2^4 - 2^0}{4} = \dfrac{15}{4}$.

51. If $f(x) = 5^x$ we have $\dfrac{f(x+h) - f(x)}{h} = \dfrac{5^{x+h} - 5^x}{h} = \dfrac{5^x 5^h - 5^x}{h} = \dfrac{5^x\left(5^h - 1\right)}{h} = 5^x\left(\dfrac{5^h - 1}{h}\right)$.

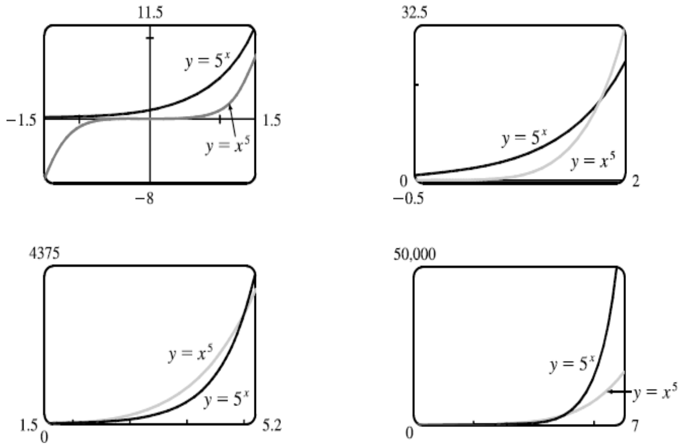
52. From the graph we see that the functions $y = e^{|x|}$ and $y = e$ intersect in two points and that $e^{|x|} \ge 1$ for all x.
The intersections points are (1, e) and –1, e), so $1 \le e^{|x|} \le e$ for $|x| \le 1$ or $-1 \le x \le 1$.



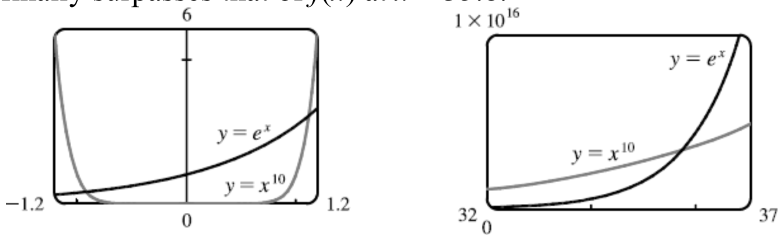
53. $e^x \cdot x^2 - 4e^x = e^x\left(x^2 - 4\right)$. Then because e^x is always positive, we know this function will be positive when $x^2 - 4 > 0$. But $x^2 - 4 = (x-2)(x+2) > 0$ when $x > 2$ or $x < -2$.

54. Suppose the month is February (in a non-leap year). Your payment on the 28th day under plan II would be $2^{28-1} = 2^{27} = 134{,}217{,}728$ cent, or \$1,342,177.28. Clearly the second method of payment results in a larger amount for any month.

55. We see from the graphs that for x less than about 1.8, $g(x)=5^x > f(x)=x^5$, and then near the point (1.765, 17.125), the curves intersect. Then $f(x) > g(x)$ from $x \approx 1.765$ until $x = 5$. At (5, 3125) there is another point of intersection, and for $x > 5$ we see that $g(x) > f(x)$. In fact, g increases much more rapidly than f beyond that point.



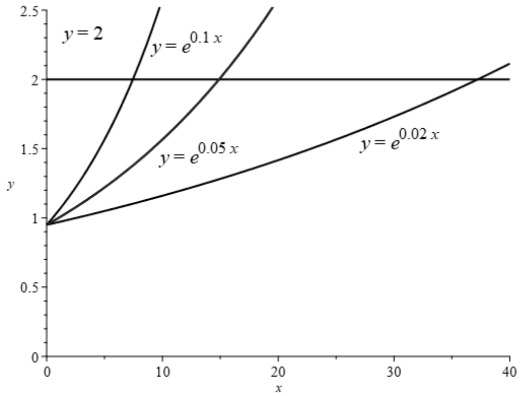
56. The graph of $g(x)$ finally surpasses that of $f(x)$ at $x \approx 35.8$.



57. We graph $y = e^x$, and $y = 1{,}000{,}000{,}000$ and determine where $e^x \approx 1\times10^9$. This seems to be true at $x \approx 20.723$, so $e^x > 1\times10^9$ for $x > 20.723$.

58. (a) Using the Rule of 72, if the interest rate is 10% your money should double in approximately 72/10 = 7.2 years. If your interest rate is 5%, your money should double in approximately72/5 = 14.4 years. If your interest rate is 2%, your money should double in approximately 72/2 = 36 years.

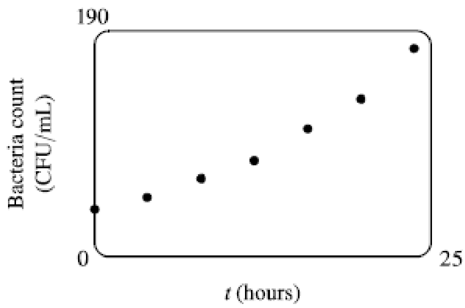
(b) Using technology to solve $0.95e^{rt} = 2$ when r = 10% suggests your money should double in 7.444 years. When r = 0.05, your money should double in 14.889 years, and when r = 0.02, your money should double in 37.222 years.



59. (a) Here is a scatterplot:

(b) Using technology, we obtain the exponential function $f(t)=36.89301(1.06614)^t$.

(c) Using technology, we find that the bacteria count doubles from 37 to 74 in about 10.868 hours.

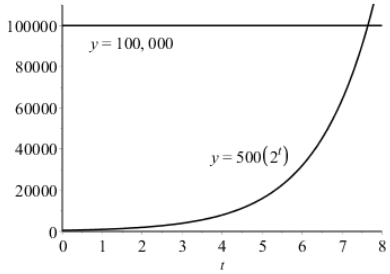


60. (a) Three hours is 3 doubling periods (each doubling period is 1 hour) so there would be 4,000 bacteria after 3 hours.

(b) In t hours there will be t doubling periods. The initial population is 500 so at time t the number of bacteria will be $y = 500\cdot 2^t$.

(c) After 40 minutes there will be $500\cdot 2^{(40/60)} = 500\cdot 2^{(2/3)} \approx 794$ bacteria.

(d) From the graphs of $y_1 = 500 \cdot 2^t$ and $y_2 = 100,000$ we see that the curves intersect at about t ≈ 7.64, so the population reaches 100,000 in about 7.64 hours.

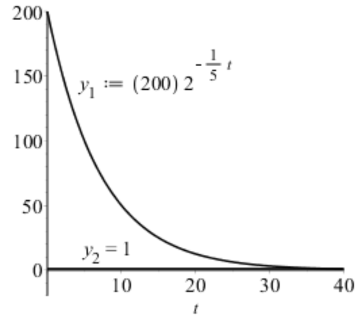


61. (a) Fifteen days is 3 half-life periods (the half-life is 5 days). So $200\left(\frac{1}{2}\right)^3 = 25$ mg.

(b) There would be $t/5$ doubling periods after t days. The initial population is 200 mg so after t days the amount of ^{210}Bi is $y = 200\left(\frac{1}{2}\right)^{t/5} = 200 \cdot 2^{-t/5}$ mg.

(c) t = 3 weeks is 21 days $\Rightarrow$ $y = 200 \cdot 2^{-21/5} \approx 10.882$ mg.

(d) We graph $y_1 = 200 \cdot 2^{-t/5}$ and $y_2 = 1$. The two curves intersect at $t \approx 38.219$, so the mass will be reduced to 1 mg in about 38.219 days.



62. (a) Sixty hours is 4 half-life periods. So $2 \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{8}$ g.

(b) In t hours, there will be t/15 half-life periods. The initial mass is 2 g, so the mass y at time t is $y = 2 \cdot \left(\frac{1}{2}\right)^{t/15}$.

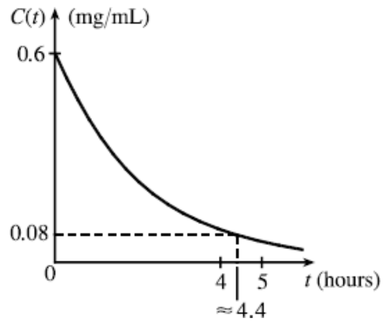
(c) 4 days = 4·24 = 96 hours. So after 4 days the amount of ^{24}Na remaining is $2 \cdot \left(\frac{1}{2}\right)^{96/15} \approx 0.024$ g.

(d) $y = 0.01 \Rightarrow t = 114.658$ hours.

63. From Table 1.6 we see that $V(1) = 76$. Using the graph in Figure 1.87, we estimate that $V = 38$ (half of 76) when $t \approx 4.5$. This gives us a half-life of 4.5 – 1 = 3.5 days.

64. (a) The exponential decay model has form $C(t) = a\left(\frac{1}{2}\right)^{t/1.5}$, where t is the number of hours after midnight and $C(t)$ is the BAC. When $t = 0$, $C(t) =$ 0.6, so $0.6 = a\left(\frac{1}{2}\right)^0 \Leftrightarrow a = 0.6$. Thus the model is $C(t) = 0.6\left(\frac{1}{2}\right)^{t/1.5}$.

(b) From the graph, we estimate that the BAC is 0.08 mg/mL when $t \approx 4.4$ hours. (Note that the legal limit is often 0.08%, which is not 0.08 mg/mL.)



65. Let $t = 0$ correspond to 1950 to get the model $P = ab^t$, where $a \approx 2614.086$ and $b \approx 1.10693$. To estimate the population in 1993, let $t = 43$ to obtain $P \approx 5381$ million. To predict the population in 2020, let $t = 70$ to find $P \approx 8466$ million.

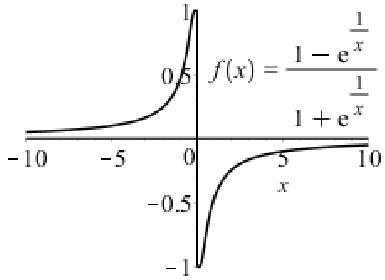
66. Let $t = 0$ correspond to 1900 to get the model $P = ab^t$, where $a \approx 80.8469$ and $b \approx 1.01269$. To estimate the population in 1925, let $t = 25$ to obtain $P \approx 111$ million. To predict the population in 2020, let $t = 120$ to find $P \approx 367$ million.

67. We begin by simplifying $x^a \cdot \left(x^{a+1}\right)^a \cdot \left(x^a\right)^{1-a} = x^a \cdot x^{a(a+1)} \cdot x^{a(1-a)} = x^a \cdot x^{a^2+a} \cdot x^{a-a^2} = x^{3a}$.
Therefore $k = 3a$.

68. If m, n, and p are positive integers and $3^m \cdot 3^n \cdot 3^p = 81$ then $3^{m+n+p} = 3^4 \Rightarrow m+n+p=4$. The maximum value of p will occur when m and n are the smallest positive integers (1) and in this case $p = 4 - m - n = 4 - 1 - 1 = 2$.

69. If $p^m \cdot p^6 = p^{12}$ then $p^m = \dfrac{p^{12}}{p^6} = p^6 \Rightarrow m = 6$. If $\left(p^3\right)^n = p^{21}$ then $p^{3n} = p^{21} \Rightarrow 3n = 21 \Rightarrow n = 7$.
Therefore the value of $n - m = 7 - 6 = 1$.

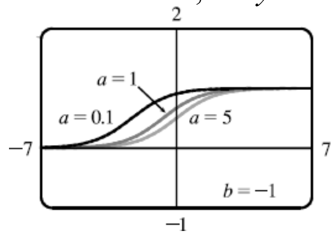
70. From the graph it appears that f is an odd function (f is undefined for $x = 0$). To confirm this, we must show that $f(-x) = -f(x)$:



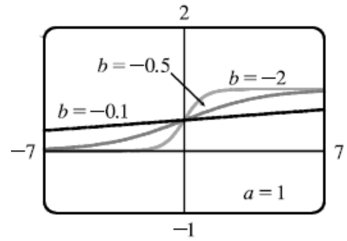
$$f(-x) = \frac{1-e^{1/(-x)}}{1+e^{1/(-x)}} = \frac{1-e^{(-1/x)}}{1+e^{(-1/x)}} = \frac{1-\dfrac{1}{e^{1/x}}}{1+\dfrac{1}{e^{1/x}}} \cdot \frac{e^{1/x}}{e^{1/x}} = \frac{e^{1/x}-1}{e^{1/x}+1}$$

$= -\dfrac{1-e^{1/x}}{1+e^{1/x}} = -f(x)$, so f is an odd function.

