FUNDAMENTALS OF PHOTONICS

THIRD EDITION

SOLUTIONS MANUAL

FOR EXERCISES†

† A solutions manual is not available for the end-of-chapter problems

FEBRUARY 20, 2019

BAHAA E. A. SALEH
University of Central Florida
Boston University

MALVIN CARL TEICH
Boston University
Columbia University

JOHN WILEY & SONS, INC.

This edition first published 2019

Registered Office
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

Editorial Office
111 River Street, Hoboken, NJ 07030, USA

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Library of Congress Cataloging-in-Publication Data is available.
Volume Set ISBN: 9781119506874
Volume I ISBN: 9781119506867
Volume II ISBN: 9781119506898

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CHAPTER
1

RAY OPTICS

1.1 POSTULATES OF RAY OPTICS

EXERCISE 1.1-1

Proof of Snell's Law

The pathlength is given by $n_1 d_1 \sec\theta_1 + n_2 d_2 \sec\theta_2$. (1)

The pathlength is a function of θ_1 and θ_2, which are related by

$$d_1 \tan\theta_1 + d_2 \tan\theta_2 = d. \quad (2)$$

The pathlength is minimized when $\dfrac{\partial}{\partial\theta_1}[n_1 d_1 \sec\theta_1 + n_2 d_2 \sec\theta_2] = 0$,
i.e., when $n_1 d_1 \sec\theta_1 \tan\theta_1 + n_2 d_2 \sec\theta_2 \tan\theta_2 (\partial\theta_2/\partial\theta_1) = 0$. (3)

From (2), we have $\dfrac{\partial}{\partial\theta_1}[d_1 \tan\theta_1 + d_2 \tan\theta_2] = 0$,

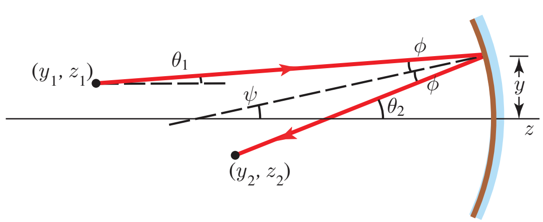
so that $d_1 \sec^2\theta_1 + d_2 \sec^2\theta_2 (\partial\theta_2/\partial\theta_1) = 0$ and $\dfrac{\partial\theta_2}{\partial\theta_1} = -\dfrac{d_1 \sec^2\theta_1}{d_2 \sec^2\theta_2}$.

Substituting into (3), we have $n_1 d_1 \sec\theta_1 \tan\theta_1 - n_2 \dfrac{d_1 \sec^2\theta_1 \tan\theta_2}{\sec\theta_2} = 0$,
whereupon $n_1 \tan\theta_1 = n_2 \sec\theta_1 \sin\theta_2$, from which $n_1 \sin\theta_1 = n_2 \sin\theta_2$, which is Snell's law.

1.2 SIMPLE OPTICAL COMPONENTS

EXERCISE 1.2-1

Image Formation by a Spherical Mirror



A ray originating at $P_1 = (y_1, z_1)$ at angle θ_1 meets the mirror at height
$y \approx y_1 + \theta_1 z_1$. (1)

The angle of incidence at the mirror is $\phi = \psi - \theta_1 \approx \dfrac{y}{-R} - \theta_1$.
The reflected ray makes angle θ_2 with the z axis:

$$\theta_2 = 2\phi + \theta_1 = 2\left[\frac{y}{-R} - \theta_1\right] + \theta_1 = \frac{2y}{-R} - \theta_1 = \frac{2(y_1 + \theta_1 z_1)}{-R} - \theta_1.$$

Substituting $f = \dfrac{-R}{2}$, we have $\theta_2 = \dfrac{y_1 + \theta_1 z_1}{f} - \theta_1$. (2)

The height y_2 can be determined from $\dfrac{y + (-y_2)}{z_2} \approx \theta_2$. (3)

