CHAPTER 2

Extract

2-1. (a) $d\Omega = \sin\theta \, d\theta \, d\phi$

$$\Omega_A = \int_{45^\circ}^{60^\circ} \int_{30^\circ}^{60^\circ} d\Omega = \int_{\pi/4}^{\pi/3} \int_{\pi/6}^{\pi/3} \sin\theta \, d\theta \, d\phi$$

$$= (\phi)\Big|_{\pi/4}^{\pi/3} (-\cos\theta)\Big|_{\pi/6}^{\pi/3}$$

$$= \left(\frac{\pi}{3} - \frac{\pi}{4}\right)(-0.5 + 0.866)$$

$$\Omega_A = \left(\frac{\pi}{12}\right)(0.366) = 0.09582 \text{ sterads}$$

$$\Omega_A = \begin{cases} 0.09582 \text{ sterads} \\ 0.09582 \left(\dfrac{180}{\pi}\right)\left(\dfrac{180}{\pi}\right) = 314.5585 \text{ (degrees)}^2 \end{cases}$$

Approximate

$$\Omega_A \simeq \left(\frac{\pi}{3} - \frac{\pi}{4}\right)\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

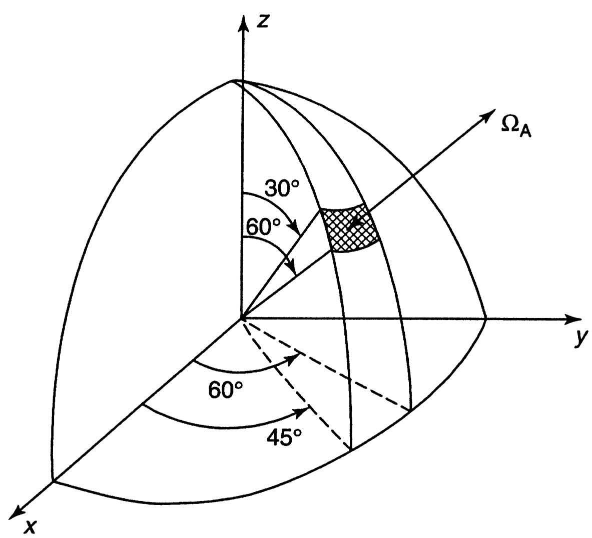
$$\simeq \left(\frac{\pi}{12}\right)\left(\frac{\pi}{6}\right) = \frac{\pi^2}{72}$$

$$\Omega_A \simeq 0.13708 \text{ sterads}$$

$$\Omega_A \simeq (60 - 45)(60 - 30)$$

$$\simeq 450 \text{ (degrees)}^2 \text{ or error of}$$

$$\left(\frac{450 - 314.5585}{314.5585}\right) \times 100 = 43.06\%$$



(b) $D_0 = \dfrac{4\pi}{\Omega_A(\text{sterads})} = \dfrac{4\pi}{0.09582} = 131.1456$ (dimensionless)

$= 10\log_{10}(131.1456) = 21.1775$ dB

or

$$D_0 = \frac{4\pi\left(\frac{180}{\pi}\right)\left(\frac{180}{\pi}\right)}{\Omega_A\ (\text{degrees})^2} = 131.1456\ (\text{dimensionless}) = 21.1775\text{ dB}$$

$$D_0 = \begin{cases} 131.1456\ (\text{dimensionless}) \\ 21.1775\ (\text{dB}) \end{cases}$$

2-2. $\underline{\mathcal{W}} = \underline{\mathcal{E}} \times \underline{\mathcal{H}} = Re[Ee^{j\omega t}] \times Re[\underline{H}e^{j\omega t}]$

Using the identity $Re[\underline{A}e^{j\omega t}] = \frac{1}{2}[\underline{E}e^{j\omega t} + \underline{E}^* e^{-j\omega t}]$

The instant Poynting vector can be written as

$$\begin{aligned}\underline{\mathcal{W}} &= \left\{\tfrac{1}{2}[\underline{E}e^{j\omega t} + \underline{E}^* e^{-j\omega t}]\right\} \times \left\{\tfrac{1}{2}[\underline{H}e^{j\omega t} + \underline{H}^* e^{-j\omega t}]\right\} \\ &= \tfrac{1}{2}\left\{\tfrac{1}{2}[\underline{E}\times\underline{H}^* + \underline{E}^*\times\underline{H}] + \tfrac{1}{2}[\underline{E}\times\underline{H}e^{j2\omega t} + \underline{E}^*\times\underline{H}^* e^{-j\omega t}]\right\} \\ &= \tfrac{1}{2}\left\{\tfrac{1}{2}[\underline{E}\times\underline{H}^* + (\underline{E}\times\underline{H}^*)^*] + \tfrac{1}{2}[\underline{E}\times\underline{H}e^{j2\omega t} + (\underline{E}\times\underline{H}e^{j\omega t})^*]\right\}\end{aligned}$$

Using the above identity again, but this time in reverse order, we can write that

$$\underline{\mathcal{W}} = \tfrac{1}{2}[\text{Re}(E\times H^*)] + \tfrac{1}{2}[\text{Re}(\underline{E}\times\underline{H}e^{j2\omega t})]$$

2-3. (a) $\underline{W}_{\text{rad}} = \frac{1}{2}[\underline{E}\times\underline{H}^*] = \dfrac{E^2}{2\eta}\hat{a}_r = \dfrac{5^2}{2(120\pi)}\hat{a}_r = 0.03315\hat{a}_r$ watts/m^2

(b)
$$\begin{aligned}P_{\text{rad}} &= \oint_s W_{\text{rad}}\, ds = \int_0^{2\pi}\int_0^{\pi}(0.03315)(r^2\sin\theta\, d\theta\, d\phi) \\ &= \int_0^{2\pi}\int_0^{\pi}(0.03315)(100)^2\cdot\sin\theta\, d\theta\, d\phi \\ &= 2\pi(0.03315)(100)^2\int_0^{\pi}\sin\theta\, d\theta = 2\pi(0.03315)(100)^2\cdot(2) \\ &= 4165.75\text{ watts}\end{aligned}$$

2-4. a. $\mathcal{U}(\theta) = \cos\theta$

$\mathcal{U}(\theta_h) = 0.5 = \cos\theta_h \Rightarrow \theta_h = \cos^{-1}(0.5) = 60°$

$\Rightarrow \Theta_h = 2(60°) = 120° = \dfrac{2\pi}{3}$ rads.

$\mathcal{U}(\theta_n) = 0 = \cos\theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90°$

$\Rightarrow \Theta_n = 2(90°) = 180° = \pi$ rads.

b. $\mathcal{U}(\theta) = \cos^2\theta$

$$\mathcal{U}(\theta_h) = 0.5 = \cos^2\theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$
$$\Rightarrow \Theta_h = 2(45) = 90^\circ = \pi/2 \text{ rads}$$
$$\mathcal{U}(\theta_n) = 0 = \cos^2\theta_n \Rightarrow \theta_n = \cos^{-1}(0) = 90^\circ$$
$$\Rightarrow \Theta_n = 2(90^\circ) = 180^\circ = \pi \text{ rads}$$

c. $\mathcal{U}(\theta) = \cos(2\theta)$

$$\mathcal{U}(\theta_h) = 0.5 = \cos(2\theta_h) \Rightarrow \theta_h = \tfrac{1}{2}\cos^{-1}(0.5) = 30^\circ$$
$$\Rightarrow \Theta_h = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$
$$\mathcal{U}(\theta_n) = 0 = \cos(2\theta_n) \Rightarrow \theta_n = \tfrac{1}{2}\cos^{-1}(0) = 45^\circ$$
$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

d. $\mathcal{U}(\theta) = \cos^2(2\theta)$

$$\mathcal{U}(\theta_h) = 0.5 = \cos^2(2\theta_h) \Rightarrow \theta_h = \tfrac{1}{2}\cos^{-1}(0.5)^{1/2} = 22.5^\circ$$
$$\Rightarrow \Theta_h = 2(22.5^\circ) = 45^\circ = \frac{\pi}{4} \text{ rads}$$
$$\mathcal{U}(\theta_n) = 0 = \cos^2(2\theta_n) \Rightarrow \theta_n = \tfrac{1}{2}\cos^{-1}(0) = 45^\circ$$
$$\Rightarrow \Theta_n = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

e. $\mathcal{U}(\theta) = \cos(3\theta)$

$$\mathcal{U}(\theta_h) = \cos(3\theta_h) = 0.5 \Rightarrow \theta_h = \frac{1}{3}\cos^{-1}(0.5) = 20^\circ$$
$$\Rightarrow \Theta_h = 2(20^\circ) = 40^\circ = 0.698 \text{ rads}$$
$$\mathcal{U}(\theta_n) = \cos(3\theta_n) = 0 \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = 30^\circ$$
$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

f. $\mathcal{U}(\theta) = \cos^2(3\theta)$

$$\mathcal{U}(\theta_h) = 0.5 = \cos^2(3\theta_h) \Rightarrow \theta_h = \frac{1}{3}\cos^{-1}(0.5)^{1/2} = 15^\circ$$
$$\Rightarrow \Theta_h = 2(15^\circ) = 30^\circ = \pi/6 \text{ rads}$$
$$\mathcal{U}(\theta_n) = 0 = \cos^2(3\theta_n) \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = 30^\circ$$
$$\Rightarrow \Theta_n = 2(30^\circ) = 60^\circ = \pi/3 \text{ rads}$$

2-5. Using the results of Problem 2-4 and a nonlinear solver to find the half power beamwidth of the radiation intensity represented by the transcentendal functions, we have that:

(a) $\mathcal{U}(\theta) = \cos\theta\cos(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 55.584^\circ \\ \text{FNBW} = 90^\circ \end{cases}$

(b) $\mathcal{U}(\theta) = \cos^2\theta\cos^2(2\theta) \Rightarrow \begin{cases} \text{HPBW} = 40.985^\circ \\ \text{FNBW} = 90^\circ \end{cases}$

(c) $\mathcal{U} = \cos\theta\cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 38.668^\circ \\ \text{FNBW} = 60^\circ \end{cases}$

(d) $\mathcal{U} = \cos^2\theta\cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 28.745^\circ \\ \text{FNBW} = 60^\circ \end{cases}$

(e) $\mathcal{U} = \cos(2\theta)\cos(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 34.942^\circ \\ \text{FNBW} = 60^\circ \end{cases}$

(f) $\mathcal{U} = \cos^2(2\theta)\cos^2(3\theta) \Rightarrow \begin{cases} \text{HPBW} = 25.583^\circ \\ \text{FNBW} = 60^\circ \end{cases}$

2-6. (a) $D_0 = \dfrac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \dfrac{4\pi(200\times 10^{-3})}{0.9(125.66\times 10^{-3})} = 22.22 = 13.47 \text{ dB}$

$G_0 = \epsilon_{cd}\cdot D_0 = 0.9(22.22) = 20 = 13.01 \text{ dB}$

(b) $D_0 = \dfrac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \dfrac{4\pi(200\times 10^{-3})}{(125.66\times 10^{-3})} = 20 = 13.01 \text{ dB}$

$G_0 = \epsilon_{cd}\cdot D_0 = 0.9\cdot(20) = 18 = 12.55 \text{ dB}$

2-7. $\boxed{\mathcal{U} = B_0\cos^2\theta}$

(a) $$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi/2} \mathcal{U}\sin\theta\, d\theta = B_0 2\pi \int_0^{\pi/2} \cos^2\theta\sin\theta\, d\theta$$

$$= 2\pi B_0 \int_0^{\pi/2} \cos^2\theta\, d\,(-\cos\theta)$$

$$P_{\text{rad}} = -2\pi B_0 \frac{\cos^3\theta}{3}\Bigg|_0^{\pi/2} = -2\pi B_0\left[\frac{-1}{3}\right] = \frac{2\pi}{3}B_0 = 10 \Rightarrow B_0 = \frac{15}{\pi}$$

$$\mathcal{U} = \frac{15}{\pi}\cos^2\theta \Rightarrow W_{\text{rad}}\Big|_{\text{max}} = \frac{U}{r^2}\Big|_{\text{max}} = \frac{15}{\pi}\frac{\cos^2\theta}{r^2}\Big|_{\text{max}}$$

$$= \frac{15}{\pi(10^3)^2} = 4.7746\times 10^{-6} \text{ watts/m}^2 @\ \theta = 0^\circ$$

$$W_{\text{rad}}\Big|_{\text{max}} = 4.7746\times 10^{-6} \text{ watts/m}^2 @\ \theta = 0^\circ$$