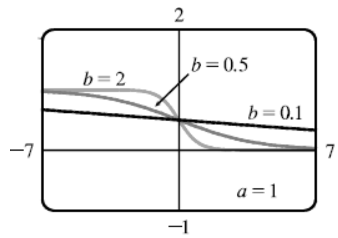
71. If we start with $b = -1$ and graph $f(x) = \dfrac{1}{1+ae^{bx}}$ for $a = 0.1$, 1, and 5, we see there is a horizontal asymptote of $y = 0$ as $x \to -\infty$ and a horizontal asymptote of $y = 1$ as $x \to \infty$. If $a = 1$, the y-intercept is As a gets smaller (closer to 0), the graph of f moves to the left.
As a gets larger, the graph of f moves to the right.



As b changes from –1 to 0, the graph of f is stretched horizontally.
As b changes through large negative values, the graph of f is compressed horizontally.



If b is positive, the graph of f is reflected through the y-axis.
Lastly, if $b = 0$, the graph of f is the horizontal line $y = 1/\,(1 + a)$.



1.6 Inverse Functions and Logarithms

EXERCISE SOLUTIONS

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1. False — Not all functions have inverses. In particular, functions that are not one-to-one do not have inverses.
2. True — A one-to-one function has exactly one range value for every element of the domain.
3. False — f and g are inverse functions only if $f\left(g\left(x\right)\right)=g\left(f\left(x\right)\right)=x$.
4. False — The expression $y=\log_b x$ is equivalent to $b^y=x$.
5. True
6. False — $\ln 8\approx 2.079$, but $\log 8\approx 0.9031$
7. True
8. True
9. False — The linear function $y=c$ does not have an inverse because it is not one-to-one.
10. True — $\sin^{-1}(-x)=-\sin^{-1}(x)$.
11. True — $\sec^{-1}(3)=\cos^{-1}\left(\frac{1}{3}\right)$.
12. The range of $y=\left(\dfrac{1}{2}\right)^{1-x}$ is (**A**) $\left(0,\infty\right)$.
13. The domain of $y=\sin^{-1}2x$ is (**D**) $\left[-\frac{1}{2},\frac{1}{2}\right]$.
14. The domain of the inverse of $y=2-\sqrt{1-x}$ is (**C**) $\left(-\infty,1\right]$.
15. If $f(x)=5x^3-x$ and g is a function such that $g(f(x))=x$, then $g(4)=1$, (**A**).
16. $\dfrac{\ln a}{\ln b}=\log_b a$, (**C**).
17. $\ln\left(x^2-y^2\right)=\ln\left[(x+y)(x-y)\right]=\ln(x+y)+\ln(x-y)$, (**C**).
18. (a) A function is one-to-one if $f(x_1)\neq f(x_2)$ whenever $x_1\neq x_2$.
    (b)The graph of a one-to-one function must pass the horizontal line test.
19. (a) $f^{-1}(y)=x \Leftrightarrow f(x)=y$ for any y in B. The domain of f^{-1} is B and the range of f^{-1} is A.
    (b) Write $y=f(x)$. Solve this equation for x in terms of y. Interchange the variables x and y to express f^{-1} as a function of x.
    (c) Reflect the graph of f about the line $y=x$.
20. Check that each point (a,b) on the graph of f is reflected across the line $y=x$ to obtain the point (b,a) on the graph of f^{-1}.
21. The function f is not one-to-one because $2\neq 6$ but $f(2)=f(6)=2$.
22. The function f is one-to-one because each x corresponds to a unique y.
23. Because there are horizontal lines that intersect the graph in more than one point, this function is not one-to-one.
24. No horizontal line intersects the graph more than once, so this function is one-to-one.
25. No horizontal line intersects the graph more than once, so this function is one-to-one.

26. Because there are horizontal lines that intersect the graph in more than one point, this function is not one-to-one.
27. The graph of $f(x)=2x-3$ is a line with slope 2. It passes the horizontal line test, so f is one-to-one.
28. If $f(x)=x^4-16,$ then $f(-1)=f(1)=-15$ so f is not one-to-one.
29. If $g(x)=1-\sin x,$ then $g(0)=g(\pi)=1,$ so g is not one-to-one.
30. The graph of $g(x)=\sqrt[3]{x}$ passes the Horizontal Line Test so g is one-to-one.
31. A football will reach every height h up to its maximum height twice: once on the way up and once on the way down. Thus f is not a one-to-one function
32. Eventually, we all stop growing and remain at a fixed height for a while. Therefore the function f is not one-to-one.
33. (a) Because f is one-to-one, if $f(6)=17,$ then $f^{-1}(17)=6.$

    (b) Because f is one-to-one, if $f^{-1}(3)=2,$ then $f(2)=3.$
34. Observe that f is a one-to-one function (f is an increasing function). By inspection, $f(1)=3$, so $f^{-1}(3)=1.$ Because f is one-to-one, $f\left(f^{-1}(2)\right)=2.$
35. Because g is an increasing function, g is one-to-one. By inspection $g(0)=4$, so $g^{-1}(4)=0.$
36. (a) The function f is one-to-one because it passes the horizontal line test.

    (b) The domain of f^{-1} is the range of f which is [–1, 3]. The range of f^{-1} is the domain of f which is [–3, 3].

    (c) $f^{-1}(2)=0$

    (d) Because $f(-1.7)\approx 0,$ $f^{-1}(0)\approx -1.7.$
37. Solve $C=\frac{5}{9}\left(F-32\right)$ for F: $\frac{9}{5}C=F-32 \Rightarrow F=\frac{9}{5}C+32.$ This provides a formula for the Fahrenheit temperature F as a function of the Celsius temperature C.

    $F\geq -459.67 \Rightarrow \frac{9}{5}C+32\geq -459.67 \Rightarrow \frac{9}{5}C\geq -491.67 \Rightarrow C\geq -273.15,$ the domain of the inverse function.
38. $m=\dfrac{m_0}{\sqrt{1-v^2/c^2}} \Rightarrow 1-\dfrac{v^2}{c^2}=\dfrac{m_0^2}{m^2} \Rightarrow \dfrac{v^2}{c^2}=1-\dfrac{m_0^2}{m^2} \Rightarrow v^2=c^2\left(1-\dfrac{m_0^2}{m^2}\right) \Rightarrow v=c\sqrt{1-\dfrac{m_0^2}{m^2}}.$

    This provides a formula for the speed of the particle v in terms of its mass, m, i.e. $v=f^{-1}(m)$.
39. $y=f(x)=1+\sqrt{2+3x} \Rightarrow y-1=\sqrt{2+3x} \Rightarrow \left(y-1\right)^2=2+3x \Rightarrow \left(y-1\right)^2-2=3x \Rightarrow$

    $x=\frac{1}{3}\left(y-1\right)^2-\frac{2}{3}.$ So $f^{-1}(x)=\frac{1}{3}\left(x-1\right)^2-\frac{2}{3}.$ The domain of f^{-1} is $x\geq 1.$
40. $y=f(x)=\dfrac{4x-1}{2x+3} \Rightarrow y(2x+3)=4x-1 \Rightarrow 2xy+3y-4x-1=2+3x \Rightarrow 3y+1=4x-2xy \Rightarrow$

    $3y+1=x(4-2y) \Rightarrow x=\dfrac{3y+1}{4-2y}.$ So $f^{-1}(x)=\dfrac{3x+1}{4-2x}.$
41. $y=f(x)=e^{2x-1} \Rightarrow \ln y=2x-1 \Rightarrow 1+\ln y=2x \Rightarrow x=\frac{1}{2}\left(1+\ln y\right).$ So, $f^{-1}(x)=\dfrac{1}{2}(1+\ln x).$
42. $y=f(x)=x^2-x \Rightarrow y=x^2-x+\frac{1}{4}-\frac{1}{4} \Rightarrow y=\left(x-\frac{1}{2}\right)^2-\frac{1}{4} \Rightarrow y+\frac{1}{4}=\left(x-\frac{1}{2}\right)^2 \Rightarrow x-\frac{1}{2}=\sqrt{y+\frac{1}{4}} \Rightarrow$

    $x=\frac{1}{2}+\sqrt{y+\frac{1}{4}}.$ So $f^{-1}(x)=\frac{1}{2}+\sqrt{x+\frac{1}{4}}.$

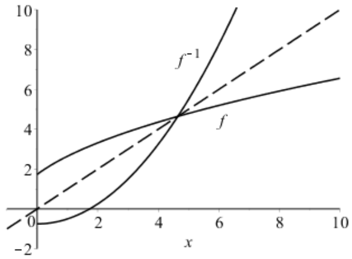
43. $y = f(x) = \ln(x+3) \Rightarrow x+3 = e^y \Rightarrow x = e^y - 3$. So $f^{-1}(x) = e^x - 3$.

44. $y = f(x) = \dfrac{1-e^{-x}}{1+e^{-x}} \Rightarrow y\left(1+e^{-x}\right) = 1-e^{-x} \Rightarrow y + ye^{-x} = 1-e^{-x} \Rightarrow ye^x + y = e^x - 1 \Rightarrow ye^x - e^x = -y-1$

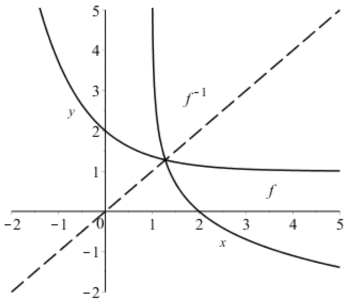
$\Rightarrow e^x = \dfrac{1+y}{1-y} \Rightarrow x = \ln\left(\dfrac{1+y}{1-y}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{1+x}{1-x}\right)$.

45. $y = f(x) = \sqrt{4x+3} \Rightarrow y^2 = 4x+3 \Rightarrow x = \dfrac{y^2-3}{4}$.

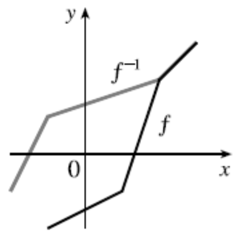
So $f^{-1}(x) = \dfrac{x^2-3}{4}$.



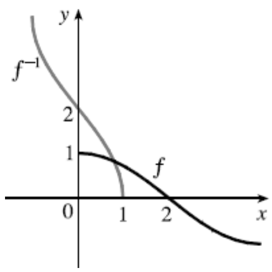
46. $y = f(x) = 1 + e^{-x} \Rightarrow y - 1 = e^{-x} \Rightarrow \ln(y-1) = -x \Rightarrow x = -\ln(y-1)$. So $f^{-1}(x) = -\ln(x-1)$.



47. Reflect the graph of f about the line $y = x$. The points $(-1, -2)$ and $(1, -1)$, $(2, 2)$ and $(3, 3)$ on f are reflected to $(-2, -1)$, $(-1, 1)$, $(2, 2)$ and $(3, 3)$ on f^{-1}.

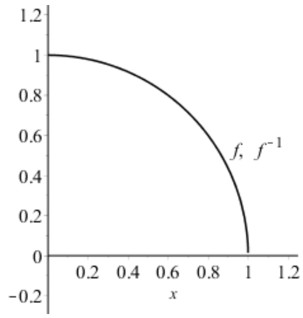


48. Reflect the graph of f about the line $y = x$.

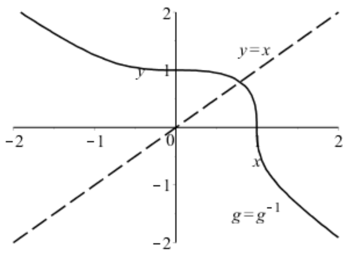


49. (a) $y = f(x) = \sqrt{1-x^2}$ $(0 \le x \le 1$ and note that $y \ge 0) \Rightarrow y^2 = 1 - x^2 \Rightarrow$
$x^2 = 1 - y^2 \Rightarrow x = \sqrt{1-y^2}$. So $f^{-1}(x) = \sqrt{1-x^2}$, $0 \le x \le 1$. In this case, f and f^{-1} are the same function.