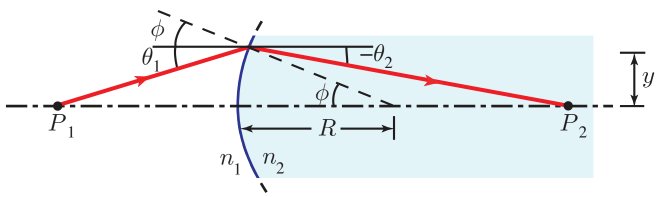
Substituting from (1) and (2) into (3), we have $y_1 + \theta_1 z_1 - y_2 = z_2 \left[\frac{y_1 + \theta_1 z_1}{f} - \theta_1 \right]$
and $y_2 = y_1 - \frac{z_2 y_1}{f} + \theta_1 \left[z_1 - \frac{z_1 z_2}{f} + z_2 \right]$.
If $\left[z_1 - \frac{z_1 z_2}{f} + z_2 \right] = 0$, or $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$, we have

$$y_2 = y_1 \left(1 - \frac{z_2}{f} \right), \qquad (4)$$

which is independent of θ_1.
From (4) it is clear that $\frac{z_2}{f} = 1 - \frac{y_2}{y_1}$, so that $y_2 = -\frac{z_2}{z_1} y_1$.

EXERCISE 1.2-2

Image Formation



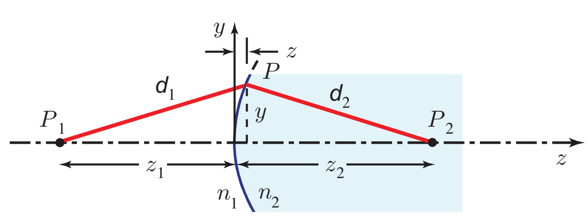
a) From Snell's law, we have $n_1 \sin(\theta_1 + \phi) = n_2 \sin[\phi - (-\theta_2)]$. Since all angles are small, the paraxial version of Snell's Law is $n_1(\theta_1 + \phi) \approx n_2(\phi + \theta_2)$, or $\theta_2 \approx (n_1/n_2)\theta_1 + [(n_1 - n_2)/n_2]\phi$.
Because $\phi \approx y/R$, we obtain $\theta_2 \approx \frac{n_1}{n_2}\theta_1 - \frac{n_2 - n_1}{n_2 R} y$, which is (1.2-8).

b) Substituting $\theta_1 \approx y/z_1$ and $(-\theta_2) \approx y/z_2$ into (1.2-8),
we have $-y/z_2 \approx \frac{(n_1/n_2)\, y}{z_1} - \frac{n_2 - n_1}{n_2 R} y$, from which (1.2-9) follows.

c) With reference to Fig. 1.2-13(b), for the ray passing through the origin 0, we have angles of incidence and refraction given by y_1/z_1 and $-y_2/z_2$, respectively, so that the paraxial Snell's Law leads to (1.2-10). Rays at other angles are also directed from P_1 to P_2, as can be shown using a derivation similar to that followed in Exercise 1.2-1.

EXERCISE 1.2-3

Aberration-Free Imaging Surface In accordance with Fermat's principle, we require



that the optical path length obey $n_1 d_1 + n_2 d_2 =$ constant $= n_1 z_1 + n_2 z_2$. This constitutes an equation defining the surface, which can be written in Cartesian coordinates as

$$n_1\sqrt{(z+z_1)^2+y^2}+n_2\sqrt{(z_2-z)^2+y^2}=n_1z_1+n_2z_2. \qquad (1)$$

Given z_1 and z_2, (1) relates y to z and thus defines the surface.

EXERCISE 1.2-4

Proof of the Thin Lens Formulas

A ray at angle θ_1 and height y refracts at the first surface in accordance with (1.2-8) and its angle is altered to $\theta = \dfrac{\theta_1}{n} - \dfrac{n-1}{nR_1}\,y\,,$ (1)
where R_1 is the radius of the first surface $(R_1 < 0)$.

At the second surface, the angle is altered again to $\theta_2 = n\theta - \dfrac{1-n}{R_2}\,y\,,$ (2)
where R_2 is the radius of the second surface $(R_2 > 0)$. We have assumed that the ray height is not altered since the lens is thin.