(b) $$\Omega_A \text{ (exact)} = \int_0^{2\pi}\int_0^{\pi} U_n\cos^2\theta\sin\theta\, d\theta\, d\phi$$

$$\Omega_A \text{ (exact)} = \frac{2\pi}{3} \text{ steradians} = 2.0944 \text{ sterads} = 6{,}875.51 \text{ (degrees)}^2$$

$$\mathcal{U} = 0.5 = \cos^2\theta_h \Rightarrow \theta_h = \cos^{-1}(0.5)^{1/2} = 45^\circ$$

$$\Rightarrow \Theta_h = 2(45^\circ) = 90^\circ = \pi/2 \text{ rads}$$

$$\Omega_A\left(\frac{\text{Kraus'}}{\text{approx}}\right) = \Theta_h^2 = (\pi/2)^2 = \frac{\pi^2}{4} = 2.4674 \text{ sterads} = 8{,}099.997 \text{ (degrees)}^2$$

(c)
$$D_0 \text{ (exact)} = \frac{4\pi}{\Omega_A \text{ (exact)}} = \frac{4\pi}{2\pi/3} = 6 = 7.782 \text{ dB}$$
$$D_0 \text{ (approx/Kraus')} = \frac{4\pi}{\Omega_A \text{ (approx)}} = \frac{4\pi}{\pi^2/4} = \frac{16}{\pi} = 5.093 = 7.0697$$

(d) G_0 Assuming lossless antenna ($P_{\text{in}} = P_{\text{rad}}$)

$$G_0 \text{ (exact)} = D_0 \text{ (exact)} = 6 = 7.782 \text{ dB}$$
$$G_0 \text{ (approx)} = D_0 \text{ (approx)} = 5.093 = 7.0697 \text{ dB}$$

$\boxed{\mathcal{U} = B_0 \cos^3\theta}$

(a)
$$P_{\text{rad}} = -2\pi B_0\left(-\frac{1}{4}\right) = \frac{\pi}{2}B_0 = 10 \Rightarrow B_0 = 20/\pi$$
$$W_{\text{rad}}\Big|_{\text{max}} = \frac{20}{\pi}\frac{1}{r^2} = \frac{20}{\pi} \times 10^{-6} = 6.366 \times 10^{-6} \text{ watts/m}^2$$

(b) Ω_A (exact) $= (\pi/2) = 1.5708$ sterads
$$\mathcal{U} = 0.5 = \cos^3\theta_h \Rightarrow \theta_h \cos^{-1}(0.5)^{1/3} = 37.467^\circ$$
$$\Rightarrow \Theta_h = 2(37.467^\circ) = 74.934^\circ = 1.30785 \text{ rads}$$
$$\Omega_A \text{ (approx)} = (1.30785)^2 = 1.71 \text{ sterads}$$

(c) D_0 (exact) $= 4\pi/\pi/2 = 8 = 9.031$ dB
$$D_0 \text{ (approx)} = \frac{4\pi}{1.71} = 7.347 = 8.66 \text{ dB}$$

(d) Assuming lossless antenna $\Rightarrow$ Gain = Directivity (see part c)

2-8. $\mathcal{U}(\theta, \phi) = \cos^n(\theta) \qquad 0 \leqslant \theta \leqslant \pi/2, 0 \leqslant \phi \leqslant 2\pi$

(a) $\mathcal{U}_n(\theta_n, \phi) = 0.5 = \cos^n(5^\circ) = [\cos(5^\circ)]^n = (0.99619)^n$
$$0.5 = (0.99619)^n$$
$$\log_{10}(0.5) = \log[(0.99619)^n] = n\log_{10}(0.99619) = n(-0.00166)$$
$$-0.30103 = -0.00166n$$

$$\boxed{n = 181.34}$$

(b) $\mathcal{U}(\theta,\phi) = \cos^{181.34}(\theta); \quad \mathcal{U}_{\max} = 1, \theta = 0^\circ$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi/2} \mathcal{U}(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi\int_0^{\pi/2}\cos^{181.34}(\theta)\sin\theta\, d\theta$$

$$= 2\pi\left[-\frac{\cos^{182.34}(\theta)}{182.34}\right] = \left[-0 + \frac{1}{182.34}\right]2\pi = \frac{2\pi}{182.34} = 0.03446$$

$$D_0 = \frac{4\pi\mathcal{U}_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{2\pi}(182.34) = 2(182.34) = 364.68$$

$$\boxed{D_0 = 364.68 = 25.62 \text{ dB}}$$

(c) *Kraus' Approximation* (2–27):

$$D_0 \simeq \frac{41{,}253}{\Theta_{1d}\Theta_{2d}} = \frac{41{,}253}{(10)(10)} = 412.53 = 26.15 \text{ dB}$$

$$\boxed{D_0 \simeq 412.53 = 26.15 \text{ dB}}$$

(d) *Tai & Pereira* (2-30b):

$$D_0 \simeq \frac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72{,}815}{2(10)^2} = \frac{72{,}815}{200} = 364.075 = 25.61 \text{ dB}$$

$$\boxed{D_0 \simeq 364.075 = 25.61 \text{ dB}}$$

2-9.

$$\mathcal{U}(\theta,\phi) = \left\{\begin{array}{ll} 1 & 0^\circ \leqslant \theta \leqslant 20^\circ \\ 0.342\ \csc(\theta) & 20^\circ \leqslant \theta \leqslant 60^\circ \\ 0 & 60^\circ \leqslant \theta \leqslant 180^\circ \end{array}\right\} 0^\circ \leqslant \phi \leqslant 360^\circ$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} u(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi\left[\int_0^{20^\circ}\sin\theta\, d\theta\right.$$

$$\left. + \int_{20^\circ}^{60^\circ} 0.342\ \csc(\theta)\times\sin\theta\, d\theta\right]$$

$$= 2\pi\left\{-\cos\theta\Big|_0^{\pi/9} + 0.342\cdot\theta\Big|_{\pi/9}^{\pi/3}\right\}$$

$$= 2\pi\left\{\left[-\cos\left(\frac{\pi}{9}\right)+1\right] + 0.342\left(\frac{\pi}{3}-\frac{\pi}{9}\right)\right\}$$

$$= 2\pi\left\{[-0.93969+1] + 0.342\pi\left(\frac{2}{9}\right)\right\}$$

$$= 2\pi\{0.06031 + 0.23876\} = 1.87912$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{1.87912} = 6.68737 = 8.25255 \text{ dB}$$

2-10. (a) $D_0 \simeq \dfrac{41{,}253}{\Theta_{1d}\Theta_{2d}} = \dfrac{41{,}253}{30(35)} = 39.29 = 15.94 \text{ dB}$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

(b) $D_0 \simeq \dfrac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \dfrac{72{,}815}{(30)^2 + (35)^2} = 34.27 = 15.35 \text{ dB}$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

2-11. $D_0 = \dfrac{4\pi U_{\max}}{P_{\text{rad}}}$

(a) $U = \sin\theta \sin\phi$ for $0 \leqslant \theta \leqslant \pi, 0 \leqslant \phi \leqslant \pi$

$U|_{\max} = 1$ and it occurs when $\theta = \phi = \pi/2$.

$$P_{\text{rad}} = \int_0^{\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = \int_0^{\pi} \sin\phi \, d\phi \int_0^{\pi} \sin^2\theta \, d\theta = 2\left(\frac{\pi}{2}\right) = \pi.$$

Thus $D_0 = \frac{4\pi(1)}{\pi} = 4 = 6.02 \text{ dB}$

The half-power beamwidths are equal to

$$\text{HPBW (az.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$
$$\text{HPBW (el.)} = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

In a similar manner, it can be shown that for

(b) $U = \sin\theta \sin^2\phi \Rightarrow D_0 = 5.09 = 7.07 \text{ dB}$

$\text{HPBW (el.)} = 120^\circ, \ \text{HPBW (az.)} = 90^\circ$

(c) $U = \sin\theta \sin^3\phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}$

$\text{HPBW (el.)} = 120^\circ, \ \text{HPBW (az.)} = 74.93^\circ$

(d) $U = \sin^2\theta \sin\phi \Rightarrow D_0 = 12\pi/8 = 4.71 = 6.73 \text{ dB}$

$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 120^\circ$

(e) $U = \sin^2\theta \sin^2\phi \Rightarrow D_0 = 6 = 7.78 \text{ dB}, \text{HPBW (az.)} = \text{HPBW (el.)} = 90^\circ$

(f) $U = \sin^2\theta \sin^3\phi \Rightarrow D_0 = 9\pi/4 = 7.07 = 8.49 \text{ dB}$

$\text{HPBW (el.)} = 90^\circ, \text{HPBW (az.)} = 74.93^\circ$

2-12. Using the half-power beamwidths found in the previous problem (Problem 2-11), the directivity for each intensity using Kraus' and Tai & Pereira's formulas is given by

$U = \sin\theta \cdot \sin\phi$;

(a) $D_0 \simeq \dfrac{41253}{\Theta_{1d}\Theta_{2d}} = \dfrac{41253}{120(120)} = 2.86 = 4.57$ dB

(b) $D_0 \simeq \dfrac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \dfrac{72{,}815}{(120)^2 + (120)^2} = 2.53 = 4.03$ dB

$U = \sin\theta \cdot \sin^2\phi$;

(a) $D_0 \simeq 3.82 = 5.82$ dB

(b) $D_0 \simeq 3.24 = 5.10$ dB

$U = \sin\theta \cdot \sin^3\phi$;

(a) $D_0 \simeq 4.59 = 6.62$ dB

(b) $D_0 \simeq 3.64 = 5.61$ dB

$U = \sin^2\theta \cdot \sin\phi$;

(a) $D_0 \simeq 3.82 = 5.82$ dB

(b) $D_0 \simeq 3.24 = 5.10$ dB

$U = \sin^2\theta \cdot \sin^2\phi$;

(a) $D_0 \simeq 5.09 = 7.07$ dB

(b) $D_0 \simeq 4.49 = 6.53$ dB

$U = \sin^2\theta \cdot \sin^3\phi$;

(a) $D_0 \simeq 6.12 = 7.87$ dB

(b) $D_0 \simeq 5.31 = 7.25$ dB

2-13. (a) $D_0 = \dfrac{4\pi}{\Theta_{1r}\Theta_{2r}} = \dfrac{4\pi}{(1.5064)^2} = 5.5377 = 7.433$ dB

(b) $D_0 = \dfrac{32\ln(2)}{\Theta_{1r}^2 + \Theta_{2r}^2} = \dfrac{32\ln(2)}{(1.5064)^2 + (1.5064)^2} = 4.88725 = 6.8906$ dB

2-14. (a) $D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{U_{\text{max}}}{U_0}$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} U \sin\theta \, d\theta = 2\pi \left\{ \int_0^{30^\circ} \sin\theta \, d\theta \right.$$

$$\left. + \int_{30^\circ}^{60^\circ} (0.5) \sin\theta \, d\theta + \int_{60^\circ}^{90^\circ} (0.1) \sin\theta \, d\theta \right\}$$

$$= 2\pi \left\{ (-\cos\theta)\Big|_0^{30^\circ} + \left(-\frac{\cos\theta}{2}\right)\Big|_{30^\circ}^{60^\circ} + (-0.1\cos\theta)\Big|_{60^\circ}^{90^\circ} \right\}$$

$$= 2\pi \left\{ (-0.866 + 1) + \left(\frac{-0.5 + 0.866}{2}\right) + \left(\frac{-0 + 0.5}{10}\right) \right\}$$

$$P_{\text{rad}} = 2\pi\{-0.866 + 1 - 0.25 + 0.433 + 0.05\} = 2\pi(0.367)$$

$$= 0.734 \cdot \pi = 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

(b) D_0 (dipole) $= 1.5 = 1.761$ dB

D_0 (above dipole) $= (7.3636 - 1.761)$ dB $= 5.6026$ dB

D_0 (above dipole) $= \frac{5.45}{1.5} = 3.633 = 5.603$ dB

2-15. (a) $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \sin^2\phi \, d\phi \cdot \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta$

$$= (\pi)\left(\frac{1}{5}\right) = \frac{\pi}{5}.$$

$$U_{\text{max}} = U(\theta = 0^\circ, \phi = \pi/2) = 1.$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi}{(\pi/5)} = 20 = 13.0 \text{ dB}$$

(b) Elevation Plane: θ varies, ϕ fixed

$\rightarrow$ Choose $\phi = \pi/2$.