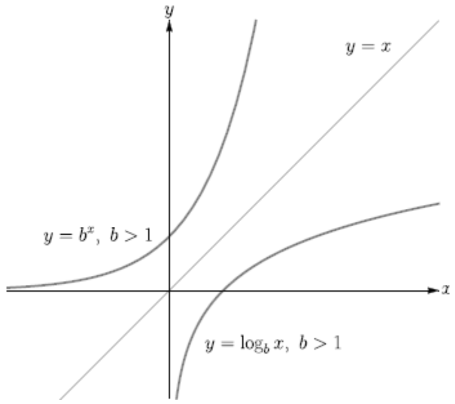
(b) The graph of f is the portion of the circle $x^2 + y^2 = 1$ with $0 \le x \le 1$ and $0 \le y \le 1$ (the quarter-circle in the first quadrant). The graph of f is symmetric with respect to the line $y = x$, so its reflection about $y = x$ is itself, that is, $f^{-1} = f$.



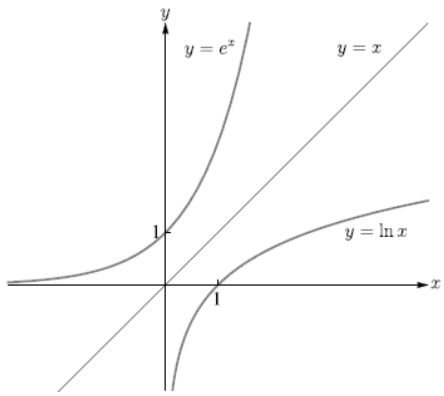
50. (a) $y = g(x) = \sqrt[3]{1-x^3} \Rightarrow y^3 = 1-x^3 \Rightarrow x^3 = 1-y^3 \Rightarrow x = \sqrt[3]{1-y^3}$.
So $g^{-1}(x) = \sqrt[3]{1-y^3}$. In this case, g and g^{-1} are the same function.
(b) The graph of g is symmetric with respect to the line $y = x$, so its reflection about $y = x$ is itself, that is, $g^{-1} = g$.



51. (a) The function $\log_b x = y$ is defined as the inverse of the exponential function with base b, that is
$\log_b x = y \Leftrightarrow b^y = x$.
(b) $(0, \infty)$
(c) $\mathbb{R}$
(d) Graph



52. (a) The natural logarithm is the logarithm with base e, denoted $\ln x$.
(b) The common logarithm is the logarithm with base 10, denoted $\log x$.
(c) Graph



53. (a) $\log_2 32 = \log_2 2^5 = 5$.

(b) $\log_8 2 = \log_8 8^{1/3} = \dfrac{1}{3}$.

54. (a) $\log_5 \dfrac{1}{125} = \log_5 \dfrac{1}{5^3} = \log_5 5^{-3} = -3$.

(b) $\ln\left(1/e^2 \right) = \ln e^{-2} = -2$.

55. (a) $\log_{10} 40 + \log_{10} 2.5 = \log_{10}\left[(40)(2.5)\right] = \log_{10} 100 = \log_{10} 10^2 = 2$

(b) $\log_8 60 - \log_8 3 - \log_8 5 = \log_8 \frac{60}{3} - \log_8 5 = \log_8 20 - \log_8 5 = \log_8 \frac{20}{5} = \log_4 = \log_8 8^{2/3} = \dfrac{2}{3}$

56. (a) $e^{-\ln 2} = \dfrac{1}{e^{\ln 2}} = \dfrac{1}{2}$ (b) $e^{\ln 2\left(\ln e^3\right)} = e^{\ln 3} = 3$

57. $\ln 10 + 2\ln 5 = \ln 10 + \ln 5^2 = \ln\left[(10)(25)\right] = \ln 250$

58. $\ln b + 2\ln c - 3\ln d = \ln b + \ln c^2 - \ln d^3 = \ln bc^2 - \ln d^3 = \ln \dfrac{bc^2}{d^3}$

59. $\frac{1}{3}\ln(x+3)^2 + \frac{1}{2}\left[\ln x - \ln\left(x^2+3x+2\right)^2\right] = \ln\left[(x+2)^3\right]^{1/3} + \frac{1}{2}\ln\dfrac{x}{\left(x^2+3x+2\right)^2} = \ln(x+2) + \ln\dfrac{\sqrt{x}}{\left(x^2+3x+2\right)^2}$

$$= \ln\frac{(x+2)\sqrt{x}}{(x+1)(x+2)} = \ln\frac{\sqrt{x}}{x+1}$$

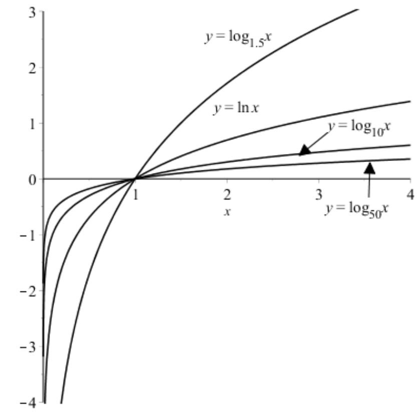
60. $\ln\left(1+e^{2x}\right) - \ln\left(1+e^{-2x}\right) = \ln\left(\dfrac{1+e^{2x}}{1+e^{-2x}}\right)$

61. $(\ln p)\left(\log_p e\right)\left(\sqrt{e}\right)^{\ln p} = (\ln p)\cdot\dfrac{\ln e}{\ln p}\cdot\left(e^{\frac{1}{2}}\right)^{\ln p} = \ln e\cdot\left(e^{\ln p}\right)^{\frac{1}{2}} = 1\cdot p^{\frac{1}{2}} = \sqrt{p}$

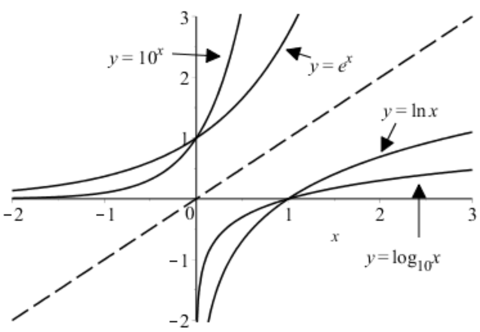
62. $\log_5 10 = \dfrac{\ln 10}{\ln 5} \approx 1.430677$

63. $\log_3 57 = \dfrac{\ln 57}{\ln 3} \approx 3.680144$

64. To graph these functions, we use $\log_{1.5} x = \dfrac{\ln x}{\ln 1.5}$ and $\log_{50} x = \dfrac{\ln x}{\ln 50}$. These graphs all approach $-\infty$ as $x \to 0^+$, and they all pass through the point (1, 0). They are all increasing, and all approach ∞ as $x \to \infty$. The functions with larger bases do so somewhat more quickly than those with smaller bases. The functions with larger bases also approach the y-axis more closely as $x \to 0^+$.



65. The plot shows that the graph of $\ln x$ is the reflection of the graph of e^x about the line $y = x$, and the graph of $\log_{10} x$ is the reflection of the graph of 10^x about the same line. The graph of 10^x increases more quickly than that of e^x. In addition, the graph of $\log_{10} x$ approaches ∞ as $x \to \infty$ more slowly than the graph of $\ln x$.

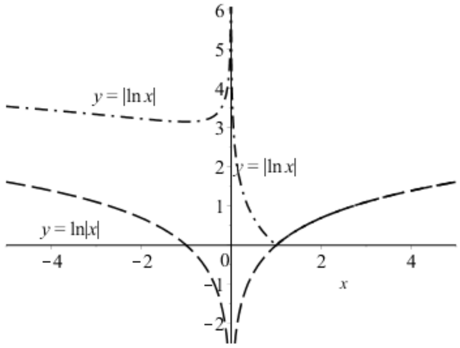


66. $\dfrac{\ln(x+h) - \ln x}{h} = \dfrac{1}{h}\ln\left(\dfrac{x+h}{h}\right) = \ln\left(\dfrac{x+h}{h}\right)^{1/h} = \ln\left(1+\dfrac{x}{h}\right)^{1/h}$

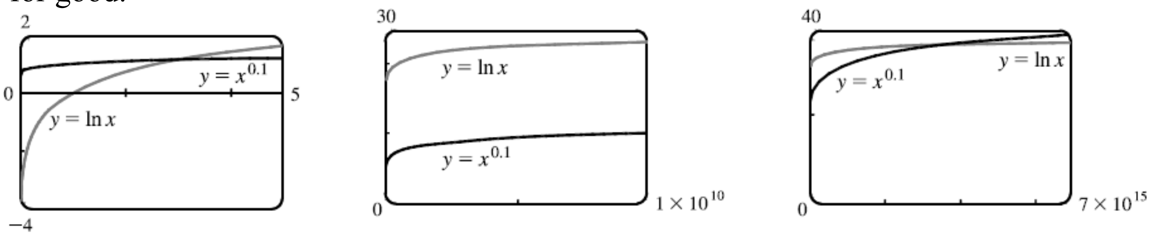
67. 3 ft = 36 in, so we need to find x so that $\log_2 x = 36 \Leftrightarrow x = 2^{36} = 68{,}719476{,}736$. In miles, this is

$$68{,}719476{,}736 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx 1{,}084{,}587.7 \text{ mi}.$$

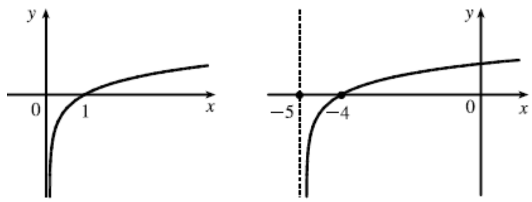
68. The graphs overlap for $x \geq 1$, and both have a vertical asymptote at $y = 0$. However, $y = |\ln x|$ is symmetric about the y-axis whereas $y = \ln|x|$ is not, and $y = |\ln x|$ approaches $-\infty$ as $x \to 0$ but $y = |\ln x| \to \infty$ as $x \to 0$.



69. From the graphs we see that $f(x) = x^{0.1} > g(x) = \ln x$ for approximately $0 < x < 3.06$, and then $g(x) > f(x)$ for roughly $3.06 < x < 3.43 \times 10^{15}$. At that point, the graph of f finally surpasses the graph of g for good.

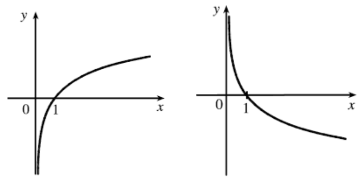


70. Shift the graph of $y = \log_{10} x$ five units to the left to obtain the graph of $y = \log_{10}(x+5)$. Note the vertical asymptote of $x = -5$.



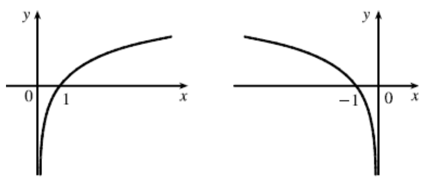
$y = \log_{10} x$ $\quad$ $y = \log_{10}(x+5)$

71. Reflect the graph of $y = \ln x$ about the x-axis to obtain the graph of $y = -\ln x$.



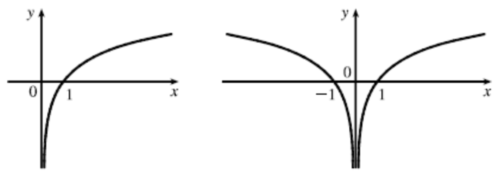
$y = \ln x$ $\quad$ $y = -\ln x$

72. Reflect the graph of $y = \ln x$ about the y-axis to obtain the graph of $y = \ln(-x)$.



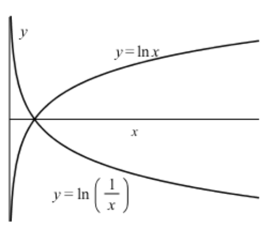
$y = \ln x$ $\quad$ $y = \ln(-x)$

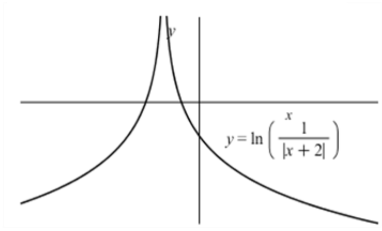
73. Reflect the portion of the graph $y = \ln x$ to the right of the y-axis about the y-axis. The graph of $y = \ln|x|$ is that reflection in addition to the original portion.