Substituting (1) into (2) we obtain:

$$\theta_2 = n\left[\frac{\theta_1}{n} - \frac{n-1}{nR_1}\,y\right] - \frac{1-n}{R_2}\,y = \theta_1 - (n-1)\,y\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

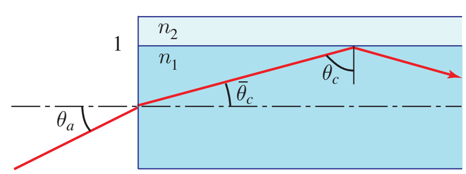
Using (1.2-11), we invoke $\theta_2 = \theta_1 - (y/f)$. (3)

If $\theta_1 = 0$, then $\theta_2 = (-y/f)$, and $z_2 \approx (y/{-\theta_2}) = f$, where f is the focal length. In general $\theta_1 \approx \dfrac{y}{z_1}$ and $-\theta_2 = \dfrac{y}{z_2}$. Therefore from (3), $\dfrac{-y}{z_2} = \dfrac{y}{z_1} - \dfrac{y}{f}$, from which (1.2-13) follows. Equation (1.2-14) can be proved by use of an approach similar to that used in Exercise 1.2-1.

EXERCISE 1.2-5

Numerical Aperture and Angle of Acceptance of an Optical Fiber

Applying Snell's law at the air/core surface:
$\sin\theta_a = n_1\sin\bar{\theta}_c = n_1\cos\theta_c$ (1)



Since $\sin\theta_c = n_2/n_1,\ \cos\theta_c = \sqrt{1-(n_2/n_1)^2}\,.$

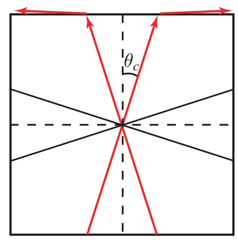
Therefore, from (1), NA $\equiv \sin\theta_a = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}\,.$

For silica glass with $n_1 = 1.475$ and $n_2 = 1.460$, the numerical aperture NA = 0.21 and the acceptance angle $\theta_a = 12.1°$.

EXERCISE 1.2-6

Light Trapped in a Light-Emitting Diode

a) The rays within the six cones of half angle $\theta_c = \sin^{-1}(1/n)$ $(= 16.1^\circ$ for GaAs) are refracted into air in all directions, as shown in the illustration. The rays outside these six cones are internally reflected. Since $\theta_c < 45^\circ$, the cones do not overlap and the reflected rays remain outside the cones and continue to reflect internally without refraction. These are the trapped rays.



b) The area of the spherical cap atop one of these cones is $A = \int_0^{\theta_c} 2\pi r \sin\theta \, r \, d\theta = 2\pi r^2(1 - \cos\theta_c)$, while the area of the entire sphere is $4\pi r^2$. Thus, the fraction of the emitted light that lies within the solid angle subtended by one of these cones is $A/4\pi r^2 = \frac{1}{2}(1 - \cos\theta_c)$ (see Sec. 18.1B). Thus, the ratio of the extracted light to the total light is $6 \times \frac{1}{2}(1 - \cos\theta_c) = 3(1 - \cos\theta_c)$ $(= 0.118$ for GaAs). Thus, 11.8% of the light is extracted for GaAs.
Note that this derivation is valid only for $\theta_c < 45^\circ$ or $n > \sqrt{2}$.

1.3 GRADED-INDEX OPTICS

EXERCISE 1.3-1

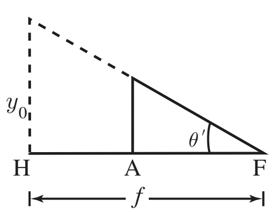
The GRIN Slab as a Lens

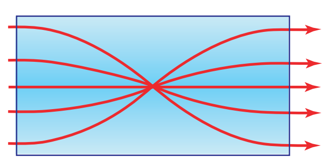
Using (1.3-11) and (1.3-12), with $\theta_0 = 0$ and $z = d$, we have $y(d) = y_0 \cos(\alpha d)$ and $\theta(d) = -y_0 \alpha \sin(\alpha d)$. Rays refract into air at an angle $\theta' \approx n_0|\theta(d)| = n_0 y_0 \alpha \sin(\alpha d)$.