$U(\theta, \phi = \pi/2) = \cos^4\theta, \quad 0 \leqslant \theta \leqslant \pi/2.$

$\cos^4\left[\frac{\text{HPBW(el.)}}{2}\right] = \frac{1}{2}$

HPBW (el.) $= 2 \cdot \cos^{-1}\{\sqrt{0.5}\}^{1/2} = 65.5^\circ$.

2-16. (a) $P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta \, d\theta \, d\phi = 2\pi$

$$\cdot \left\{ \int_0^{30^\circ} \cdot \sin\theta \, d\theta + \int_{30^\circ}^{90^\circ} \frac{\cos\theta \cdot \sin\theta}{0.866} \, d\theta \right\}$$

$$= 2\pi \left\{ \int_0^{\pi/6} \sin\theta \, d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{0.866} \cos\theta \cdot \sin\theta \, d\theta \right\}$$

$$= 2\pi \left\{ -\cos\theta\Big|_0^{\pi/6} + \frac{1}{0.866}\left(-\frac{\cos^2\theta}{2}\right)\Big|_{\pi/6}^{\pi/2} \right\} = 2\pi[-0.866 + 1 + 0.433]$$

$$= 3.5626$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi(1)}{3.5626} = 3.5273 = 5.4745 \text{ dB}$$

(b) $U = \dfrac{\cos(\theta)}{0.866} = 0.5 \Rightarrow \cos\theta = 0.5(0.866) = 0.433, \theta = \cos^{-1}(0.433) = 64.34°$

$$\Theta_{1r} = 2(64.34) = 128.68° = 2.246 \text{ rad} = \Theta_{2r}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{(2.246)^2} = 2.4912 = 3.9641 \text{ dB}$$

2-17. a. 35 dB

b. $20\log_{10}\left|\dfrac{E_{\text{max}}}{E_s}\right| = 35, \log_{10}\left|\dfrac{E_{\text{max}}}{E_s}\right| = \dfrac{35}{20} = 1.75$

$$\left|\frac{E_{\text{max}}}{E_s}\right| = 10^{1.75} = 56.234$$

2-18. a. $U = \sin\theta, U_{\text{max}} = 1, \quad P_{\text{rad}} = \displaystyle\int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$

$$= \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2$$

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.2732$$

b. HPBW $= 120°, 2\pi/3$

The directivity based on (2–33a) is equal to,

$$D_0 = \frac{101}{120° - 0.0027(120°)^2} = 1.2451$$

while that based on (2–33b) is equal to,

$$D_0 = -172.4 + 191\sqrt{0.818 + \frac{1}{120°}} = 1.2245$$

c. Computer Program $D_0 = 1.2732$

2-19. a. $U = \sin^3\theta$, $U_{\text{max}} = 1$, $P_{\text{rad}} = \displaystyle\int_0^{2\pi}\int_0^{\pi} \sin^4\theta \, d\theta \, d\phi = \frac{3}{4}\pi^2$,

$$D_0 = \frac{4\pi}{\frac{3}{4}\pi^2} = \frac{16}{3\pi} = 1.6976$$

b. HPBW = 74.93°

From (2–33a), $D_0 = \dfrac{101}{(74.93°) - 0.0027(74.93°)^2} = 1.68971$

From (2–33b), $D_0 = -172.4 + 191\sqrt{0.818 + \dfrac{1}{74.93°}} = 1.75029$

c. Computer program $D_0 = 1.693$

The value of D_0(= 1.693) is similar to that of (4–91) or 1.643

2-20. a. $U = J_1{}^2(\text{ka}\sin\theta)$,

$a = \lambda/10, \text{ka}\sin\theta = \dfrac{\pi}{5}\sin\theta. \quad \text{HPBW} = 93.10°$

From (2–33a) $D_0 = 101/[(93.10) - 0.0027(93.10)^2] = 1.449120$

From (2–33b) $D_0 = -172.4 + 191\sqrt{0.818 + \dfrac{1}{93.10}} = 1.477271$

$$a = \lambda/20, \text{ka}\sin\theta = \frac{\pi}{10}\sin\theta, \quad \text{HPBW} = 91.10°.$$

From (2-33a), $D_0 = 1.47033$, From (2-33b), $D_0 = 1.502$

b. $$a = \frac{\lambda}{10}. \quad P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} J_1^2(\text{ka}\sin\theta)\cdot\sin\theta\, d\theta\, d\phi = 0.7638045$$

$$U_{\max} = 0.0893, \; D_0 = \frac{4\pi(0.0893)}{0.7638045} = 1.469193$$

$$a = \frac{\lambda}{20}, \quad P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} J_1{}^2(\pi/10\cdot\sin\theta)\cdot\sin\theta\, d\theta\, d\phi = 0.202604$$

$$U_{\max} = 0.0240714, \quad D_0 = \frac{4\pi(0.0240714)}{0.202604} = 1.49257.$$

If the radius of loop is smaller than $\lambda/20$, the directivity approaches to 1.5.

2-21. Using the numerical techniques, the directivity for each intensity of (Prob. 2-11) with 10° uniform divisions is equal to $U = \sin\theta\cdot\sin\phi$;

(a) Midpoint; $D_0 = \dfrac{4\pi U_{\max}}{P_{\text{rad}}}$

$$U_{\max} = 1. \; P_{\text{rad}} = \frac{\pi}{18}\left(\frac{\pi}{18}\right)\sum_{j=1}^{18}\sin\phi_j\sum_{i=1}^{18}\sin^2\theta_i$$

$$\theta_i = \frac{\pi}{36} + (i-1)\frac{\pi}{18}, \quad i = 1, 2, 3, \ldots, 18$$

$$\phi_j = \frac{\pi}{36} + (j-1)\frac{\pi}{18}, \quad j = 1, 2, 3, \ldots, 18$$

$$P_{\text{rad}} = \left(\frac{\pi}{18}\right)^2(11.38656)(8.9924) = 3.119$$

$$D_0 = \frac{4\pi(1)}{3.119} = 4.03 = 6.05 \text{ dB}$$

(b) Trailing edge of each division

$$\text{Trailing edge; } \theta_i = i(\pi/18),\ i = 1, 2, 3, \ldots, 18$$
$$\phi_j = j(\pi/18),\ j = 1, 2, 3, \ldots, 18$$
$$P_{\text{rad}} = \left(\frac{\pi}{18}\right)^2 (11.25640)(8.96985) = 3.076$$
$$D_0 = \frac{4\pi(1)}{3.119} = 4.09 = 6.11 \text{ dB}$$

In a similar manner

$U = \sin\theta \cdot \sin^2\phi$;

(a) $P_{\text{rad}} = 2.463 \Rightarrow D_0 = 5.10 = 7.07$ dB

(b) $P_{\text{rad}} = 2.451 \Rightarrow D_0 = 5.13 = 7.10$ dB

$U = \sin\theta \cdot \sin^3\phi$;

(a) $P_{\text{rad}} = 2.092 \Rightarrow D_0 = 6.01 = 7.79$ dB

(b) $P_{\text{rad}} = 2.086 \Rightarrow D_0 = 6.02 = 7.80$ dB

$U = \sin^2\theta \cdot \sin\phi$;

(a) $P_{\text{rad}} = 2.469 \Rightarrow D_0 = 4.74 = 6.76$ dB

(b) $P_{\text{rad}} = 2.618 \Rightarrow D_0 = 4.80 = 6.81$ dB

$U = \sin^2\theta \cdot \sin^2\phi$;

(a) $P_{\text{rad}} = 2.092 \Rightarrow D_0 = 6.01 = 7.79$ dB

(b) $P_{\text{rad}} = 2.086 \Rightarrow D_0 = 6.02 = 7.80$ dB

$U = \sin^2\theta \cdot \sin^3\phi$;

(a) $P_{\text{rad}} = 1.777 \Rightarrow D_0 = 7.07 = 8.49$ dB

(b) $P_{\text{rad}} = 1.775 \Rightarrow D_0 = 7.08 = 8.50$ dB

2-22. Using the computer program Directivity of Chapter 2, the directivities for each radiation intensity of Problem 2-11 are equal to

a. $U = \sin\theta \sin\phi$; $P_{\text{rad}} = 3.1318$

$$U_{\max} = 1; \quad D_0 = \frac{4\pi \cdot U_{\max}}{3.1318} = 4.0125 \Rightarrow 6.034 \text{ dB}$$

b. $U = \sin\theta \cdot \sin^2\phi$; $P_{\text{rad}} = 2.4590$

$$U_{\max} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.4590} = 5.110358 \Rightarrow 7.0845 \text{ dB}$$

c. $U = \sin\theta \cdot \sin^3\phi;\ P_{\text{rad}} = 2.0870$

$$U_{\max} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02124 \Rightarrow 7.80 \text{ dB}$$

d. $U = \sin^2\theta \sin\phi;\ P_{\text{rad}} = 2.6579$

$$U_{\max} = 1; \quad D_0 = \frac{4\pi \cdot 1}{2.6579} = 4.72793 \Rightarrow 6.746 \text{ dB}$$

e. $U = \sin^2\theta \cdot \sin^2\phi;\ P_{\text{rad}} = 2.0870$

$$D_0 = \frac{4\pi \cdot 1}{2.0870} = 6.02126 \Rightarrow 7.7968 \text{ dB}$$

f. $U = \sin^2\theta \cdot \sin^3\phi;\ P_{\text{rad}} = 1.7714$

$$D_0 = \frac{4\pi \cdot 1}{1.7714} = 7.09403 \Rightarrow 8.5089 \text{ dB}$$

2-23. (a) $E|_{\max} = \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]|_{\max} = 1$ at $\theta = 0°$.

$$0.707E_{\max} = 0.707 \cdot (1) = \cos\left[\frac{\pi}{4}(\cos\theta_1 - 1)\right]$$

$$\frac{\pi}{4}(\cos\theta_1 - 1) = \pm\frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(2) = \text{does not exist} \\ \cos^{-1}(0) = 90° = \frac{\pi}{2} \text{ rad.} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2\left(\frac{\pi}{2}\right) = \pi$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

(b) Using the computer program of Chapter 2

$$D_0 = 2.00789 = 3.027 \text{ dB}$$

Since the pattern is not very narrow, the answer obtained using Kraus' approximate formula is not as accurate.