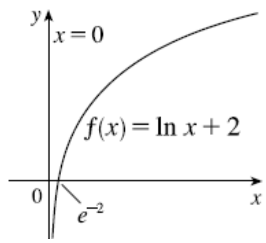
$y = \ln x$ $\quad$ $y = \ln|x|$

74. Start with a graph of $y = \ln x$ and reflect across the x-axis to obtain the graph of $y = \ln\dfrac{1}{x}$. Then reflect the portion of this graph in Quadrant I acrros the y-axis and combine with that portion to obtain the graph of $y = \ln\dfrac{1}{|x|}$. Finally, shift the graph 2 units to the left.

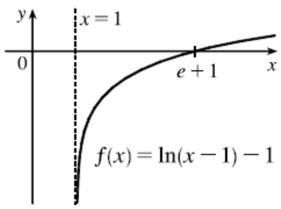




75. (a) The domain of $f(x) = \ln x + 1$ is $x > 0$ and the range is $\mathbb{R}$.
(b) $y = 0 \Rightarrow \ln x + 1 = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$
(c) Shift the graph of $y = \ln x$ up one unit.



76. (a) The domain of $f(x) = \ln(x-1) - 1$ is $x > 1$ and the range is $\mathbb{R}$.
(b) $y = 0 \Rightarrow \ln(x-1) = 1 \Rightarrow x - 1 = e^1 \Rightarrow x = e + 1$
(c) Shift the graph of $y = \ln x$ one unit to the right and down one unit.



77. $e^{7-4x} = 6 \Leftrightarrow 7 - 4x = \ln 6 \Leftrightarrow 7 - \ln 6 = 4x \Leftrightarrow x = \frac{1}{4}(7 - \ln 6)$

78. $\ln(3x - 10) = 2 \Leftrightarrow 3x - 10 = e^2 \Leftrightarrow 3x = e^2 + 10 \Leftrightarrow x = \frac{1}{3}(e^2 + 10)$

79. $\ln(x^2 - 1) = 3 \Leftrightarrow x^2 - 1 = e^3 \Leftrightarrow x^2 = e^3 + 1 \Leftrightarrow x = \pm\sqrt{e^3 + 1}$

80. $e^{2x} - 3e^x + 2 = 0 \Leftrightarrow (e^x - 1)(e^x - 2) = 0 \Leftrightarrow e^x = 1$ or $e^x = 2 \Leftrightarrow x = \ln 1$ or $x = \ln 2$, so $x = 0$ or $x =$ ln 2.

81. $2^{x-5} = 3 \Leftrightarrow x - 5 = \log_2 3 \Leftrightarrow x = \log_2 3 + 5$ or

$2^{x-5} = 3 \Leftrightarrow \ln(2^{x-5}) = \ln 3 \Leftrightarrow (x-5)\ln 2 = \ln 3 \Leftrightarrow x - 5 = \dfrac{\ln 3}{\ln 2} \Leftrightarrow x = 5 + \dfrac{\ln 3}{\ln 2}$

82. $\ln x + \ln(x-1) = 1 \Leftrightarrow \ln(x\cdot(x-1)) = 1 \Leftrightarrow x(x-1) = e^1 \Leftrightarrow x^2 - x - e = 0$. The quadratic formula (with $a = 1$, $b = -1$, and $c = -e$) gives $x = \frac{1}{2}\left(1 \pm \sqrt{1+4e}\right)$, but we reject the negative root since the natural log is not defined for $x < 0$. So $x = \frac{1}{2}\left(1+\sqrt{1+4e}\right)$.

83. $\ln x + \ln(x-1) = 1 \Leftrightarrow \ln(\ln x) = 1 \Leftrightarrow e^{\ln(\ln x)} = e \Leftrightarrow \ln x = e \Leftrightarrow e^{\ln x} = e^e \Leftrightarrow x = e^e$.

84. $e^{ax} = Ce^{bx} \Leftrightarrow \ln\left(e^{ax}\right) = \ln\left[C\left(e^{ax}\right)\right] \Leftrightarrow ax = \ln C + \ln e^{bx} \Leftrightarrow ax = \ln C + bx \Leftrightarrow ax - bx = \ln C$

$\Leftrightarrow (a-b)x = \ln C \Leftrightarrow x = \dfrac{\ln C}{a-b}$

85. $\log(x-5) - \ln 6 > 0 \Rightarrow \log(x-5) > \ln 6 \Rightarrow x - 5 > 10^{\ln 6} \Rightarrow x > 5 + 10^{\ln 6}$

86. $\log_4(x+6) + \log_4(x-3) \geq 1 \Rightarrow \log_4\left((x+6)(x-3)\right) \geq 1 \Rightarrow (x+6)(x-3) \geq 4^1 \Rightarrow$

$x^2 + 3x - 18 \geq 4 \Rightarrow x^2 + 3x - 22 \geq 0 \Rightarrow \left(x + \left(\frac{3+\sqrt{97}}{2}\right)\right)\left(x - \left(\frac{3+\sqrt{97}}{2}\right)\right) \geq 0 \Rightarrow x \geq \dfrac{-3+\sqrt{97}}{2}$.

87. $2\ln x = \left(\ln x\right)^{-1} + 1 \Rightarrow 2\ln x - \ln x - 1 = 0 \Rightarrow (2\ln x + 1)(\ln x - 1) = 0 \Rightarrow \ln x = -\dfrac{1}{2}$ or $\ln x = 1 \Rightarrow$

$x = e^{-1/2}$ or $x = e$

88. $\ln(\cos x) \leq 1 \Rightarrow \cos x \leq e$, but since $|\cos(x)| \leq 1 < e \approx 2.71828$, this inequality is true for all given x.

89. $|\ln x| < 1 \Leftrightarrow -1 < \ln x < 1 \Leftrightarrow e^{-1} < x < e^1 \; or \; x \in \left(\frac{1}{e}, e\right)$

90. $\ln x < 0 \Rightarrow x < e^0 \Rightarrow x < 1$. Since the domain of $f(x) = \ln x$ is $x > 0$, the solution of the original inequality is $0 < x < 1$.

91. $e^x > 5 \Rightarrow \ln e^x > \ln 5 \Rightarrow x > \ln 5$

92. $1 < e^{3x-1} < 2 \Rightarrow \ln < 3x - 1 < \ln 2 \Rightarrow 0 < 3x - 1 < \ln 2 \Rightarrow 1 < 3x < 1 + \ln 2 \Rightarrow \frac{1}{3} < x < \frac{1}{3}\left(1+\ln 2\right)$

93. $1 - 2\ln x < 3 \Rightarrow -2\ln x < 2 \Rightarrow \ln x > -1 \Rightarrow x > e^{-1}$

94. (a) We need $e^x - 3 > 0 \Leftrightarrow e^x > 3 \Leftrightarrow x > \ln 3$. Thus the domain of $f(x) = \ln\left(e^x - 3\right)$ is $\left(\ln 3, \infty\right)$.

(b) $y = \ln\left(e^x - 3\right) \Rightarrow e^y = e^x - 3 \Rightarrow e^x = e^y + 3 \Rightarrow x = \ln\left(e^y + 3\right)$, so $f^{-1}(x) = \ln\left(e^x + 3\right)$.

Now $e^x + 3 > 0 \Rightarrow e^x > -3$, which is true for any real x, so the domain of f^{-1} is $\mathbb{R}$.

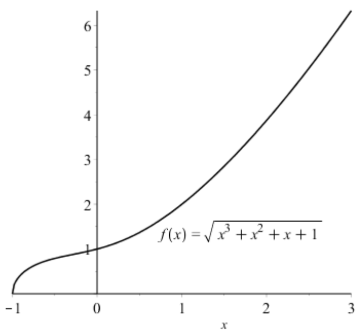
95. (a) $e^{\ln 300} = 300$; $\ln\left(e^{300}\right) = 300$

(b) A calculator indicates that $e^{\ln 300} = 300$ but gives an error message for $\ln\left(e^{300}\right)$ because e^{300} is larger than most calculators can evaluate.

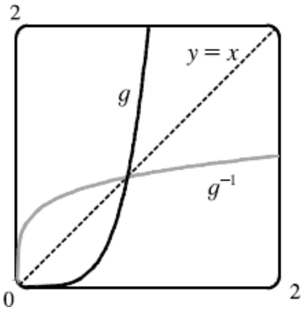
96. The graph of the function $y = f(x) = \sqrt{x^3 + x^2 + x + 1}$ is increasing, so f is one-to-one.
Maple gives two complex expressions as well as

$y = f^{-1}(x) = \frac{1}{6}\dfrac{M^{2/3} - 8 - 2M^{1/3}}{2M^{1/3}}$, where

$M = 108x^2 + 12\sqrt{48 - 120x^2 + 81x^4} - 80$.



97. (a) The graph of $g(x) = x^6 + x^4,\ x \geq 0,$ is shown.



(b) If we use Maple to solve $x = y^6 + y^4$ for y, we obtain two real solutions: $\pm\dfrac{\sqrt{6}}{6}\dfrac{\sqrt{C^{1/3}\left(C^{2/3} - 2C^{1/3} + 4\right)}}{C^{1/3}}$, where $C = 108x + 12\sqrt{3}\sqrt{x(27x - 4)}$, and the inverse for $y = x^6 + x^4\ (x \geq 0)$ is the positive solution, whose domain is $\left[\frac{4}{27}, \infty\right)$.

98. (a) $n = f(t) = 100 \cdot 2^{t/3} \Rightarrow \dfrac{n}{100} = 2^{t/3} \Rightarrow \log_2\left(\dfrac{n}{100}\right) = \dfrac{t}{3} \Rightarrow t = 3\log_2\left(\dfrac{n}{100}\right)$. We could rewrite this as $t = f^{-1}(n) = 3 \cdot \dfrac{\ln(n/100)}{\ln 2}$. This function tells us how long it will take to obtain n bacteria.

(b) $n = 50{,}000 \Rightarrow t = f^{-1}(50{,}000) = 3 \cdot \dfrac{\ln\left(\frac{50000}{100}\right)}{\ln 2} = 3\left(\dfrac{\ln 500}{\ln 2}\right) \approx 26.9$ hours

99. (a) $Q = Q_0\left(1 - e^{-t/a}\right) \Rightarrow \dfrac{Q}{Q_0} = 1 - e^{-t/a} \Rightarrow e^{-t/a} = 1 - \dfrac{Q}{Q_0} \Rightarrow -\dfrac{t}{a} = \ln\left(1 - \dfrac{Q}{Q_0}\right) \Rightarrow t = -a\ln\left(1 - Q/Q_0\right)$.

This gives us the time t necessary to obtain a given charge.

(b) $Q = 0.9Q_0$ and $a = 2 \Rightarrow t = -2\ln\left(1 - 0.9Q_0/Q_0\right) = -2\ln 0.1 \approx 4.6$ seconds.

100. $\cos^{-1}(-1) = \pi$ because $\cos\pi = -1$ and π is in the interval $[0, \pi]$.