Therefore, $\overline{\mathrm{AF}} \approx \dfrac{y(d)}{\theta'} = \dfrac{y_0 \cos(\alpha d)}{n_o y_0 \alpha \sin(\alpha d)} = \dfrac{1}{n_0 \alpha \tan(\alpha d)}$ and
$f = \dfrac{y_0}{\theta'} = \dfrac{1}{n_0 \alpha \sin(\alpha d)}$, so that

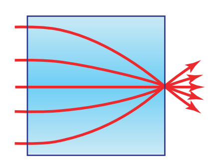
$$\overline{\mathrm{AH}} = f - \overline{\mathrm{AF}} = \frac{1}{n_0\alpha}\left[\frac{1}{\sin(\alpha d)} - \frac{1}{\tan(\alpha d)}\right] = \frac{1}{n_0\alpha}\,\frac{1 - \cos(\alpha d)}{\sin(\alpha d)}$$
$$= \frac{1}{n_0\alpha}\,\frac{2\sin^2(\alpha d/2)}{2\sin(\alpha d/2)\cos(\alpha d/2)} = \frac{1}{n_0\alpha}\tan(\alpha d/2).$$

Trajectories:





$d = \pi/\alpha$



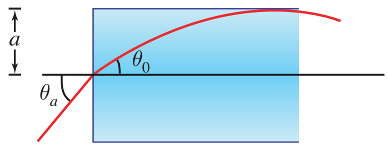
$d = \pi/2\alpha$

EXERCISE 1.3-2

Numerical Aperture of the Graded-Index Fiber

Using (1.3-11) with $y_0 = 0$, we obtain $y(z) = (\theta_0/\alpha)\sin(\alpha z)$. The ray traces a sinusoidal trajectory with amplitude θ_0/α that must not exceed the radius a. Thus $\theta_0/\alpha = a$.
The acceptance angle is therefore $\theta_a \approx n_0\theta_0 = n_0\alpha a$.

For a step-index fiber (Exercise 1.2-5),
$\theta_a = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1+n_2)(n_1-n_2)}$.
If $n_1 \approx n_2$, $\theta_a \approx \sqrt{2n_1(n_1-n_2)}$.
If $n_1 = n_0$ and $n_2 = n_0(1-\alpha^2a^2/2)$,
$\theta_a \approx \sqrt{2n_0(\alpha^2a^2n_0/2)} = \alpha\, a\, n_0$, which is the same acceptance angle as for the graded-index fiber.



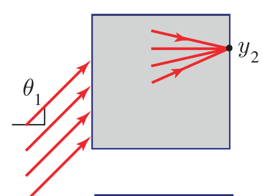
1.4 MATRIX OPTICS

EXERCISE 1.4-1

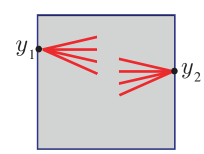
Special Forms of the Ray-Transfer Matrix

Using the basic equations
$y_2 = Ay_1 + B\,\theta_1$ and $\theta_2 = C\,y_1 + D\,\theta_1$, we obtain:

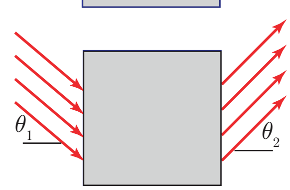
• If $A = 0$, then $y_2 = B\theta_1$, i.e., for a given θ_1, we see that y_2 is the same regardless of y_1.
This is a focusing system.



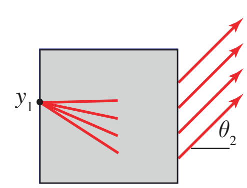
• If $B = 0$, then $y_2 = A\,y_1$, i.e., for a given y_1, we see that y_2 is the same regardless of θ_1.
This is an imaging system.



• If $C = 0$, then $\theta_2 = D\,\theta_1$, i.e., we see that all parallel rays remain parallel.



• If $D = 0$, then $\theta_2 = C\,y_1$, i.e., we see that all rays originating from a point become parallel.



EXERCISE 1.4-2

A Set of Parallel Transparent Plates

The first plate has ray transfer matrix: $\begin{bmatrix}1 & d_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1/n_1\end{bmatrix}=\begin{bmatrix}1 & d_1/n_1\\0 & 1/n_1\end{bmatrix}$.

The second plate has ray transfer matrix: $\begin{bmatrix}1 & d_2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & n_1/n_2\end{bmatrix}=\begin{bmatrix}1 & d_2 n_1/n_2\\0 & n_1/n_2\end{bmatrix}$.