2-24. a. $E|_{\max} = \cos\left(\frac{\pi}{4}(\cos\theta + 1)\right)|_{\max} = 1$ at $\theta = \pi$.

$$0.707 \cdot = \cos\left(\frac{\pi}{4}(\cos\theta_1 + 1)\right)$$

$$\frac{\pi}{4}(\cos\theta_1 + 1) = \pm\frac{\pi}{4} \Rightarrow \theta_1 = \begin{cases} \cos^{-1}(-2) \rightarrow \text{does not exist.} \\ \cos^{-1}(0) \rightarrow 90° \rightarrow \frac{\pi}{2} \text{ rad} \end{cases}$$

$$\Theta_{1r} = \Theta_{2r} = 2\left(\frac{\pi}{2}\right) = \pi.$$

$$D_0 \simeq \frac{4\pi}{\pi^2} = \frac{4}{\pi} = 1.273 = 1.049 \text{ dB}$$

b. Computer Program

$D_0 = 2.00789 = 3.027$ dB

2-25. a. $$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi/2} U_0 \sin(\pi\sin\theta)\cdot\sin\theta\, d\theta\, d\phi = 2\pi\cdot U_0\cdot\frac{\pi}{2}J_1(\pi) = U_0\pi^2 J_1(\pi)$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi U_0}{U_0\pi^2 J_1(\pi)} = \frac{4}{\pi}\cdot\frac{1}{J_1(\pi)} = 4.4735$$

$$\leftarrow \frac{\pi}{2}J_1(\pi) = 0.44707273561622$$

b. Computer Program

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi/2} U_0 \sin(\pi\sin\theta)\sin\theta\, d\theta\, d\phi = 2\pi\cdot(0.44707273561618)$$

$$D_0 = 4.4735$$

2-26. (a) Using the computer program of Chapter 2.

$$D_0 = 14.0707 = 11.48 \text{ dB}$$

(b) $$U|_{\max} = \left[\frac{\sin(\pi\sin\theta)}{\pi\sin\theta}\right]^2_{\max} = 1 \quad \text{when } \theta = 0°.$$

$$U = \tfrac{1}{2}U_{\max} = \tfrac{1}{2}(1) = \left[\frac{\sin(\pi\sin\theta_1)}{\pi\sin\theta_1}\right]^2$$

Iteratively we obtain $\theta_1 = 26.3°$. Therefore

$$\Theta_{1d} = \Theta_{2d} = 2(26.3°) = 52.6°.$$

and $D_0 \simeq \dfrac{41{,}253}{(52.6)^2} = 14.91 = 11.73$ dB using the Kraus' formula

(c) For Tai and Pereira's formula

$$D_0 = \frac{72{,}815}{2\cdot\Theta_{1d}^2} = \frac{72{,}815}{2(52.6)^2} = 13.16 = 11.19 \text{ dB}$$

2-27. $$U = \frac{1}{2\eta}|E|^2 = \frac{1}{2\eta}\sin\theta\cos^2\phi \Rightarrow U_{\max} = \frac{1}{2\eta}$$

(a) $$P_{\text{rad}} = 2\cdot\int_0^{\pi/2}\int_0^{\pi}\frac{1}{2\eta}\sin^2\theta\cos^2\phi\, d\theta\, d\phi = \frac{1}{\eta}\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8\eta}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi\left(\frac{1}{2\eta}\right)}{\frac{\pi^2}{8\eta}} = \frac{16}{\pi} = 5.09 = 7.07 \text{ dB}$$

(b) $U_{\max} = \frac{1}{2\eta}$ at $\theta = \pi/2, \phi = 0$

In the elevation plane through the maximum $\phi = 0$ and $U = \frac{1}{2\eta}\sin\theta$.
The 3-dB point occurs when

$$U = 0.5\,U_{\max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\sin\theta_1 \Rightarrow \theta_1 = \sin^{-1}(0.5) = 30^\circ$$

Therefore $\Theta_{1d} = 2(90 - 30) = 120^\circ$

In the azimuth plane through the maximum $\theta = \pi/2$ and $U = \frac{1}{2\eta}\cos^2\phi$.

The 3-dB point occurs when $U = 0.5\,U_{\max} = 0.5\left(\frac{1}{2\eta}\right) = \frac{1}{2\eta}\cos^2\theta_1$
$\Rightarrow \phi_1 = \cos^{-1}(0.707) = 45^\circ$, $\Theta_{2d} = 2(90^\circ - 45^\circ) = 90^\circ$.

Therefore using Kraus' formula $D_0 \simeq \frac{41{,}253}{120 \cdot (90)} = 3.82 = 5.82$ dB

(c) Using Tai and Pereira's formula

$$D_0 \simeq \frac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \frac{72{,}815}{(120)^2 + (90)^2} = 3.24 = 5.10 \text{ dB}$$

(d) Using the computer program of Chapter 2.

$$D_0 = 5.16425 = 7.13 \text{ dB}$$

2-28. $\mathcal{U} = \left[\frac{J_1(\text{ka}\sin\theta)}{\sin\theta}\right]^2 = (\text{ka})^2\left[\frac{J_1(\text{ka}\sin\theta)}{\text{ka}\sin\theta}\right]^2 = \mathcal{U}_0\left[\frac{J_1(\text{ka}\sin\theta)}{\text{ka}\sin\theta}\right]^2$

(a) $\mathcal{U}_{\max} = \mathcal{U}_0\left(\frac{1}{2}\right)^2 = \frac{\mathcal{U}_0}{4}$ and it occurs when $\text{ka}\sin\theta = 0 \Rightarrow \theta = 0^\circ$.
The 3-dB point is obtained using

$$\mathcal{U} = \tfrac{1}{2}\,\mathcal{U}_{\max} = \frac{\mathcal{U}_0}{8} = \mathcal{U}_0\left[\frac{J_1(\text{ka}\sin\theta)}{\text{ka}\sin\theta}\right]^2 \Rightarrow \frac{J_1(\text{ka}\sin\theta)}{\text{ka}\sin\theta} = 0.3535$$

with the aid of the $J_1(x)/x$ tables of Appendix V.

$$x = \text{ka}\sin\theta_1 = 1.61 \Rightarrow \theta_1 = \sin^{-1}(1.61/2\pi) = 14.847^\circ$$
$$\Rightarrow \Theta_{1r} = 29.694^\circ$$

(b) Since $\Theta_{1r} = \Theta_{2r} = 29.694^\circ$, the directivity is equal to

$$D_0 \simeq \frac{41{,}253}{(29.694)^2} = 46.79 = 16.70 \text{ dB}$$

2-29. $G_0 = 16 \text{ dB} \Rightarrow 16 = 10\log_{10} G_0(\text{dimensionless}) \Rightarrow G_0(\text{dim}) = 10^{1.6} = 39.81$

$r = 100 \text{ meters} = 10{,}000 \text{ cm} = 10^4 \text{ cm}$

$P_{\text{rad}} = e_{cd}P_{\text{in}} = (1)P_{\text{in}} = 8 \text{ watts}$

$f = 1{,}900 \text{ MHz} \Rightarrow \lambda = 30 \times 10^9/1.9 \times 10^9 = 15.789 \text{ cm}$

(a)
$$W_0 = \frac{P_{\text{rad}}}{4\pi r^2} = \frac{8}{4\pi(10^4)^2} = \frac{8}{4\pi \times 10^8}$$
$$= \frac{2}{\pi} \times 10^{-8} = 0.6366 \times 10^{-8} \text{ watts/cm}^2$$
$$W_0 = 0.6366 \times 10^{-8} = 6.366 \times 10^{-9} \text{ watts/cm}^2$$
$$W_{\text{max}} = W_0 G_0(\text{dim}) = 6.366 \times 10^{-9}(39.81) = 253.438 \times 10^{-9}.$$

$$\boxed{W_{\text{max}} = 253.438 \times 10^{-9} \text{ watts/cm}^2}$$

(b) $D_0(\lambda/4 \text{ monopole}) = 1.643$
$$A_{em} = \frac{\lambda^2}{4\pi}D_0 = \frac{\lambda^2}{4\pi}(1.643) = \frac{1.643(15.789)^2}{4\pi} = 32.5938 \text{ cm}^2$$
$$A_{em} = 32.5938 \text{ cm}^2$$
$$P(\text{received}) = W_{\text{max}}\, A_{em} = (253.438 \times 10^{-9})(32.5938)$$

$$\boxed{P(\text{received}) = 8.2606 \times 10^{-6} \text{ watts}}$$

2-30. (a) Linear because $\Delta\phi = 0$.

(b) Linear because $\Delta\phi = 0$.

(c) Circular because

1. $E_x = E_y$
2. $\Delta\phi = \pi/2$.

CCW because E_y leads E_x. AR $= 1, \tau = 90°$

(d) Circular because

1. $E_x = E_y$
2. $\Delta\phi = -\pi/2$

CW because E_y lags E_x. AR $= 1, \tau = 90°$

(e) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$. CCW because E_y leads E_x. AR = OA/OB

Letting $E_x = E_y = E_0$

$$\left.\begin{array}{l} \text{OA} = E_0[0.5(1+1+\sqrt{2})]^{1/2} = 1.30656E_0 \\ \text{OB} = E_0[0.5(1+1-\sqrt{2})]^{1/2} = 0.541196E_0 \end{array}\right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

$$\tau = 90^\circ - \tfrac{1}{2}\tan^{-1}\left[\frac{2(1)\cos(45^\circ)}{1-1}\right] = 90^\circ - \tfrac{1}{2}\tan^{-1}\left(\frac{1.414}{0}\right)$$

$$= 90^\circ - \tfrac{1}{2}(90^\circ) = 45^\circ$$

(f) Elliptical because $\Delta\phi$ is not multiples of $\pi/2$ CW because E_y lags E_x.

$$\text{From above}\quad \left.\begin{array}{l} \text{OA} = 1.30656E_0 \\ \text{OB} = 0.541196E_0 \end{array}\right\} \Rightarrow \text{AR} = \frac{1.30656}{0.541196} = 2.414$$

From above $\tau = 90^\circ - \frac{1}{2}(90^\circ) = 45^\circ$

(g) Elliptical because

1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.

CCW because E_y leads E_x.

$$\left.\begin{array}{l} \text{OA} = E_y\left\{\tfrac{1}{2}[0.25+1+0.75]\right\}^{1/2} = E_y \\ \text{OB} = E_y\left\{\tfrac{1}{2}[0.25+1-0.75]\right\}^{1/2} = 0.5E_y \end{array}\right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2.$$

$$\tau = 90^\circ - \tfrac{1}{2}\tan^{-1}\left(\frac{0}{-0.75}\right) = 90^\circ - \tfrac{1}{2}(180^\circ) = 0^\circ$$

(h) Elliptical because

1. $E_x \neq E_y$
2. $\Delta\phi$ is not zero or multiples of π.

CW because E_y lags E_x.

$$\text{From above}\quad \left.\begin{array}{l} \text{OA} = E_y \\ \text{OB} = 0.5E_y \end{array}\right\} \Rightarrow \text{AR} = \frac{1}{0.5} = 2$$

$$\tau = 90^\circ - \tfrac{1}{2}(180^\circ) = 0^\circ.$$

2-31. $\mathcal{E}_x(z,t) = \text{Re}[E_x e^{j(\omega t + kz + \phi_x)}] = E_x\cos(\omega t + kz + \phi_x)$

$\mathcal{E}_y(z,t) = \text{Re}[E_y e^{j(\omega t + kz + \phi_y)}] = E_y\cos(\omega t + kz + \phi_y)$

where E_x and E_y are real positive constants.