101. $\sin^{-1}(0.5) = \dfrac{\pi}{6}$ because $\sin\dfrac{\pi}{6} = 0.5$ and $\dfrac{\pi}{6}$ is in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

102. $\tan^{-1}\left(\sqrt{3}\right) = \dfrac{\pi}{3}$

103. $\arctan(-1) = -\dfrac{\pi}{4}$

104. $\csc^{-1}\sqrt{2} = \dfrac{\pi}{4}$

105. $\arcsin 1 = \dfrac{\pi}{2}$

106. $\sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right) = -\dfrac{\pi}{4}$

107. $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{6}$

108. $\cot^{-1}\left(-\sqrt{3}\right) = \dfrac{5\pi}{6}$

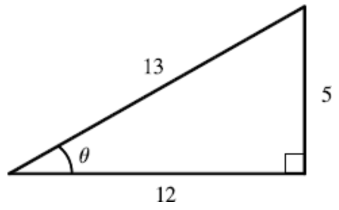
109. $\sec^{-1} 2 = \dfrac{\pi}{3}$

110. $\arcsin\left(\sin\left(\dfrac{5\pi}{4}\right)\right) = \arcsin\left(-\dfrac{1}{\sqrt{2}}\right) = -\dfrac{\pi}{4}$

111. Let $\theta = \sin^{-1}\left(\dfrac{5}{13}\right)$ (see figure to the right).

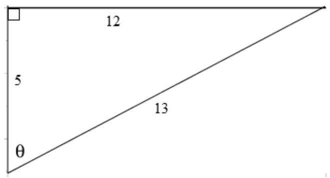
Then $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$$

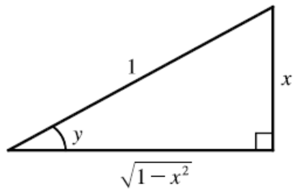


112. Let $\theta = \cos^{-1}\left(\dfrac{12}{13}\right)$ (see figure below). Then

$\tan\left(\cos^{-1}\frac{5}{13}\right) = \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{12}{13}}{\frac{5}{13}} = \dfrac{12}{5}.$

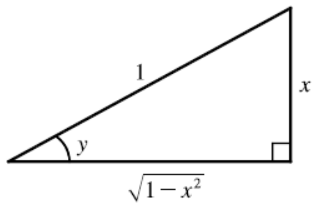


113. $\tan\left(\sin^{-1} x\right) = \tan\left(\dfrac{\pi}{2}\right) = \dfrac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$ is undefined because $\cos\frac{\pi}{2} = 0.$

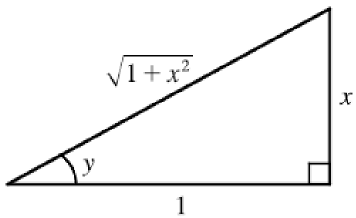


114. Let $y = \sin^{-1} x$, so $\sin y = x$, and from the triangle, we see that $\cos(\sin^{-1} x) = \cos y = \sqrt{1-x^2}$.

115. Let $y = \sin^{-1} x$. Then $\sin y = x$, so from the triangle (which illustrates the case when $y > 0$), we see that $\tan\left(\sin^{-1} x\right) = \tan y = \dfrac{x}{\sqrt{1-x^2}}$.

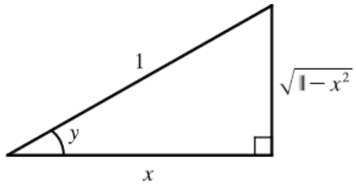


116. Let $y = \tan^{-1} x$. Then $\tan y = x$, so from the triangle (which illustrates the case when $y > 0$), we see that $\sin\left(\tan^{-1} x\right) = \sin y = \dfrac{x}{\sqrt{1+x^2}}$.

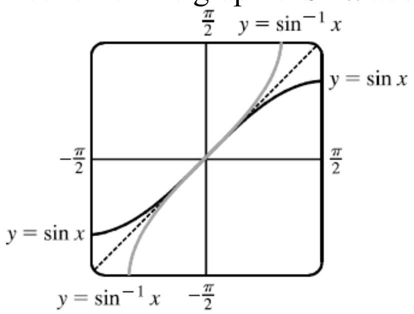


117. Let $y = \arccos x$. Then $\cos y = x$, so from the triangle (which illustrates the case when $y > 0$), we see that $\sin\left(2\arccos x\right) = \sin 2y = 2\sin y\cos y$

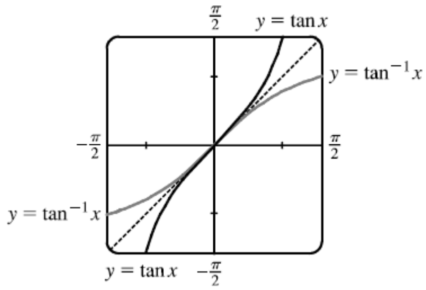
$$= 2\left(\sqrt{1-x^2}\right)(x) = 2x\sqrt{1-x^2}.$$



118. The graph of $\sin^{-1} x$ is the reflection of the graph of $\sin x$ about the line $y = x$.



119. The graph of $\tan^{-1} x$ is the reflection of the graph of $\tan x$ about the line $y = x$.

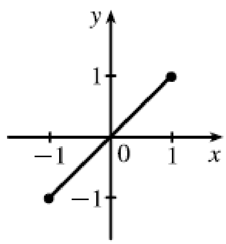


120. $g(x) = \sin^{-1}(3x+1)$

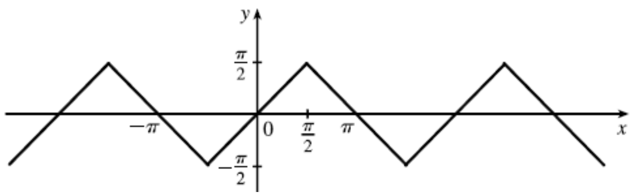
Domain $(g) = \{x \mid -1 \le 3x+1 \le 1\} = \{x \mid -2 \le 3x \le 0\} = \{x \mid -\frac{2}{3} \le x \le 0\} = \left[-\frac{2}{3}, 0\right]$.

Range $(g) = \{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}\} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

121. (a) $f(x) = \sin\left(\sin^{-1} x\right)$ Since one function undoes what the other one does, we get the identify function, $y = x$, on the restricted domain $-1 \le x \le 1$.



(b) $f(x) = \sin\left(\sin^{-1} x\right)$ This is similar to part (a), but with domain Equations for g on intervals of the form $\left(-\frac{\pi}{2} + \pi n, \frac{\pi}{2} + \pi n\right)$, for any integer n, can be found using $g(x) = (-1)^n x + (-1)^{n+1} n\pi$. The sine function is monotonic on each of these intervals, and hence, so is g (but in a linear fashion).

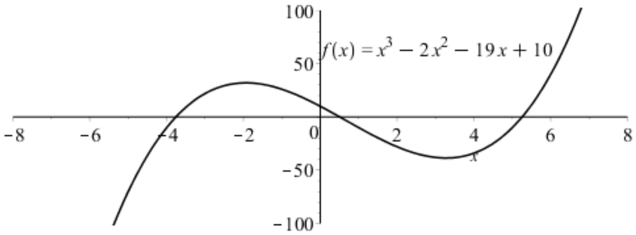


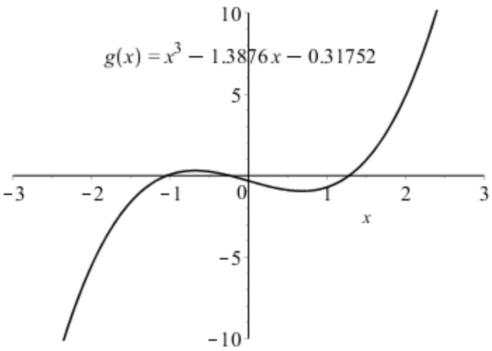
122. (a) If the point (x, y) is on the graph of $y = f(x)$, then the point $(x-c, y)$ is that point shifted c units to the left. Since f is one-to-one, the point (y, x) is on the graph of $y = f^{-1}(x)$ and the point corresponding to $(x-c, y)$ on the graph of f is $(y, x-c)$ on the graph of f^{-1}. Thus, the curve's reflection is shifted *down* the same number of units as the curve itself is shifted to the left. So an expression for the inverse function is $g^{-1}(x) = f^{-1}(x) - c$.

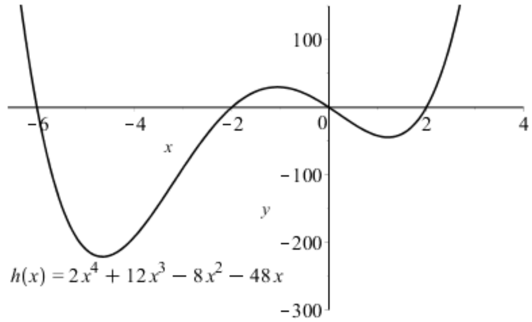
(b) If we compress (or stretch) a curve horizontally, the curve's reflection in the line $y = x$ is compressed (or stretched) *vertically* by the same factor. Using this geometric principle, we see that the inverse of $h(x) = f(cx)$ can be expressed as $h^{-1}(x) = (1/c)\, f^{-1}(x)$.

1.7 Technology in AP Calculus

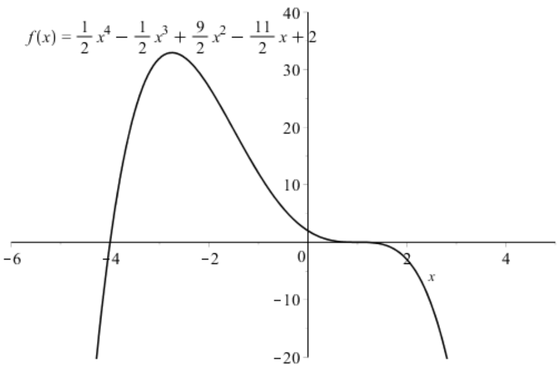
EXERCISE SOLUTIONS

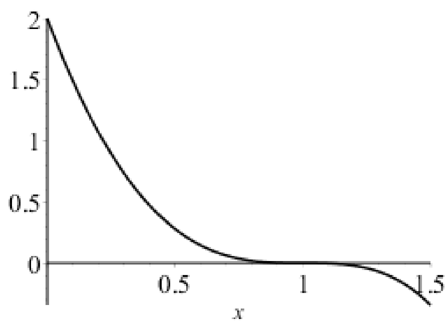
1. 

2. 

3. 

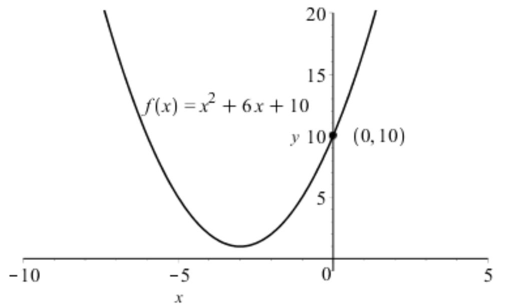
4. You will need to zoom in around $x = 1$ to see that zero:





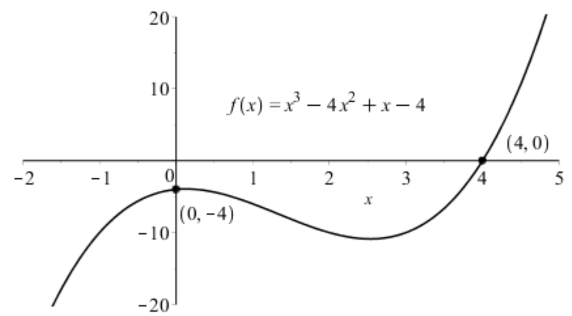
5. The functions $x^2 + 5x + 5$ and $\sin x$ do not intersect on the interval $[0, 2\pi)$.

6. The functions $-2\cos\left(x-\frac{\pi}{4}\right)$ and $3x+8$ do not intersect on the interval $[0,2\pi)$.
7. The only solution of $\cos x = e^{2x-1}$ on the interval $[0,2\pi)$ is $(0.448, 0.901)$.
8. The only solution of $\log_{1/2} x = \tan x$ on the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is $(0.614, 0.704)$.
9. The solutions of $\sin x + \cos x = 1$ in $[-2\pi, 2\pi]$ are $(-2\pi,1), (-\frac{3\pi}{2},1), (0,1), \left(\frac{\pi}{2},0\right)$, and $(2\pi,1)$.
10. The solutions of $\dfrac{1}{\sin x} - 2 = 3$ in the interval $[-2\pi, 2\pi]$ are $x = -6.082, -3.343, 0.201$, and $x = 2.940$.
11. The only solution of $2\log x + 4 = 10$ is $(1000,10)$.
12. The functions $y = x^2$ and $y = 2^x$ intersect in the points $(4,16), (2,4)$ and $(-0.767, 9.588)$.
13. (a)A quadratic function could intersect $y = \sin x$ in at most 3 points.
 (b) A cubic function could intersect $y = \sin x$ in at most 3 points.
 (c) An nth degree polynomial could intersect $y = \sin x$ in at most 3 points.
(d) A linear function could intersect $y = \sin x$ in infinitely many points.
14. (a) The points of intersection of $f(x) = x^2 - 3x - 2$ and $g(x) = 2x - 3$ are approximately $(0.208, -2.582)$ and $(4.791, 6.583)$.
 (b) The functions $f(x) = \ln x$ and $g(x) = x^2 - 1$ intersect in the points $(0.451, -0.797)$ and $(1,1)$.
 (c) The functions $f(x) = x^2$ and $g(x) = 1 + \sqrt{x-1}$ intersect in the points $(1,1)$ and $(1.206, 1.453)$.
15. Using the approximation $\pi \approx 3.142$, the volume would be calculated at $4.189\overline{3}r^3$.. Using a stored value of π, the volume would be $4.188790204r^3$. The difference is $0.000543r^3$.
16. (a)



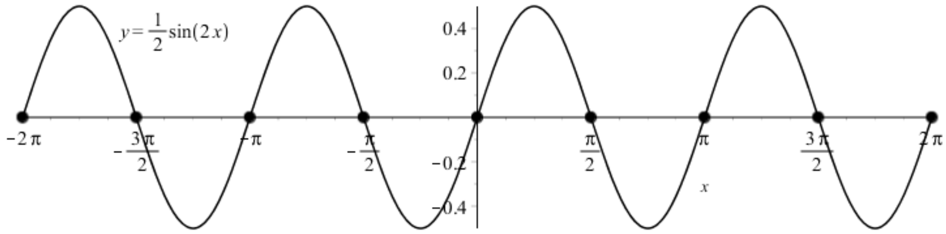
 (b) There are no zeros for this function.
 (c) The y-intercept is (0, 10).