The first and second plates together have a ray transfer matrix:

$$\begin{bmatrix}1 & d_2 n_1/n_2\\0 & n_1/n_2\end{bmatrix}\begin{bmatrix}1 & d_1/n_1\\0 & 1/n_1\end{bmatrix}=\begin{bmatrix}1 & d_1/n_1+d_2/n_2\\0 & 1/n_2\end{bmatrix}.$$

Similarly N plates have a ray transfer matrix: $\begin{bmatrix}1 & \sum_i d_i/n_i\\0 & 1/n_N\end{bmatrix}$.

Including the interface between the N^{th} plate and air, the overall ray transfer matrix becomes:

$$\begin{bmatrix}1 & 0\\0 & n_N\end{bmatrix}\begin{bmatrix}1 & \sum_i d_i/n_i\\0 & 1/n_N\end{bmatrix}=\begin{bmatrix}1 & \sum_i d_i/n_i\\0 & 1\end{bmatrix}.$$

The ray transfer matrix of an inhomogeneous plate with refractive index $n(z)$ and width d is:

$$\begin{bmatrix}1 & \int_0^d dz/n(z)\\0 & 1\end{bmatrix}.$$

EXERCISE 1.4-3

A Gap Followed by a Thin Lens

$$\mathbf{M}=\begin{bmatrix}1 & 0\\-1/f & 1\end{bmatrix}\begin{bmatrix}1 & d\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & d\\-1/f & 1-d/f\end{bmatrix}.$$

EXERCISE 1.4-4

Imaging with a Thin Lens

$$\mathbf{M}=\begin{bmatrix}1 & d_2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & d_1\\-1/f & 1-d_1/f\end{bmatrix}=\begin{bmatrix}1-d_2/f & d_1+d_2(1-d_1/f)\\-1/f & 1-d_1/f\end{bmatrix}.$$

For imaging, the matrix element B must vanish (see Exercise 1.4-1), so that $d_1+d_2(1-d_1/f)=0$. Dividing this by $d_1 d_2$ yields $1/d_2+1/d_1-1/f=0$.

For all parallel rays to be focused onto a single point, the matrix element A must vanish (see Exercise 1.4-1), so that $1-d_2/f=0$ or $d_2=f$.

EXERCISE 1.4-5

Imaging with a Thick Lens

a) This system is composed of 5 subsystems:
1) A distance d_1 in air, followed by
2) An air/glass refracting surface, followed by
3) A distance d in glass, followed by
4) An glass/air refracting surface, followed by
5) A distance d_2 in air.

The ray transfer matrix of subsystem 2) is:

$$\begin{bmatrix} 1 & 0 \\ -(n-1)/nR & 1/n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/nf_1 & 1/n \end{bmatrix}, \qquad \text{where } f_1 = R/(n-1).$$

The ray transfer matrix of subsystems 2) and 3) is:

$$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/nf_1 & 1/n \end{bmatrix} = \begin{bmatrix} 1-d/nf_1 & d/n \\ -1/nf_1 & 1/n \end{bmatrix}.$$

The ray transfer matrix of subsystems 2), 3), and 4) (the lens) is:

$$\begin{bmatrix} 1 & 0 \\ -(n-1)/R & n \end{bmatrix} \begin{bmatrix} 1-d/nf_1 & d/n \\ -1/nf_1 & 1/n \end{bmatrix} = \begin{bmatrix} 1-d/nf_1 & d/n \\ -(1-d/nf_1)/f_1 - 1/f_1 & -d/nf_1 + 1 \end{bmatrix}.$$

The ray transfer matrix of the entire system is:

$$\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1-d/nf_1 & d/n \\ -2/f_1 + d/nf_1^2 & 1-d/nf_1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}.$$

For this system to be an imaging system, the B element of its ray transfer matrix must vanish, *i.e.*, $B = d_1(1-d/nf_1) + d/n + d_2\left[d_1(-2/f_1 + d/nf_1^2) + (1-d/nf_1)\right] = 0.$

Grouping together the terms proportional to d_1, d_2, and $d_1 d_2$, we have
$(d_1+d_2)(1-d/nf_1) - d_1 d_2(2/f_0 - d/nf_1^2) + d/n = 0.$ (1)