Choosing $z = 0$ and letting $\Delta\phi = \phi_y - \phi_x = \phi_y - 0 = \phi$

$$\begin{array}{l} \mathcal{E}_x(t) = E_x\cos(\omega t) \\ \mathcal{E}_y(t) = E_y\cos(\omega t + \phi) \end{array} \qquad (1)$$

and

$$\mathcal{E}(t) = \sqrt{\mathcal{E}_x{}^2 + \mathcal{E}_y{}^2} = \sqrt{E_x{}^2 \cos^2(\omega t) + E_y{}^2 \cos^2(\omega t + \phi)} \tag{2}$$

The maximum and minimum values of (2) are the major and minor axes of the polarization ellipse. Squaring (2) and using the half-angle identity, equation (2) can be written as

$$\mathcal{E}^2(t) = \tfrac{1}{2}\{E_x{}^2 + E_y{}^2 + E_x{}^2 \cos(2\omega t) + E_y{}^2 \cos^2[2(\omega t + \phi)]\} \tag{3}$$

Since E_x and E_y are constants, the maximum and minimum values of (3) occur when $f(t) = E_x{}^2 \cos(2\omega t) + E_y{}^2 \cos[2(\omega t + \phi)]$ is maximum or minimum. These are found by differentiating (4) and setting it equal to zero. Thus

$$\frac{df}{d(2\omega t)} = -E_x{}^2 \sin(2\omega t) - E_y{}^2 \sin[2(\omega t + \phi)] = 0 \tag{4}$$

or

$$\begin{aligned} E_x^2 \sin(2\omega t) &= -E_y^2 \sin[2(\omega t + \phi)] \\ &= -E_y^2\{\sin 2\omega t \cos 2\phi + \cos 2\omega t \sin 2\phi\} \end{aligned} \tag{5}$$

Dividing (5) by $\cos(2\omega t)$ yields

$$E_x^2 \tan(2\omega t) = -E_y^2 \tan[2\omega t] \cos(2\phi) + \sin(2\phi)]$$

or

$$\tan(2\omega t) = \frac{-E_y^2 \sin(2\phi)}{E_x^2 + E_y^2 \cos(2\phi)}$$

from which we obtain that

$$\cos(2\omega t) = \frac{E_x^2 + E_y^2 \cos(2\phi)}{\pm\rho} \tag{6}$$

$$\cos(2\omega t + 2\phi) = \frac{E_y^2 + E_x^2 \cos(2\phi)}{\pm\rho} \tag{7}$$

where

$$\rho = \sqrt{E_x^4 + E_y^4 + 2E_x^2 E_y^2 \cos(2\phi)} \tag{8}$$

Substituting (6)–(8) into (3) yields

$$\mathcal{E}^2 = \tfrac{1}{2}\left[E_x^2 + E_y^2 \pm \frac{1}{\rho}(\rho^2)\right]$$

whose maximum value is

$$\mathcal{E}_{\max} = \text{OA} = \left\{\tfrac{1}{2}[E_x^2 + E_y^2 + (E_x^4 + E_y^4 + 2E_x^2E_y^2 \cos 2\phi)^{1/2}]\right\}^{1/2}$$

$$\mathcal{E}_{\min} = \text{OB} = \left\{\tfrac{1}{2}[E_x^2 + E_y^2 - (E_x^4 + E_y^4 + 2E_x^2E_y^2 \cos 2\phi)^{1/2}]\right\}^{1/2}$$

The tilt angle τ can be obtained by expanding (1) and writing the two as

$$\frac{\mathcal{E}_x^2}{E_x^2} - \frac{2\mathcal{E}_x \mathcal{E}_y \cos\phi}{E_x\ E_y} + \frac{\mathcal{E}_y^2}{E_y^2} = \sin^2\phi \tag{9}$$

which is the equation of a tilted ellipse. Choosing a coordinate system whose principal axes coincide with the major and minor axes of the tilted ellipse, we can write that

$$\begin{aligned} \mathcal{E}_x &= \mathcal{E}_x' \sin(z) - \mathcal{E}_y' \cos(z) \\ \mathcal{E}_y &= \mathcal{E}_x' \cos(z) + \mathcal{E}_y' \sin(z) \end{aligned} \tag{10}$$

where $\mathcal{E}_x'$ and $\mathcal{E}_y'$ are the new field values along the new principal axes x', y', z'. Substituting (10) into (9) yields

$$\frac{2\mathcal{E}_x' \mathcal{E}_y' \cos(z)\sin(z)}{E_x^2} - \frac{2\mathcal{E}_x' \mathcal{E}_y' \cos(z)\sin(z)}{E_y^2} - \frac{2\mathcal{E}_x' \mathcal{E}_y' \cos\phi}{E_x\ E_y}(\sin^2 z - \cos^2 z) = 0$$

which when solved for the tilt angle τ reduces to

$$\tan\left[2\left(\frac{\pi}{2} - \tau\right)\right] = \frac{2E_x\ E_y \cos\phi}{E_x^2 - E_y^2}$$

or

$$\tau = \frac{\pi}{2} - \tfrac{1}{2}\tan^{-1}\left(\frac{2E_x\ E_y \cos\phi}{E_x^2 - E_y^2}\right)$$

For more details on the tilt angle derivation, see J.D. Kraus, Antennas, McGraw - Hill, 1950, pp. 464–476.

2-32. (a)

$$\begin{aligned} \hat{\rho}_w &= \hat{a}_x \cos\phi_1 + \hat{a}_y \sin\phi_1 \\ \hat{\rho}_a &= \hat{a}_x \cos\phi_2 + \hat{a}_y \sin\phi_2 \\ \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |(\hat{a}_x \cos\phi_1 + \hat{a}_y \sin\phi_1) \cdot (\hat{a}_x \cos\phi_2 + \hat{a}_y \sin\phi_2)|^2 \\ &= |\cos\phi_1 \cos\phi_2 + \sin\phi_1 \sin\phi_2|^2 = |\cos(\phi_1 - \phi_2)|^2 \end{aligned}$$

(b)

$$\begin{aligned} \hat{\rho}_w &= \hat{a}_x \sin\theta_1 \cos\phi_1 + \hat{a}_y \sin\theta_1 \sin\phi_1 + \hat{a}_z \cos\theta_1 \\ \hat{\rho}_a &= \hat{a}_x \sin\theta_2 \cos\phi_2 + \hat{a}_y \sin\theta_2 \sin\phi_2 + \hat{a}_z \cos\theta_2 \\ \text{PLF} &= |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\sin\theta_1 \cos\phi_1 \sin\theta_2 \cos\phi_2 + \sin\theta_1 \sin\phi_1 \sin\theta_2 \cdot \sin\phi_2 \\ &\qquad + \cos\theta_1 \cdot \cos\theta_2|^2 \\ \text{PLF} &= |\sin\theta_1 \cdot \sin\theta_2(\cos\phi_1 \cdot \cos\phi_2 + \sin\phi_1 \sin\phi_2) + \cos\theta_1 \cos\theta_2|^2 \\ \text{PLF} &= |\sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2) + \cos\theta_1 \cos\theta_2|^2 \end{aligned}$$

2-33. Assuming electric field is x-polarized

(a) $\underline{E}_w = \hat{a}_x E_1 e^{-jkz} \Rightarrow \hat{\rho}_w = \hat{a}_x$

$$\underline{E}_a = (\hat{a}_\theta - j\hat{a}_\phi)E_0 f(r,\theta,\phi) \Rightarrow \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}\right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \tfrac{1}{2}|\hat{a}_x \cdot \hat{a}_\theta - j\hat{a}_x \cdot \hat{a}_\phi|^2$$

since

$$\hat{a}_\theta = \hat{a}_x \cos\theta\cos\phi + \hat{a}_y \cos\theta\sin\phi - \hat{a}_z \sin\theta$$

$$\hat{a}_\phi = -\hat{a}_x \sin\phi + \hat{a}_y \cos\phi$$

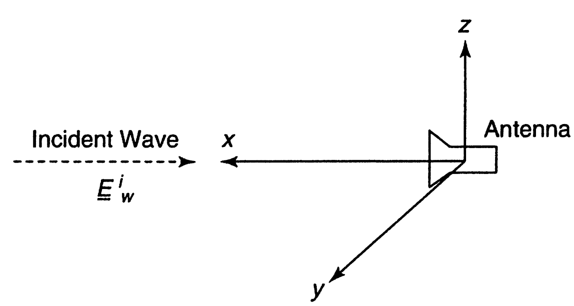
$$\text{PLF} = \tfrac{1}{2}(\cos^2\theta\cos^2\phi + \sin^2\phi)$$

(b) when $\underline{E}_a = (\hat{a}_\theta + j\hat{a}_\theta)E_0 f(r,\theta,\phi)$, PLF is also

$$\text{PLF} = \tfrac{1}{2}(\cos^2\theta\cos^2\phi + \sin^2\phi)$$

A more general, but also more complex, expression can be derived when the incident electric field is of the form $\underline{E}_w = (a\hat{a}_x + b\hat{a}_y)e^{-jkz}$ where a, b are real constants. It can be shown (using the same procedure) that

$$\text{PLF} = \frac{1}{\sqrt{2(a^2+b^2)}}[(a\cos\theta\cos\phi + b\sin\theta\sin\phi)^2 + (a\sin\phi - b\cos\phi)^2]^{1/2}$$



2-34. (a) $\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}$

1. **Elliptical polarization; $AR = \frac{3}{1} = 3$; Left Hand (CCW)**
    a. 2 components orthogonal to direction of propagation
    b. Not of same magnitude
    c. 90° phase difference between them
    d. y component is leading the z component *or* z component is lagging the y component

(b) $\underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}$

1. **Linear polarization; $AR = \infty$; No rotation**
    a. 2 components orthogonal to direction of propagation.
    b. Not of the same magnitude
    c. 0° phase difference between them,

(c) PLF $= |\hat{\rho}_w \cdot \hat{\rho}_a|^2$

$$\underline{E}_w = E_0(j\hat{a}_y + 3\hat{a}_z)e^{+jkx} = E_0 \underbrace{\left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)}_{\hat{\rho}_w} \sqrt{10}e^{+jkx}$$

$$\hat{\rho}_w = \left(\frac{j\hat{a}_y + 3\hat{a}_z}{\sqrt{10}}\right)$$

$$\underline{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx} = E_0 \underbrace{\left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)}_{\hat{\rho}_a} \sqrt{5}e^{-jkx}$$

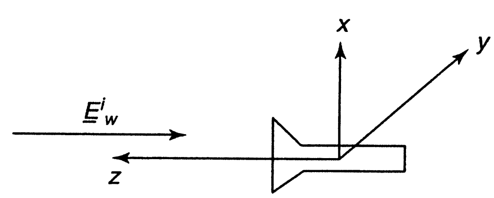
$$\hat{\rho}_a = \left(\frac{\hat{a}_y + 2\hat{a}_z}{\sqrt{5}}\right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\frac{(j\hat{a}_y + 3\hat{a}_z)}{\sqrt{10}} \cdot \frac{(\hat{a}_y + 2\hat{a}_z)}{\sqrt{5}}\right|^2 = \frac{|j+6|^2}{50} = \frac{37}{50}$$

$$\text{PLF} = \frac{37}{50} = \boxed{0.740 = -1.31 \text{ dB}}$$

2-35. $\underline{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0e^{+jkz}$

$$\underline{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1\frac{e^{-jkr}}{r}\Big|_{\theta=0^\circ\ z-\text{axis}} = (\hat{a}_x + 2\hat{a}_y)E_1\frac{e^{-jkz}}{z}$$



(a) $\underline{E}_w^i = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E_0e^{+jkz}$

Circular: 2 components, same amplitude, 90° phase difference

(b) Clockwise (y component is leading the x component)

(c) $\underline{E}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}\right)\sqrt{5}E_1\frac{e^{-jkz}}{z}$

Linear: 2 components, 0° phase difference

(d) No rotation

(e) $\hat{\rho}_w = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right), \quad \hat{\rho}_a = \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}\right)$