17. (a)



 (b) The only zero of this function is (4, 0).
 (c) The y-intercept is (0, –4).

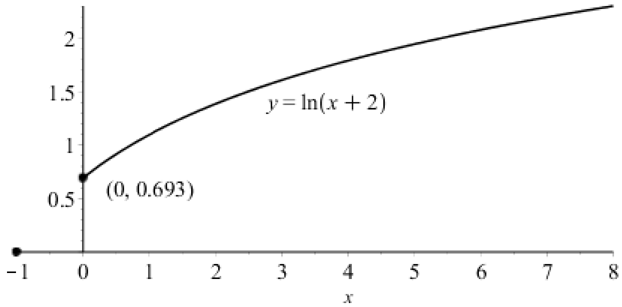
18. (a)



(b) The zeros are $(-2\pi,0),(-\frac{3\pi}{2},0),(-\pi,0),(-\frac{\pi}{2},0),(0,0),(\frac{\pi}{2},0),(\pi,0),(\frac{3\pi}{2},0),$ and $(2\pi,0)$.

(c) The y-intercept is (0,0).

19. (a)



(b) There are no zeros for $x \geq 0$.

(c) The y-intercept is $(0, \ln(2)) \approx (0, 0.693)$

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1 Review

EXERCISE SOLUTIONS

1. (a) A function f is a rule that assigns to each element x in a set D (the domain) exactly one element, called $f(x)$, in a set E. The set of inputs for the function, D, is the domain. The range of f is the set of all possible values of $f(x)$ as x varies throughout the domain.

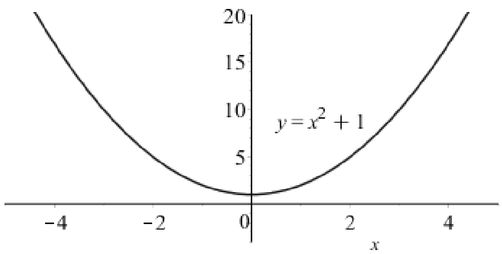
   (b) To obtain the graph of the function f, plot the ordered pairs of points, $(x, f(x))$.

   (c) A curve is the graph of a function if it passes the Vertical Line Test – that is, if no vertical line intersects the curve more than once.

2. There are four ways to represent a function:
   - Verbally: $C(w)$ is the cost of mailing a large envelope of weight w.
   - Numerically (with a table of values):

w (ounces)	$C(w)$ (dollars)
$0 < w \le 1$	0.98
$1 < w \le 2$	1.19
$2 < w \le 3$	1.40
$3 < w \le 4$	1.61
$4 < w \le 5$	1.82
⋮	⋮

   - Visually (with a graph):



   - Algebraically: $y = f(x) = x^2 + 1$

3. (a) f is an even function if for every x in its domain, $f(-x) = f(x)$. The graph of an even function is symmetric with respect to the y-axis. Examples of even functions include $y = x^2$, $y = -x^2$, and $y = \cos x$.

   (b) f is an odd function if for every x in its domain, $f(-x) = -f(x)$. The graph of an even function is symmetric about the origin. Examples of even functions include $y = x^3$, $y = x^5 + x$, and $y = \sin x$.

4. A function f is increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

5. A mathematical model is a mathematical description of a real-world phenomenon. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

6. (a) $y = 2x + 5$ is a linear function.

   (b) $y = x^7$ is a power function.

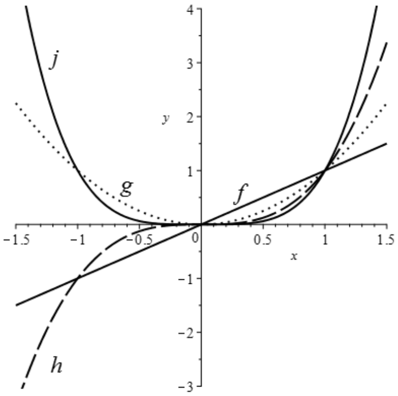
   (c) $y = e^x$ is an exponential function.

(d) $y = x^2 - 3x + 13$ is a quadratic function.

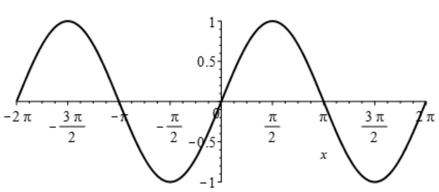
(e) $y = x^5 - 3x^4 + 7x - 12$ is a polynomial of degree 5.

(f) $y = \dfrac{x-3}{x+7}$ is a rational function.

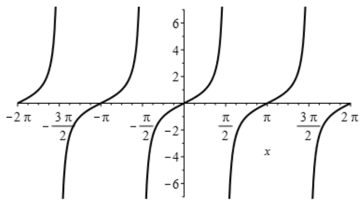
7.



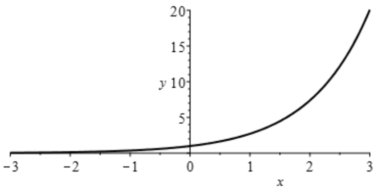
8. (a) $y = \sin x$



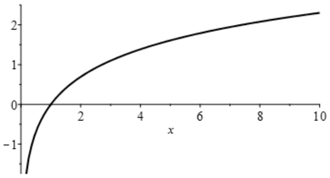
(b) $y = \tan x$



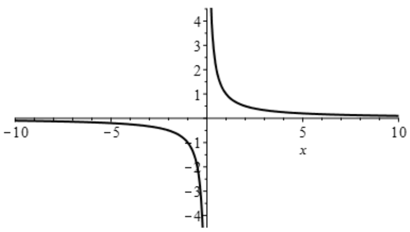
(c) $y = e^x$



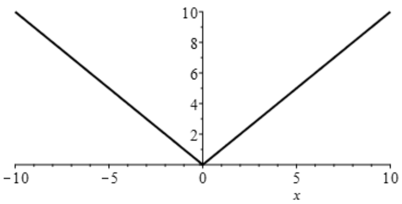
(d) $y = \ln x$



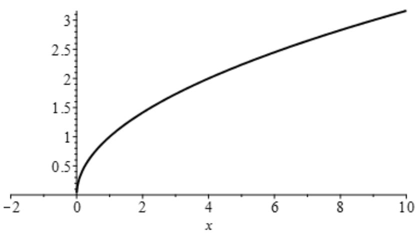
(e) $y = 1/x$



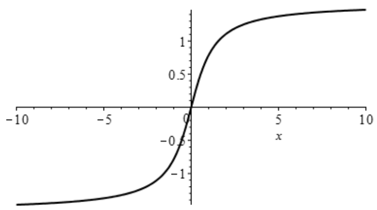
(f) $y = |x|$



(g) $y = \sqrt{x}$



(h) $y = \tan^{-1} x$



9. (a) The domain of $f+g$ is $A \cap B$.
   (b) The domain of fg is $A \cap B$.
   (c) The domain of f/g is $\{x \mid x \in A \cap B \text{ and } g(x) \neq 0\}$.
10. The composite of f and g is $(f \circ g)(x) = f(g(x))$ and its domain is the set of all x in the domain of g such that $g(x)$ is in the domain of f.
11. (a) If the graph of f is shifted up 2 units, the equation would become $y = f(x)+2$.
    (b) If the graph of f is shifted down 2 units, the equation would become $y = f(x)-2$.
    (c) If the graph of f is shifted 2 units to the right, the equation would become $y = f(x-2)$.
    (d) If the graph of f is shifted 2 units to the left, the equation would become $y = f(x+2)$.
    (e) If the graph of f is shifted reflected about the x-axis, the equation would become $y = -f(x)$.
    (f) If the graph of f is shifted reflected about the y-axis, the equation would become $y = f(-x)$.
    (g) If the graph of f is stretched vertically by a factor of 2, the equation would become $y = 2f(x)$.
    (h) If the graph of f is shrunk vertically by a factor of 2, the equation would become $y = \frac{1}{2}f(x)$.
    (i) If the graph of f is stretched horizontally by a factor of 2, the equation would become $y = f\left(\frac{1}{2}x\right)$.
    (j) If the graph of f is shrunk horizontally by a factor of 2, the equation would become $y = f(2x)$.
12. (a) If $f(x) = x^2$ is shifted up 1 unit, the function would become $f(x) = x^2+1$.
    (b) If $f(x) = x^2$ is shifted 1 unit to the left, the function would become $f(x) = (x+1)^2$.
    (c) If $f(x) = x^2$ is shifted 2 units down and 1 unit to the right, the function would become $f(x) = (x-1)^2 - 2 = x^2 - 2x - 2$.
    (d) If $f(x) = x^2$ is shifted 2 units up, then reflected across the x-axis, the function would become $f(x) = -x^2 - 2$.
    (e) If $f(x) = x^2$ is shifted down 1 unit, w units to the right, and then reflected across the y-axis, the function would become $f(x) = (-x-1)^2 - w = x^2 + 2x + 1 - w$.
13. (a) A function is one-to-one if it never takes on the same value twice; if no horizontal line intersects the graph more than once the function is one-to-one.
    (b) If f is a one-to-one function with domain A and range B, then its inverse function has domain B and range A and is defined by $f^{-1}(y) = x \iff f(x) = y$ for any y in B. The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$.
14. (a) For $-1 \leq x \leq 1$, $\sin^{-1} x$ is an angle between $-\pi/2$ and $\pi/2$ whose sine is x. The inverse sine function has domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$.
    (b) For $-1 \leq x \leq 1$, $\cos^{-1} x$ is an angle between 0 and π whose cosine is x. The inverse cosine function has domain $[-1, 1]$ and range $[0, \pi]$.
    (c) $\tan^{-1} x$ is an angle between $-\pi/2$ and $\pi/2$ whose tangent is x. The inverse cosine function has domain $\mathbb{R}$ and range $[-\pi/2, \pi/2]$.
15. False   Let $f(x) = x^2, s = -1,$ and $t = 1$. Then $f(s+t) = (-1+1)^2 = 0$,
    but $f(s) + f(t) = (-1)^2 + 1^2 = 2 \neq 0$.
16. False   Let $f(x) = x^2$. Then $f(-2) = 4 = f(2)$, but $-2 \neq 2$.
17. False   Let $f(x) = x^2$. Then $f(3x) = (3x)^2 = 9x^2$ and $3 \cdot f(x) = 3x^2$. So $f(3x) \neq 3 \cdot f(x)$.

18. True — If and f is a decreasing function, then the y-values get smaller as we move from left to right.

19. False — Let $f(x) = x^2$, and $g(x) = 2x$. Then $(f \circ g)(x) = f(g(x)) = f(2x) = (2x)^2 = 4x^2$ and $(g \circ f)(x) = g(f(x)) = g(x^2) = 2x^2$. So $f \circ g \neq g \circ f$.

20. False — Let $f(x) = x^3$. Then f is one-to-one and $f^{-1}(x) = \sqrt[3]{x}$. But $1/f(x) = 1/x^3$, which is not equal to $f^{-1}(x)$.