Using the definitions
$1/f = 2/f_1 - d/nf_1^2$ (2)
and $h = (fd/nf_1)$, (3)

(1) becomes: $(d_1+d_2)(1-h/f) - d_1 d_2/f + d/n = 0.$ (4)

We now rewrite (4) in terms of z_1 and z_2 by substituting $d_1 = z_1 - h$ and $d_2 = z_2 - h$.
The results is: $z_1 + z_2 - z_1 z_2/f + b = 0,$ (5)

where $$\begin{aligned} b &= d/n - h^2/f - 2h(1-h/f) = d/n + h^2/f - 2h \\ &= d/n + (h/f)(h-2f). \end{aligned} \qquad (6)$$

If $b=0$, (5) gives the desired result, $1/z_1 + 1/z_2 = 1/f$. To prove that $b=0$, we use (2) and (3) to write $1/f = (2f-h)/f_1 f$, from which $2f - h = f_1$. Substituting this into (6), we obtain $b = d/n - hf_1/f$. We now use (3) to write $d/n = hf_1/f$, so that $b = hf_1/f - hf_1/f = 0$, as promised.

b) We show below that a ray parallel to the optical axis at height y_1 must pass through the point F_2, a distance $f-h$ from the right surface of the lens, regardless of the height y_1. This can be easily shown if we consider the ray transfer matrix of the system composed of the thick lens (subsystems 2, 3, and 4 above) followed by a distance

$f-h$ in air. This composite system has ray transfer matrix

$$\begin{bmatrix} 1 & f-h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1-d/nf_1 & d/n \\ -2/f_1+d/nf_1^2 & 1-d/nf_1 \end{bmatrix}.$$

If the element $A=0$, then $y_2=B\theta_1$ so that for $\theta_1=0$ (for rays parallel to the optical axis), we have $y_2=0$, i.e., the rays pass through the point F_2.

We now examine $A=(1-d/nf_1)+(f-h)(-2/f_1+d/nf_1^2)$, and show that it is 0. Using (2), we have $A=(1-h/f)+(f-h)(-2+h/f)/f_1$. Using the relation $2f-h=f_1$, we obtain $A=(1-h/f)+(f-h)/(-f)=0$, as promised.

EXERCISE 1.4-6

A Periodic Set of Pairs of Different Lenses

Here, the unit cell is composed of 2 subsystems, each comprising a distance d of free space followed by a lens. The ray transfer matrix of the unit cell is therefore given by the product

$$\begin{bmatrix} 1 & d \\ -1/f_2 & 1-d/f_2 \end{bmatrix} \begin{bmatrix} 1 & d \\ -1/f_1 & 1-d/f_1 \end{bmatrix}.$$

The A and D elements of this product are:

$$A=1-d/f_1, \quad D=-d/f_2+(1-d/f_2)(1-d/f_1)$$

so that

$$b=(A+D)/2=1-d/f_1-d/f_2+d^2/2f_1f_2=2(1-d/2f_1)(1-d/2f_2)-1.$$

The condition $|b|\le 1$ is equivalent to $-1\le b\le 1$ or $0\le b+1\le 2$, which leads to the desired condition

$$0\le(1-d/2f_1)(1-d/2f_2)\le 1.$$

EXERCISE 1.4-7

An Optical Resonator

The resonator may be regarded as a periodic system whose unit system is a single round trip between the pair of mirrors. In a resonator of length d, a paraxial ray starting at the position y_0 travels a distance d in free space, is reflected from the mirror 2, then travels again backward through the same distance of free space, and finally is reflected from the mirror 1 at position y_1. The process is repeated periodically. The unit cell therefore consists of a cascade of two subsystems, each comprising propagation in free space followed by reflection from a mirror. The condition of stability may determined by writing the ray transfer matrix of the unit cell, as in the previous exercise. Since a mirror with radius of curvature R has the same ray transfer matrix as a lens with focal length f, if $f=-R/2$, the stability condition determined for the periodic set of pairs of lenses considered in the previous exercise may be directly used to obtain:

$$0\le(1+d/R_1)(1+d/R_2)\le 1.$$

The same result is set forth in (11.2-5).