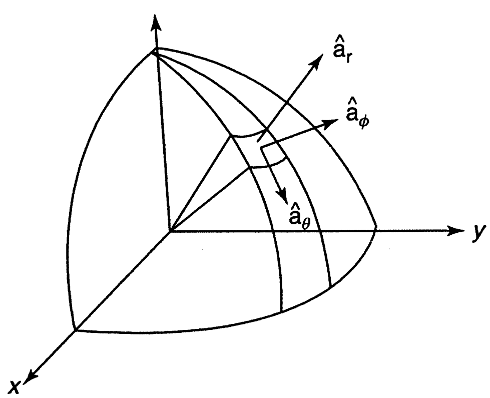
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left[\left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right) \cdot \left(\frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}\right)\right]^2 = \frac{|1+j2|^2}{10} = \frac{5}{10}$$

$$\text{PLF} = \frac{5}{10} = 0.5 = 10\log_{10}(0.5) = -3 \text{ dB}$$

2-36. (a) $\underline{E}_a = E_0(j\hat{a}_\theta + 2\hat{a}_\phi)f_0(\theta_0,\phi_0)\frac{e^{-jkr}}{r} = E_0 \underbrace{\left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right)}_{\hat{\rho}_a} \sqrt{5} f_0(\theta_0,\phi_0)\frac{e^{-jkr}}{r}$

$$\hat{\rho}_a = \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right)$$

Elliptical, CW



(b) $\underline{E}_w = E_1(2\hat{a}_\theta + j\hat{a}_\phi)f_1(\theta_0,\phi_0)\frac{e^{+jkr}}{r}$

$$= E_1 \underbrace{\left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)}_{\hat{\rho}_w} \sqrt{5} f_1(\theta_0,\phi_0)\frac{e^{+jkr}}{r}$$

$$\hat{\rho}_w = \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)$$

Elliptical, CW

(c) $\text{PLF} = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left|\left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right) \cdot \left(\frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}}\right)\right|^2 = \left|\frac{2j + j2}{\sqrt{25}}\right|^2 = \left|\frac{4j}{\sqrt{25}}\right|^2$

$$\boxed{\text{PLF} = \frac{16}{25} = 0.64 = -1.938 \text{ dB}}$$

2-37. (a) $\underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz} \Rightarrow \hat{\rho}_w = \frac{1}{\sqrt{2}}(\hat{a}_x \pm j\hat{a}_y)$

$\underline{E}_a \simeq E_1(\hat{a}_\theta - j\hat{a}_\phi)f(r,\theta,\phi) \Rightarrow \hat{\rho}_a = \frac{1}{\sqrt{2}}(\hat{a}_\theta - j\hat{a}_\phi)$

$\text{PLF} = \frac{1}{2}|(\hat{a}_x \pm j\hat{a}_y)\cdot(\hat{a}_\theta - j\hat{a}_\phi)|^2 = \frac{1}{2}|(\hat{a}_x\cdot\hat{a}_\theta \pm \hat{a}_y\cdot\hat{a}_\phi) - j(\hat{a}_x\hat{a}_\phi \mp \hat{a}_y\hat{a}_\theta)|^2$

Converting the spherical unit vectors to rectangular, as it was done in Problem 2.32, leads to

$$\text{PLF} = \tfrac{1}{2}(\cos\theta \pm 1)^2$$

(b) When

$$\underline{E}_w = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-jkz}$$

$\underline{E}_a \simeq E_1(\hat{a}_\theta + j\hat{a}_\phi)f(r,\theta,\phi)$ the PLF is equal to

$$\text{PLF} = \tfrac{1}{2}(\cos\theta \mp 1)^2$$

2-38. $\underline{E}_w = (\hat{a}_\theta\cos\phi - \hat{a}_\phi\sin\phi\cos\theta)f(r,\theta,\phi)$ or

$$\underline{E}_w = \left[\frac{\hat{a}_\theta\cos\phi - \hat{a}_\phi\sin\phi\cos\theta}{\sqrt{\cos^2\phi + \sin^2\phi\cos^2\theta}}\right]\sqrt{\cos^2\phi + \sin^2\phi\cos^2\theta}\cdot f(r,\theta,\phi)$$

Thus $\hat{\rho}_w = \dfrac{\hat{a}_\theta\cos\phi - \hat{a}_\phi\sin\phi\cos\theta}{\sqrt{\cos^2\phi + \sin^2\phi\cos^2\theta}}$

and

$$\text{PLF} = |\hat{\rho}_w\cdot\hat{\rho}_a|^2 = \left|\left(\frac{\hat{a}_\theta\cos\phi - \hat{a}_\phi\sin\phi\cos\theta}{\sqrt{\cos^2\phi + \sin^2\phi\cos^2\theta}}\right)\cdot\hat{a}_x\right|^2$$

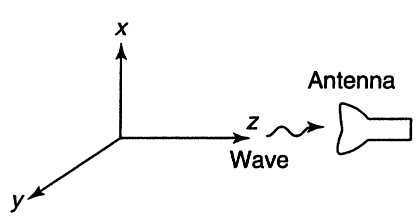
Transforming the rectangular unit vector to spherical using

$$\hat{a}_x = \hat{a}_r\sin\theta\cos\phi + \hat{a}_\theta\cos\theta\cos\phi - \hat{a}_\phi\sin\phi$$

the PLF reduces to $\text{PLF} = \dfrac{\cos^2\theta}{\cos^2\phi + \sin^2\phi\cos^2\theta}$

The same answer is obtained by transforming the spherical unit vectors to rectangular, as was done in Prob. 2-32.

2-39. $\underline{E}_a \simeq (2\hat{a}_x \pm j\hat{a}_y)f(r,\theta,\phi) = \left(\dfrac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}\right)\sqrt{5}f(r,\theta,\phi)$



(a) $\hat{\rho}_w = \left(\dfrac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right) \Rightarrow$ Wave is Right Hand (RH)

$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}\right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

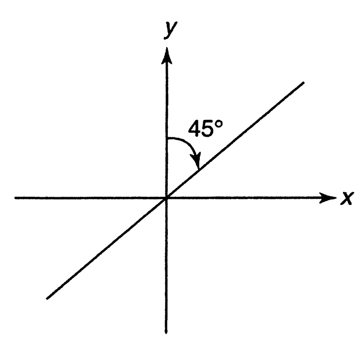
$$= \begin{cases} \dfrac{9}{10} = -0.4576 \text{ dB using the } + \text{ sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \dfrac{1}{10} = -10 \text{ dB using the } - \text{ sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$

(b) $\hat{\rho}_w = \left(\dfrac{\hat{a}x + j\hat{a}y}{\sqrt{2}}\right) \Rightarrow$ Wave is Left Hand (LH)

$$\hat{\rho}_a = \left(\frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}\right)$$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= \begin{cases} \dfrac{1}{10} = -10 \text{ dB using the } + \text{ sign} & \text{(Antenna is LH in receiving mode and RH in transmitting)} \\ \dfrac{9}{10} = -0.4545 \text{ dB using the } - \text{ sign} & \text{(Antenna is RH in receiving mode and LH in transmitting)} \end{cases}$$



2-40. for $\hat{\rho}_w$

$$\hat{\rho}_w = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}; \text{PLF} = \left|\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \cdot \frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}}\right|^2$$

$$\text{PLF} = \frac{1}{34}|(\hat{a}_x \cdot 4\hat{a}_x) + (\hat{a}_y \cdot j\hat{a}_y)|^2 = \frac{1}{34}|4 + j|^2$$

$$= 0.5$$

2-41. (a) RHCP; $\hat{\rho}_a = \dfrac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$

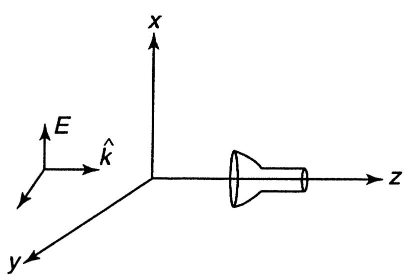
$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right|^2 = 0.9 = -0.46 \text{ (dB)}$$

(b) LHCP; $\hat{\rho}_a = \dfrac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\frac{2\hat{a}_x + j\hat{a}_y}{\sqrt{5}} \cdot \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right|^2 = 0.1 = -10.0 \text{ (dB)}$$

2-42. $\underline{E}^i = (\hat{a}_x - j\hat{a}_y)E_0 e^{-jkz} = \left(\dfrac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E_0 e^{-jkz}$

$$\hat{\rho}_w = \frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}} \qquad \text{CW}$$



(a) $\underline{E}^a = (\hat{a}_x + j\hat{a}_y)E_1 e^{+jkz}$

$= \left(\dfrac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E_1 e^{+jkz}$

$\hat{\rho}_a = \dfrac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \qquad \text{CW}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right) \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)\right|^2 = \left(\frac{1-j^2}{2}\right)^2 = 1$$

PLF $= 1 = 0$ dB

(b) $\underline{E}^a = \left(\dfrac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E_1 e^{+jkz}$

$\hat{\rho}_a = \dfrac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right) \cdot \left(\frac{\hat{a}_x - j\hat{a}_y}{\sqrt{2}}\right)\right|^2 = \left|\frac{1+j^2}{2}\right|^2 = 0$$

PLF $= 0 = -\infty$ dB

2-43. $\underline{E}^i = \hat{a}_x E_0 e^{-jkz}$, $\hat{\rho}_w = \hat{a}_x$

$$\underline{E}^a = (\hat{a}_x + j\hat{a}_y)E_1 e^{+jkz} = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)\sqrt{2}E_1 e^{+jkz}$$

$$\hat{\rho}_a = \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)$$

(a) $A_{em} = \frac{\lambda^2}{4\pi} e_o D_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2$

$(\leftarrow e_o D_0 = G_0)$

At 10 GHz $\Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2}$

$$G_0 = 10 = 10\log_{10} G_0(\text{dim}) \Rightarrow G_0(\text{dim}) = 10^1 = 10$$

$$A_{em} = \frac{\lambda^2}{4\pi} G_0 |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \frac{(3 \times 10^{-2})^2}{4\pi}(10)\left|\hat{a}_x \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}\right)\right|^2$$

$$= \frac{9 \times 10^{-4}}{4\pi}(10)\left(\tfrac{1}{2}\right) = \frac{9 \times 10^{-3}}{4\pi}\left(\tfrac{1}{2}\right) = (0.7162 \times 10^{-3})\left(\tfrac{1}{2}\right)$$

$$A_{em} = 0.3581 \times 10^{-3} \text{ m}^2$$

(b) $P_T = A_{em}\, W^i = (0.3581 \times 10^{-3})(10 \times 10^{-3}) = 3.581 \times 10^{-6}$ watts

$P_T = 3.581 \times 10^{-6}$ watts

2-44. $\underline{E}_a = (2\hat{a}_x \pm j\hat{a}_y)Ee^{-jkz}$

$$\hat{\rho}_a = \frac{2\hat{a}_x \pm j\hat{a}_y}{\sqrt{5}}$$

(a) $\underline{E}_w = \hat{a}_x E_w \Rightarrow \hat{\rho}_w = \hat{a}_x$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\frac{2}{\sqrt{5}}\right|^2 = \frac{4}{5} = 0.8 = -0.9691 \text{ dB}$$

(b) $\underline{E}_w = \hat{a}_y E_w \Rightarrow \hat{\rho}_w = \hat{a}_y$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \left|\frac{1}{\sqrt{5}}\right|^2 = \frac{1}{5} = 0.2 = -6.9897 \text{ dB}$$

2-45. (a) $E_y = E_y' + E_y'' = 3\cos\omega t + 2\cos\omega t = 5\cos\omega t$

$$E_x = E_x' + E_x'' = 7\cos\left(\omega t + \frac{\pi}{2}\right) + 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$= -7\sin\omega t + 3\sin\omega t = -4\sin\omega t$$

$$\text{AR} = \frac{5}{4} = 1.25$$

(b) At $\omega t = 0, \quad \vec{E} = 5\hat{a}_y$

At $\omega t = \pi/2 \Rightarrow \vec{E} = -4\hat{a}_x \Rightarrow$ Rotation in CCW

2-46. (a) PLF $= \frac{1}{2}$ independent of $\psi \longrightarrow$ must have CP

$\therefore$ AR $= 1$.