21. True — We can always divide by e^x because $e^x \neq 0$ for every x.

22. True — The function $\ln x$ is increasing on $(0, \infty)$.

23. False — Let $x = e$. Then $(\ln x)^6 = (\ln e)^6 = 1^6 = 1$, but $6 \ln x = 6 \ln e = 6 \cdot 1 = 6 \neq 1$. It is true, however, that $\ln(x^6) = 6 \ln x$ for $x > 0$.

24. False — Let $x = e^2$ and $a = e$. Then $\dfrac{\ln x}{\ln a} = \dfrac{\ln e^2}{\ln e} = \dfrac{2 \ln e}{\ln e} = 2$ and $\ln \dfrac{x}{a} = \ln \dfrac{e^2}{e} = \ln e = 1$, so in general the statement is false. What *is* true however, is that $\ln \dfrac{x}{a} = \ln x - \ln a$.

25. False — It is true that $\tan \frac{3\pi}{4} = -1$, but since the range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$, we must have $\tan^{-1}(-1) = -\frac{\pi}{4}$.

26. False — For example, $\tan^{-1} 20$ is defined, but $\sin^{-1} 20$ and $\cos^{-1} 20$ are not.

27. False — For example, if $x = -3$, then $\sqrt{(-3)^2} = \sqrt{9} = 3$, not -3.

28. (a) When $x = 2, y \approx 2.7$, thus $f(2) \approx 2.7$.

(b) $f(x) = 3 \Rightarrow x \approx 2.3, 5.6$

(c) The domain of f is $[-6, 6]$.

(d) The range of f is $[-4, 4]$.

(e) f is increasing on $[-4, 4]$.

(f) f is not one-to-one because it fails the Horizontal Line Test.

(g) f is odd because its graph is symmetric about the origin.

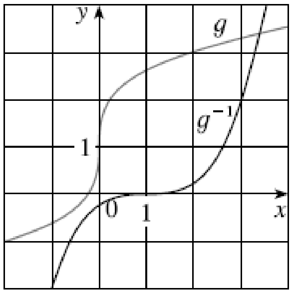
29. (a) When $x = 2, y = 3$. Thus $g(2) = 3$.

(b) g is one-to-one because it passes the Horizontal Line Test.

(c) When $y = 2$, $x \approx 0.2$, so $g^{-1}(2) \approx 0.2$.

(d) The range of g is $[-1, 3.5]$, which is the same as the domain of g^{-1}.

(e) We reflect the graph of g through the line $y = x$ to obtain the graph of g^{-1}.

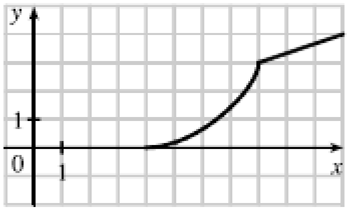


30. $$\frac{f(a+h) - f(a)}{h} = \frac{(a^2 + 2ah + h^2 - 2a - 2h + 3) - (a^2 - 2a + 3)}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2$$

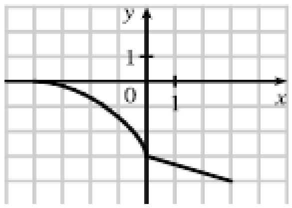
31. $f(x) = \dfrac{2}{3x - 1}$ Domain: $\{x \mid x \neq \frac{1}{3}\}$, Range: $\{y \mid y \neq 0\}$

32. $g(x)=\sqrt{16-x^4}$ Domain: $[-2,2]$, Range: $[0,4]$
33. $h(x)=\ln(x+6)$ Domain: $(-6,\infty)$, Range: $\mathbb{R}$
34. $F(t)=3+\cos 2t$ Domain: $\mathbb{R}$, Range: $[2,4]$
35. $f(x)=\dfrac{3}{x+2}$ Domain: $\{x \mid x \neq -2\}$, Range: $\{y \mid y \neq 0\}$
36. $f(x)=3^x+2$ Domain: $\mathbb{R}$, Range: $\{y \mid y > 0\}$
37. $f(x)=\dfrac{\sin x}{x}$ Domain: $\{x \mid x \neq 0\}$, Range: $\left[-\frac{2}{3\pi},1\right]$
38. $f(x)=\tan(x+1)$ Domain: $\{x \mid x \neq (k+1)(\pi-2)\}$ for $k \in \mathbb{Z}$, Range: $\mathbb{R}$
39. (a) To obtain the graph of $y=f(x)+8$, we shift the graph of $y=f(x)$ up 8 units.
(b) To obtain the graph of $y=f(x+8)$, we shift the graph of $y=f(x)$ left 8 units.
(c) To obtain the graph of $y=1+2f(x)$, we stretch the graph of $y=f(x)$ vertically by a factor of 2, and then shift the resulting graph up 1 unit.
(d) To obtain the graph of $y=f(x-2)-2$, we shift the graph of $y=f(x)$ right 2 units (for the "–2" inside the parentheses), and then shift the resulting graph 2 units down.
(e) To obtain the graph of $y=-f(x)$, we reflect the graph of $y=f(x)$ across the x-axis.
(f) To obtain the graph of $y=f^{-1}(x)$, we reflect the graph of $y=f(x)$ about the line $y=x$.

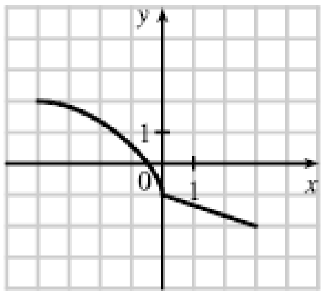
40. (a) To obtain the graph of $y=f(x-8)$, we shift the graph of $y=f(x)$ right 8 units.



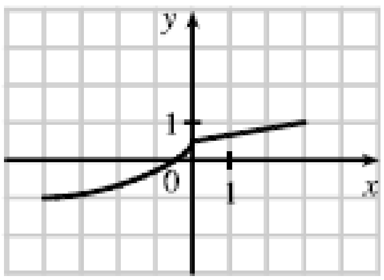
(b) To obtain the graph of $y=\frac{1}{2}f(x)-1$, we reflect the graph of $y=f(x)$ about the x-axis.



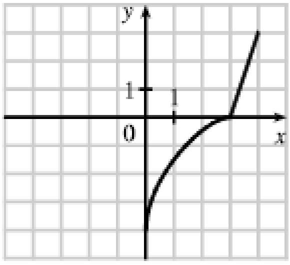
(c) To obtain the graph of $y=2-f(x)$, we reflect the graph of $y=f(x)$ about the x-axis, and then shift the resulting graph up 2 units.



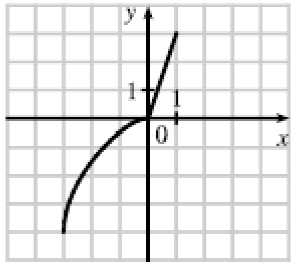
(d) To obtain the graph of $y=\frac{1}{2}f(x)-1$, we shrink the graph of $y=f(x)$ by a factor of 2, and then shift the resulting graph down 1 unit.



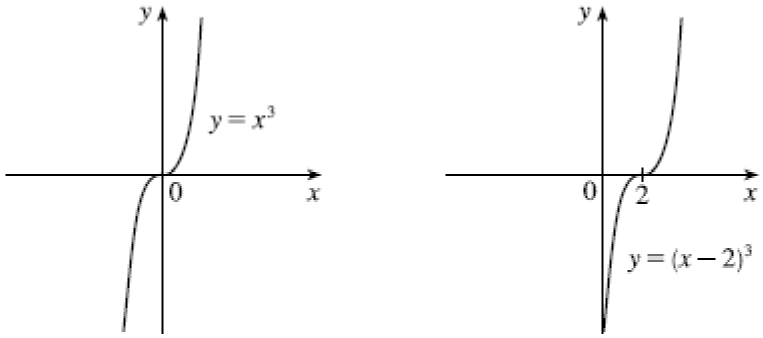
(e) To obtain the graph of $y = f^{-1}(x)$, reflect the graph of $y = f(x)$ about the line $y = x$.



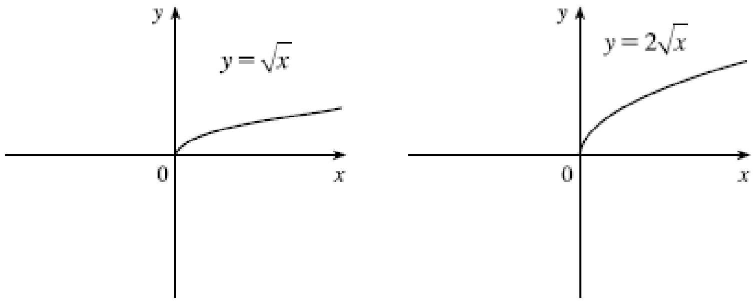
(f) To obtain the graph of $y = f^{-1}(x+3)$, we reflect the graph of $y = f(x)$ about the line $y = x$, and then shift the resulting graph left 3 units.



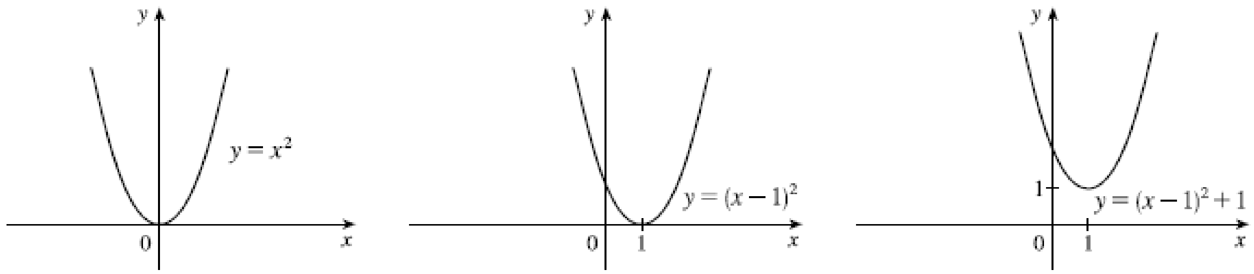
41. Start with the graph of $y = x^3$ and shift 2 units to the right.



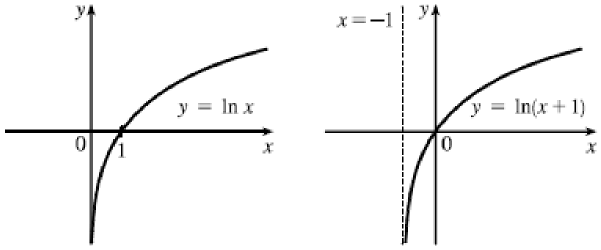
42. Start with the graph of $y = \sqrt{x}$ and stretch vertically by a factor of 2.



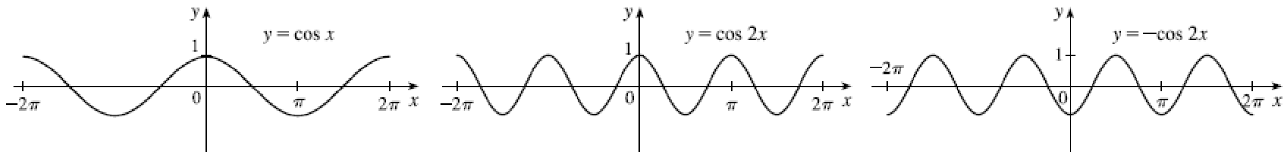
43. First note that $y = x^2 - 2x + 2 = (x^2 - 2x + 1) + 1 = (x-1)^2 + 1$. Then start with the graph of $y = x^2$, shift 1 unit to the right, and shift 1 unit up.



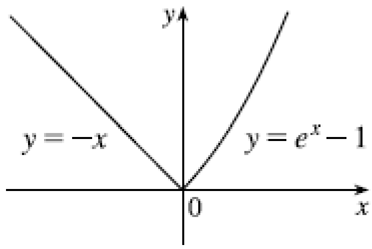
44. Start with the graph of $y = \ln x$ and shift left 1 unit.



45. Start with the graph of $y = \cos x$, shrink horizontally by a factor of 2, and reflect about the x-axis.



46. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ e^x - 1 & \text{if } x \ge 0 \end{cases}$. On $(-\infty, 0)$, graph $y = -x$ (the line with slope -1 and y-intercept 0) with an open endpoint (0,0). On $[0, \infty)$, graph $y = e^x - 1$ (shift the graph of $y = e^x$ down 1 unit) with closed endpoint (0, 0).



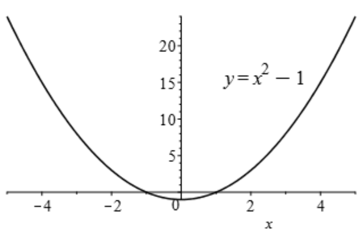
47. (a) The terms of f are a mixture of odd and even powers of x, so f is neither even nor odd.
(b) The terms of f are all odd powers of x, so f is odd.
(c) $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$, so f is even.
(d) $f(-x) = 1 + \sin(-x) = 1 - \sin x$. Because $f(-x) \ne f(x)$ and $f(-x) \ne -f(x)$, f is neither even nor odd.

48. (a) $\dfrac{2^a 2^b 3^a}{2^{-a}} = 3^a 2^a 2^b 2^a = 3^a 2^{2a+b}$

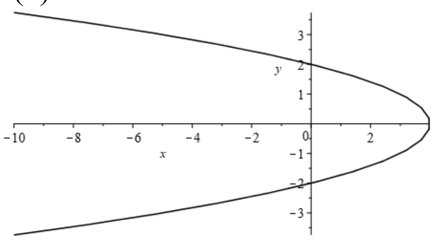
(b) $e^{\ln x - \ln y} = e^{\ln\left(\frac{x}{y}\right)} = \dfrac{x}{y}$

(c) $2\ln a + 3\ln b - 4\ln c = \ln a^2 + \ln b^3 - \ln c^4 = \ln\left(\dfrac{a^2 b^3}{c^4}\right)$

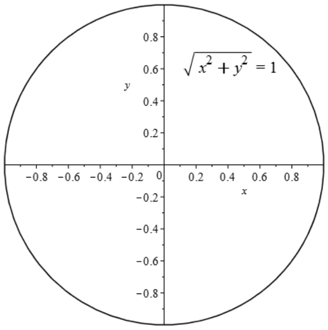
49. (a) is a function



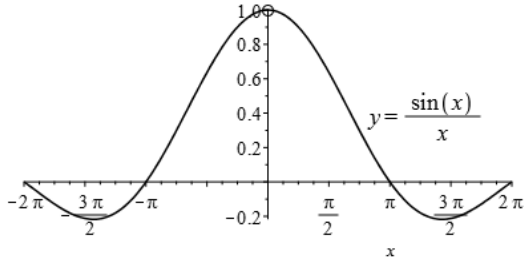
(b) not a function



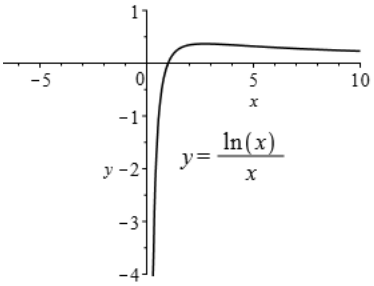
(c) not a function



(d) is a function



(e) is a function

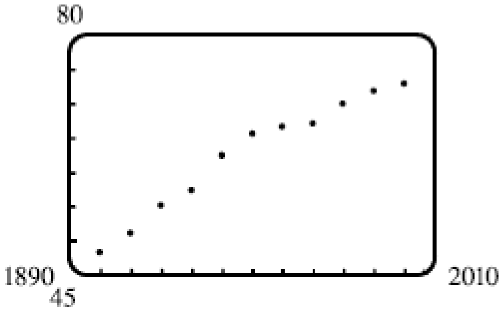


50. (a) $2a+3$ (b) $2a+2$ (c) $2a+2$ (d) $2a+2$
51. The largest element in the range of $f(x)=\sqrt{-x^2+4x+21}$ is (**B**) 5.
52. If $g(x)\frac{x^2-16}{x-4}$ and $f(x)=x+4$, then (**D**) Every point on the graph of g is on the graph of f, but not conversely.
53. Of the functions listed, (**D**) is an even function.
54. If (2, 6) is a point on the graph of $y=f(x)$, and $g(x)=-f(x+3)$, then (**B**) (–1, –6) is on the graph of $y=g(x)$.
55. If 4 is an x-intercept for the graph a function h, and $g(x)=h(-2x)+5$, the point (**A**) (–2, 5) lies on the graph of g.
56. For the line segment from (–2, 2) to (–1, 0), the slope is $\frac{0-2}{-1+2}=-2$, and an equation is $y-0=-2(x+1)$ or equivalently, $y=-2x-2$. The circle has equation $x^2+y^2=1$; the top half has equation $y=\sqrt{1-x^2}$ (we have solved for positive y). Thus $f(x)=\begin{cases}-2x-2 & \text{if } -2\le x\le -1\\ \sqrt{1-x^2} & \text{if } -1<x\le 1\end{cases}$.
57. (a) $(f\circ g)(x)=f(g(x))=f\left(x^2-9\right)=\ln\left(x^2-9\right)$; The domain is $(-\infty,-3)\cup(3,\infty)$.
    (b) $(g\circ f)(x)=g(f(x))=g(\ln x)=(\ln x)^2-9$; The domain is $(0,\infty)$.
    (c) $(f\circ f)(x)=f(f(x))=f(\ln x)=\ln(\ln x)$; The domain is $(1,\infty)$.
    (d) $(g\circ g)(x)=g(g(x))=g\left(x^2-9\right)=\left(x^2-9\right)^2-9=x^4-18x^2+72$; The domain is $\mathbb{R}$.

58. One possible combination is $f(x)=\dfrac{1}{x}, g(x)=\sqrt{1+x}, h(x)=\sqrt{x}$. Then $(f\circ g\circ h)(x)=$

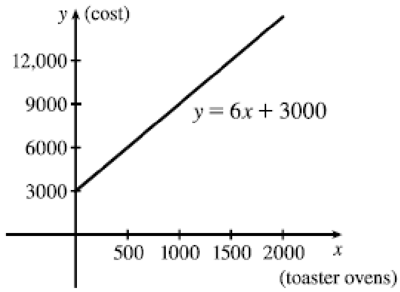
$$f(g(h(x)))=f\left(g\left(\sqrt{x}\right)\right)=f\left(\sqrt{1+\sqrt{x}}\right)=\frac{1}{\sqrt{1+\sqrt{x}}}=F(x).$$

59. Many models are plausible. Your choice depends on whether you think medical advances will keep increasing life expectancy, or if there is bound to be a leveling-off of life expectancy. A linear model, $y=0.2493x-423.4818$, gives us an estimate of 77.6 years for the year 2010.



60. (a) Let x denote the number of toaster ovens produced in one week and y the associated cost. Using the points (1000, 9000) and (1500, 12,000), we get an equation of a line:

$$y-9000=\frac{12,000-9000}{1500-100}(x-1000) \Rightarrow y=6(x-1000)+9000 \Rightarrow y=6x+3000.$$



(b) The slope of 6 means that each additional toaster oven produced adds \$6 to the weekly production cost.

(c) The y-intercept of 3000 represents the overhead cost – the cost incurred without producing anything.

61. We need to know the value of x such that $f(x)=2x+\ln x=2$. Since $x=1$ gives us $y=2, f^{-1}(2)=1$.

62. $y=\dfrac{x+1}{2x+1}$. Interchanging x and y gives us $x=\dfrac{y+1}{2y+1} \Rightarrow 2xy+x=y+1 \Rightarrow 2xy-y=1-x \Rightarrow$

$$y(2x-1)=1-x \Rightarrow y=\frac{1-x}{2x-1}=f^{-1}(x).$$

63. (a) $e^{2\ln 3}=\left(e^{\ln 3}\right)^2=3^2=9$

(b) $\log_{10}25+\log_{10}4=\log_{10}(25\cdot 4)=\log_{10}100=\log_{10}10^2=2$

(c) $\tan\left(\arcsin\frac{1}{2}\right)=\tan\frac{\pi}{6}=\dfrac{1}{\sqrt{3}}$

(d) Let $\theta=\cos^{-1}\frac{4}{5}$, so $\cos\theta=\frac{4}{5}$. Then $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)=\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{\frac{9}{25}}=\dfrac{3}{5}$

64. (a) $e^x=5 \Rightarrow x=\ln 5$

(b) $\ln x = 2 \Rightarrow x = e^2$

(c) $e^{e^x} = 2 \Rightarrow e^x = \ln 2 \Rightarrow x = \ln(\ln 2)$

(d) $\tan^{-1} x = 1 \Rightarrow \tan(\tan^{-1} x) = \tan 1 \Rightarrow x = \tan 1 \ (\approx 1.5574)$

(e) $e^{3x} = 2e^x \Rightarrow \ln(e^{3x}) = \ln(2e^x) \Rightarrow 3x = \ln 2 + x \Rightarrow 2x = \ln 2 \Rightarrow x = \dfrac{\ln 2}{2}$

(f) $\dfrac{\ln x}{x} = 3$ There is no solution because, for all x, $\ln x < x$ so $\dfrac{\ln x}{x} < 1$.

(g) $\ln(\ln x) = 5 \Rightarrow \ln x = e^5 \Rightarrow x = e^{e^5} \ (\approx 2.851 \times 10^{64})$

(h) $2\sin x - \cos x = 1 \Rightarrow x = k\pi, k \arctan\left(\frac{4}{3}\right), k \in \mathbb{Z}$.

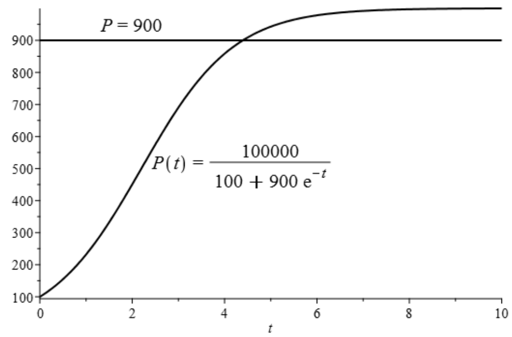
65. (a) After 4 days, ½ gram remains; after 8 days, ¼ g remains, after 12 days, ⅛ g remains; and after 16 days, $\frac{1}{16}$ g remains.

(b) $m(4) = \dfrac{1}{2}, m(8) = \dfrac{1}{2^2}, m(12) = \dfrac{1}{2^3}, m(16) = \dfrac{1}{2^4}$. From the pattern, we see that $m(t) = 2^{-t/4}$.

(c) $m(t) = 2^{-t/4} \Rightarrow \log_2 m = -t/4 \Rightarrow t = -4\log_2 m$; This is the time elapsed when there are m grams of ^{100}Pd.

(d) $m = 0.01 \Rightarrow t = -4\log_2 0.01 = -4\left(\dfrac{\ln 0.01}{\ln 2}\right) \approx 26.6$ days.

66. (a) Based on the graph, the population reaches 900 after about 4.3 years.



(b) Interchanging t and P we have

$$t = \frac{100{,}000}{100 + 900e^{-P}} \Rightarrow 100 + 900e^{-P} = \frac{100{,}000}{t} \Rightarrow 900e^{-P} = 100{,}000t^{-1} - 100 \Rightarrow e^{-P} = \frac{1000t^{-1} - 1}{9}$$

$$\Rightarrow -P = \ln\left(\tfrac{1}{9}\left(1000t^{-1} - 1\right)\right) \Rightarrow P = -\ln\left(\tfrac{1}{9}\left(1000t^{-1} - 1\right)\right) \text{ so } P^{-1}(t) = -\ln\left(\tfrac{1000}{9t} - \tfrac{1}{9}\right).$$

This function tells us the yea, in which the population reaches the population value P.

(c) $P^{-1}(900) = -\ln\left(\frac{1000}{9\cdot 900} - \frac{1}{9}\right) = -\ln\left(\frac{100}{81} - \frac{1}{9}\right) = -\ln\left(\frac{1}{81}\right) = -4\ln 3 \approx -4.394$ years

67. (a) $f(-x) = 6e^{-(-x)^2} = 6e^{-(x)^2} = f(x)$ so f is an even function

(b) The average rate of change of f over [1, 3] is

$$\frac{f(3) - f(1)}{3 - 1} = \frac{6e^{-9} - 6e^{-1}}{2} = 3e^{-9} - 6e^{-1} \approx -1.103$$

(c) Graph.