(b) Polarization will be elliptical with major axies aligned with x-axis.

guess: AR $= 2$

verify: $\hat{\rho}_w = (2\hat{a}_x + ja_y)/\sqrt{5}$

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\frac{2\cos\psi + j\sin\psi}{\sqrt{5}}|^2 = \frac{4\cos^2\psi + \sin^2\psi}{5}$$

$\psi = 0$: PLF $= 0.8$

$\psi = 90°$: PLF $= 0.2$

(c) PLF $= 1$ at $\psi = 45°$ and $225°$

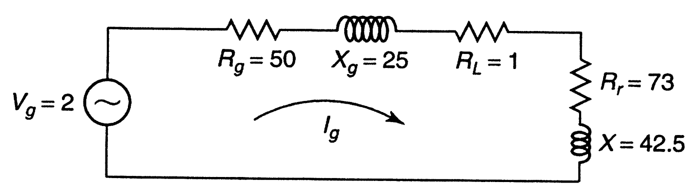
PLF $= 0$ at $\psi = 135°$ and $315°$

Polarization must be linear with that angle of $45°$

$\therefore$ AR $= \infty$

2-47. $$I_g = \frac{2}{(50 + 1 + 73) + j(25 + 42.5)} = \frac{2}{124 + j67.5}$$

$$= (12.442 - j6.7724) \times 10^{-3} = 14.166 \times 10^{-3}\angle - 28.56°$$



(a) $P_s = \frac{1}{2}\text{Re}(V_g \cdot I_g^*) = Re(12.442 + j6.7724) \times 10^{-3} = 12.442 \times 10^{-3}$ W

(b) $P_r = \frac{1}{2}|I_g|^2 R_r = 7.325 \times 10^{-3}$ W

(c) $P_L = \frac{1}{2}|I_g|^2 R_L = 0.1003 \times 10^{-3}$ W

The remaining supplied power is dissipated as heat in the internal resistor of the generator, or

$$P_g = \frac{1}{2}|I_g|^2 R_g = 5.0169 \times 10^{-3} \text{ W}$$

Thus

$$P_r + P_L + P_g = (7.325 + 0.1003 + 5.0169) \times 10^{-3} = 12.4422 \times 10^{-3} = P_s$$

2-48. The impedance transfer equation of

$$Z_{\text{in}} = Z_c \left[\frac{Z_L + jZ_c \tan(kl)}{Z_c + jZ_L \tan(kl)}\right]$$

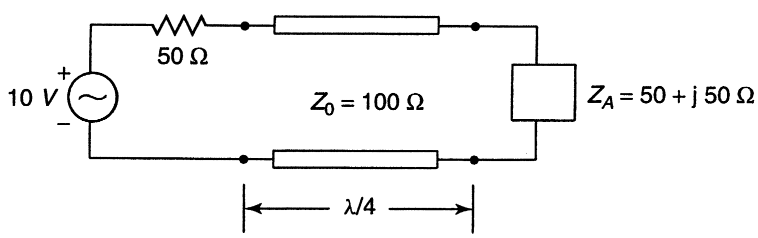
reduces for $l = \lambda/2$ to $Z_{\text{in}} = Z_L$

Therefore the equivalent load impedance at the terminals of the generator is the same as that for Problem 2-47.

Thus the supplied, radiated, and dissipated powers are the same as those of Problem 2-47.

2-49. (a) $Z_{\text{in}} = \dfrac{(100)^2}{50 + j50} = \dfrac{10000}{5000} = (50 - j50) = 100 - j100\ \Omega$

$$I_g = \frac{10}{150 - j100} = \frac{10}{180.3\angle -33.7^\circ} = 0.05546\angle 33.7^\circ A$$



(b) $P_s = \frac{1}{2}\text{Re}\{V_g I_g^*\} = \frac{1}{2} \times 10 \times 0.05546 \times \cos(33.7^\circ)$
$= 0.231$ W

(c) $P_A = \frac{1}{2}|I_g|^2\text{Re}\{Z_{\text{in}}\} = \frac{1}{2} \times (0.05546)^2 \times 100 = 0.1538$ W
$P_{\text{rad}} = e_{cd}P_A = 0.96 \times 0.1538 = 0.148$ W

2-50.

$$\text{Gain} = \frac{P_{\text{rad}}}{P_{\text{accepted}}}\text{Directivity}$$

$$\text{Realized Gain} = \frac{P_{\text{rad}}}{P_{\text{available}}}\text{Directivity}$$

$$\frac{\text{Gain}}{\text{Realized Gain}} = \frac{P_{\text{available}}}{P_{\text{accepted}}}$$

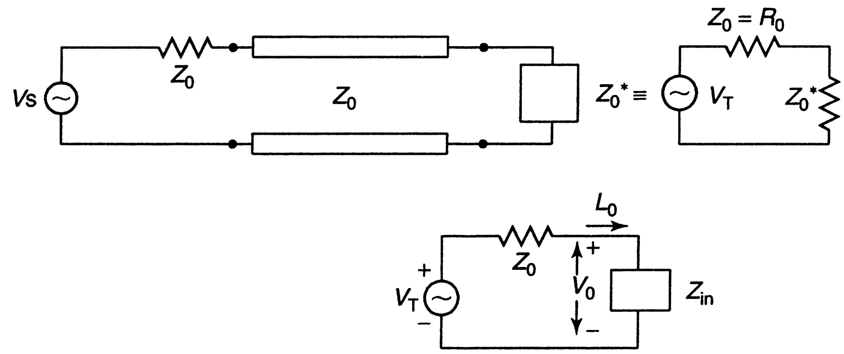


Fig. 1.

$$
\begin{aligned}
P_{\text{available}} &= \tfrac{1}{2}\frac{\left(\frac{V_s}{\sqrt{2}}\right)^2}{Z_0} = \frac{V_s^2}{4Z_0} \\
V(x) &= A(e^{-jkx} + \Gamma(0)e^{jkx}) \\
I(x) &= \frac{A}{Z_0}(e^{-jkx} - \Gamma(0)e^{jkx}) \\
V(0) &= A(1+\Gamma(0)) \\
I(0) &= \frac{A}{Z_0}(1-\Gamma(0))
\end{aligned}
$$

From Fig. 1;

$$
\begin{aligned}
&- V_s + I(0)Z_0 + V(0) = 0 \\
&- V_s + \frac{A}{Z_0}(1-\Gamma(0))Z_0 + A(1+\Gamma(0)) = 0 \\
&- V_s + A - A\Gamma(0) + A + A\Gamma(0) = 0 \\
&2A = V_s \rightarrow A = \frac{V_s}{2}
\end{aligned}
$$

$$
\begin{aligned}
P_{\text{accepted}} &= \text{Re}[V(0)I^*(0)] \\
V(0) &= \frac{V_s}{2}(1+\Gamma(0)) \\
I(0) &= \frac{V_s}{2Z_0}(1-\Gamma(0)) \\
\Gamma(0) &= \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}
\end{aligned}
$$

$$\Rightarrow V(0) = \frac{V_s}{2}\left(1 + \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}\right)$$

$$= \frac{V_s}{2}\left(1 + \frac{R_{\text{in}} + jX_{\text{in}} - Z_0}{R_{\text{in}} + jX_{\text{in}} + Z_0}\right)$$

$$= \frac{V_s}{2}\left(\frac{R_{\text{in}} + jX_{\text{in}} + Z_0 + R_{\text{in}} + jX_{\text{in}} - Z_0}{R_{\text{in}} + jX_{\text{in}} + Z_0}\right)$$

$$V(0) = \frac{V_s(R_{\text{in}} + jX_{\text{in}})}{R_{\text{in}} + jX_{\text{in}} + Z_0}$$

$$I(0) = \frac{V_s}{2Z_0}\left(1 - \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}\right) = \frac{V_s}{2Z_0}\left(\frac{Z_{\text{in}} + Z_0 - Z_{\text{in}} + Z_0}{Z_{\text{in}} + Z_0}\right)$$

$$I(0) = \frac{V_s}{Z_{\text{in}} + Z_0} = \frac{V_s}{R_{\text{in}} + jX_{\text{in}} + Z_0}$$

$$\text{Re}[V(0)I(0)^*] = \text{Re}\left[\frac{V_sR_{\text{in}} + jV_sX_{\text{in}}}{R_{\text{in}} + Z_0 + jX_{\text{in}}} \times \frac{V_s}{R_{\text{in}} + Z_0 - jX_{\text{in}}}\right]$$

$$P_{\text{accepted}} = \text{Re}\left(\frac{V_s^2(R_{\text{in}} + jX_{\text{in}})}{(R_{\text{in}} + Z_0)^2 + X_{\text{in}}^2}\right) = \frac{V_s^2 R_{\text{in}}}{(R_{\text{in}} + Z_0)^2 + X_{\text{in}}^2}$$

$$\frac{\text{Gain}}{\text{Realized Gain}} = \frac{\dfrac{V_s^2}{4Z_0}}{\dfrac{V_s^2 R_{\text{in}}}{(R_{\text{in}} + Z_0)^2 + X_{\text{in}}^2}} = \frac{(R_{\text{in}} + Z_0)^2 + X_{\text{in}}^2}{4Z_0R_{\text{in}}}$$

2-51. (a) $R_L = R_{hf}(\underline{2\text{-}90\text{b}}) = \frac{l}{C}\sqrt{\frac{\omega\mu_o}{2\sigma}}$

$$= \frac{\lambda/60}{2\pi(\lambda/200)} \cdot \sqrt{\frac{2\pi \times 10^9(4\pi \times 10^{-7})}{2(5.7 \times 10^7)}}$$

$$= 0.4415 \times 10^{-2} = 0.004415 \text{ (ohms)}$$

(b) $R_r(\underline{4-19}) = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{60}\right)^2 = 0.21932$

$\Rightarrow R_{\text{in}} = R_r = 0.21932$ ohms (because of assumed constant current)

(c) $e_{cd}(\underline{2-90}) = \frac{R_r}{R_L + R_r} = \frac{0.21932}{0.21932 + 0.004415} = 0.98027$

$e_{cd} = 98.027\%$

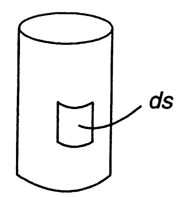
(d)
$$Z_L = (R_L + R_{\text{in}}) + jX_{\text{in}} = (0.21932 + 0.004415) + jX_{\text{in}}$$
$$= 0.2237 + jX_{\text{in}}$$

$$X_{\text{in}} \simeq -120\frac{\ln(l/2a) - 1}{\tan\left(\frac{kl}{2}\right)} = -120\frac{\left[\ln\left(\frac{\lambda/60}{\lambda/100}\right) - 1\right]}{\tan\left(\frac{2\pi}{2\lambda}\frac{\lambda}{60}\right)}$$
$$= -120 \cdot \left[\frac{0.51003 - 1}{0.05241}\right] = +1{,}120.03$$

$$|\Gamma| = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{(0.2237 + j1{,}120.03) - 50}{(0.2237 + j1{,}120.03) + 50} = 0.9999$$

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.9999}{1 - 0.9999} = 9{,}999 \simeq \infty.$$

2-52. Radiation Efficiency of a dipole



$I_z(z) = I_0 \cos\left[\frac{\pi}{l}z'\right], -l/2 \leqslant z' \leqslant l/2$

$$H_\phi(r = a)|_{\text{at the surface}} = \frac{I_0}{2\pi a}\cos\left[\frac{\pi}{l}z\right]$$

$ds = a\, d\phi\, dz \Rightarrow$ differential patch of area.

$dw \Rightarrow$ power loss into this patch.

$dw = \frac{1}{2}|H_\phi|^2 R_s a\, d\phi\, dz$

(time avs) ($\leftarrow R_s =$ skin resistance)

$$dW = \left(\frac{I_0}{2\pi a}\right)^2 \cdot \frac{R_s}{2}\cos^2\left[\frac{\pi}{l}Z\right] a\, d\phi\, dz$$

$$W(\text{total loss}) = \int_{-l/2}^{l/2}\int_{\phi=0}^{2\pi} \frac{{I_0}^2 R_s}{8\pi^2 \cdot a^2}\cos^2\left[\frac{\pi}{l}Z\right] a\, d\phi\, dz$$

$$W = \frac{{I_0}^2}{8\pi^2 a^2} \cdot 2\pi a \cdot R_s \int_{-l/2}^{l/2} \cos^2\left[\frac{\pi}{l}z\right] dz = \frac{{I_0}^2}{4\pi}\frac{l \cdot R_s}{a} \cdot \frac{1}{2}$$
$$= \tfrac{1}{2}{I_0}^2 R_L$$
$$R_L = \frac{1}{2} \cdot \frac{lR_s}{2\pi a}$$

2-53. $E = \begin{cases} 1 & 0 < \theta \leqslant 45^\circ \\ 0 & 45^\circ < \theta \leqslant 90^\circ \\ \frac{1}{2} & 90^\circ < \theta \leqslant 180^\circ \end{cases}$

(a)

$$U = \frac{r^2 E^2}{2\eta} = \frac{r^2 |E|^2}{\eta}, \qquad U_{\max} = \frac{r^2}{\eta} = \frac{1}{120\pi}$$

$$P_{\text{rad}} = \frac{r^2}{\eta} \int_0^{2\pi} d\phi \left[\int_0^{45^\circ} \sin\theta \, d\theta + \int_{90^\circ}^{180^\circ} \frac{1}{4} \sin\theta \, d\theta \right]$$

$$= \frac{r^2}{\eta} [2\pi] \left[-\cos\theta \Big|_0^{45^\circ} + \frac{1}{4} (-\cos\theta) \Big|_{90^\circ}^{180^\circ} \right]$$

$$= \frac{2r^2\pi}{\eta} \left[-\cos 45^\circ + \cos 0^\circ - \frac{1}{4} \cos 180^\circ + \frac{1}{4} \cos 90^\circ \right]$$

$$P_{\text{rad}} = 0.54289 \frac{2\pi r^2}{\eta}$$

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi \frac{r^2}{\eta}}{0.54289 (2\pi) r^2 / \eta} = 3.684$$

(b) When the field is equal to 10 v/m, for $\theta = 0^\circ$.

$$\Rightarrow E = \begin{cases} 10 \text{ v/m} & 0 < \theta \leqslant 45^\circ \\ 0 & 45^\circ < \theta \leqslant 90^\circ \\ \frac{1}{2} \times 10 \text{ v/m} & 90^\circ < \theta \leqslant 180^\circ \end{cases}$$

$$P_{\text{rad}} = \frac{r^2}{\eta} \left[\int_0^{2\pi} \left\{ \int_0^{45^\circ} |E|^2 \sin\theta \, d\theta + \int_{90^\circ}^{180^\circ} |E|^2 \sin\theta \, d\theta \right\} d\phi \right]$$

$$P_{\text{rad}} = r^2 (0.54289) \left(\frac{2\pi}{\eta} \right) |10|^2 = 36{,}193$$

$$P_{\text{rad}} = \tfrac{1}{2} |I|^2 R_r = |I_{\text{rms}}|^2 \cdot R_r$$

$$\Rightarrow R_r = \frac{36{,}193}{|I_{\text{rms}}|^2} = \frac{36{,}193}{25} = 1{,}447.72$$

2-54.
```
Input parameters:
-----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360
```

```
Output parameters:
------------------
Radiated power (watts) = 0.1566
Partial Directivity (theta) (dimensionless) = 80.2511
Partial Directivity (theta) (dB) = 19.0445
Partial Directivity (phi) (dimensionless) = 80.2511
Partial Directivity (phi) (dB) = 19.0445
Directivity (dimensionless) = 80.2511
Directivity (dB) = 19.0445
```

Using Table 12.1

$$a = 3\lambda, b = 2\lambda$$

$$D_0 = 4\pi\left(\frac{ab}{\lambda^2}\right) = 4\pi(6) = 24\pi$$

$$D_0 = 75.398 = 18.774 \text{ dB}$$

Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$ then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-55.
```
Input parameters:
-----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360

Output parameters:
------------------
Radiated power (watts) = 0.0330
Partial Directivity (theta) (dimensionless) = 62.4635
Partial Directivity (theta) (dB) = 17.9563
Partial Directivity (phi) (dimensionless) = 62.4635
Partial Directivity (phi) (dB) = 17.9563
Directivity (dimensionless) = 62.4635
Directivity (dB) = 17.9563
```

Using Table 12.1

$$a = 3\lambda, b = 2\lambda$$

$$D_0 = 0.81\left(4\pi\frac{ab}{\lambda^2}\right) = 0.81(24\pi)$$

$$= 61.072 = 17.858 \text{ dB}$$

Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$, then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-56. Input parameters:

```
----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360

Output parameters:
-----------------
Radiated power (watts) = 0.4863
Partial Directivity (theta) (dimensionless) = 4.2443
Partial Directivity (theta) (dB) = 6.2780
Partial Directivity (phi) (dimensionless) = 4.2443
Partial Directivity (phi) (dB) = 6.2780
Directivity (dimensionless) = 4.2443
Directivity (dB) = 6.2780
```

Using Table 12.1

$$\begin{aligned} f = 10 \text{ GHz} \Rightarrow \lambda = 3 \text{ cm} \Rightarrow a &= \frac{2.286}{3}\lambda = 0.762\lambda \\ b &= \frac{1.016}{3}\lambda = 0.3387\lambda \\ D_0 &= 0.81\left(4\pi\frac{ab}{\lambda^2}\right) = 0.81(4\pi)(0.762)(0.3387) \\ &= 2.627 = 4.194 \text{ dB} \end{aligned}$$

Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$, then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-57. Input parameters:

```
----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360
```

```
Output parameters:
------------------
Radiated power (watts) = 0.0338
Partial Directivity (theta) (dimensionless) = 92.9470
Partial Directivity (theta) (dB) = 19.6824
Partial Directivity (phi) (dimensionless) = 92.9470
Partial Directivity (phi) (dB) = 19.6824
Directivity (dimensionless) = 92.9470
Directivity (dB) = 19.6824
```

Using Table 12.2

$$a = 1.5\lambda$$

$$D_0 = \frac{4\pi}{\lambda^2}(\pi a^2) = \left(\frac{2\pi a}{\lambda}\right)^2 = 9\pi^2$$

$$D_0 = 88.826 = 19.485 \text{ dB}$$

Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$, then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-58. Input parameters:

```
-----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360

Output parameters:
------------------
Radiated power (watts) = 0.0418
Partial Directivity (theta) (dimensionless) = 75.1735
Partial Directivity (theta) (dB) = 18.7606
Partial Directivity (phi) (dimensionless) = 75.1735
Partial Directivity (phi) (dB) = 18.7606
Directivity (dimensionless) = 75.1735
Directivity (dB) = 18.7606
```

Using Table 12.2

$$a = 1.5\lambda$$

$$D_0 = 0.836\left(\frac{2\pi a}{\lambda}\right)^2 = 0.836(9\pi^2)$$

$$D_0 = 74.2589 = 18.71 \text{ dB}$$

Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$, then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-59. Input parameters:

```
-----------------
The lower bound of theta in degrees = 0
The upper bound of theta in degrees = 90
The lower bound of phi in degrees = 0
The upper bound of phi in degrees = 360

Output parameters:
------------------
Radiated power (watts) = 0.4952
Partial Directivity (theta) (dimensionless) = 6.3439
Partial Directivity (theta) (dB) = 8.0236
Partial Directivity (phi) (dimensionless) = 6.3439
Partial Directivity (phi) (dB) = 8.0236
Directivity (dimensionless) = 6.3439
Directivity (dB) = 8.0236
```

Using Table 12.2

$$f = 10 \text{ GHZ} \Rightarrow \lambda = 3 \text{ cm} \Rightarrow a = \frac{1.143}{3}\lambda = 0.381\lambda$$

$$D_0 = 0.836\left(\frac{2\pi a}{\lambda}\right)^2 = 0.836[2\pi(0.381)]^2$$

$$D_0 = 4.791 = 6.804 \text{ dB}$$

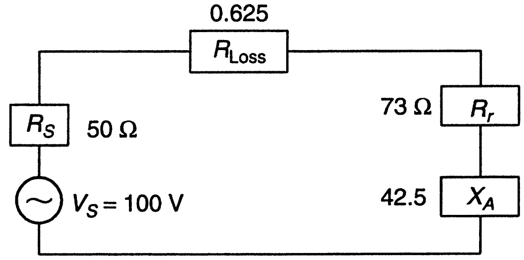
Since the maximum $|E_\theta| = |E_\phi| = |\underline{E}|$, then the maximum directivity

$$D_0 = D_\theta = D_\phi$$

2-60. $f = 150$ MHz, $\lambda = 2$m

$\Rightarrow$ 1 m dipole is $\frac{\lambda}{2}$ in electrical length

$\Rightarrow R_r = 73\ \Omega, Z_{\text{in}} = 73 + j42.5\ \Omega$



a. $I_{\text{ant}} = \dfrac{V_s}{50 + 73 + 0.625 + j42.5} = 0.765\angle - 18.97^\circ A$

b. $P_{\text{dissip}} = P_{\text{Loss}} = \frac{1}{2}|I_{\text{ant}}|^2 \cdot R_{\text{Loss}} = 189$ mW

c. $P_{\text{rad}} = \frac{1}{2}|I_{\text{ant}}|^2 \cdot R_r = 21.36\ W$

d. $E_{cd} = \dfrac{R_r}{R_r + R_{\text{Loss}}} = \dfrac{73}{73 + 0.625} = 99\%$

2-61. $\underline{E} = \hat{a}_\theta E_\theta \simeq \hat{a}_\theta j\eta \dfrac{kI_0 l}{4\pi r} e^{-jkr} \sin\theta = -j\eta \dfrac{kI_0 e^{-jkr}}{4\pi r} [\underbrace{-\hat{a}_\theta l \sin\theta}_{\underline{l}_e}]$

a. $\underline{l}_e = -\hat{a}_\theta l \sin\theta$

b. $|l_e|_{\max} = |-\hat{a}_\theta l \sin\theta|_{\max} = l \quad @\ \theta = 90^\circ$

c. $|l_e|_{\max}/l = 1$

2-62.

$$\underline{E} = \hat{a}_\theta E_\theta = \hat{a}_\theta j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$= j\eta \frac{kI_0 e^{-jkr}}{4\pi r}\left[-\hat{a}_\theta \frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{k\sin\theta}\right]$$

$$= j\eta \frac{kI_0 e^{-jkr}}{4\pi r}\left[\underbrace{-\hat{a}_\theta \frac{\lambda}{\pi} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}}_{\underline{l}_e}\right]$$

$$\underline{l}_e = -\hat{a}_\theta \frac{\lambda}{\pi} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} = -\hat{a}_\theta 0.3183\lambda \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$|\underline{l}_e|_{\max} = \left|-\hat{a}_\theta 0.3183\lambda \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|_{\max} = 0.3183\lambda \quad @\ \theta = 90^\circ$$

$$\frac{|l_e|_{\max}}{l} = \frac{0.3183\lambda}{\lambda/2} = 0.6366 = 63.66\% \quad @\ \theta = 90^\circ$$

2-63.

$$\underline{l}_e = -\hat{a}_\theta l \sin\theta, l = \lambda/50, f = 10 \text{ GHz} \Rightarrow \lambda = 3 \text{ cm}$$

$$W = \frac{1}{2\eta}|\underline{E}|^2 = 10^{-3} \text{ W/cm} \Rightarrow |E| = \sqrt{2\eta W}$$

$$= \sqrt{2(377)(10^{-3})} = 0.8683 \text{ V/cm}$$

$$V_{oc}|_{\max} = |\underline{E}^i||\underline{l}_e|_{\max} = (0.8683)\left(\frac{\lambda}{50}\right) = 52.1 \times 10^{-3} \text{ Votts}$$

2-64. Since$|\underline{l}_e|_{\max} = l/2 \Rightarrow |V_{oc}|_{\max} = \frac{1}{2}(V_{oc}$ of dipole with uniform current)

$V_{oc}|_{\max} = \frac{1}{2}(52.1 \times 10^{-3}) = 26.05 \times 10^{-3}$ Votts (see Problem 2-63)

2-65. $|\underline{l}_e|_{\max} = 0.3183\lambda \Rightarrow |V_{oc}| = |l_e|_{\max}|E^i|$. From Problem 2-63 solution

$|V_{oc}| = 0.8683(0.3183\lambda) = 0.27638\lambda = 0.27638(3) = 0.82914$ Votts

2-66. Using equation (2-94), the effective aperture of an antenna can be written as

$$A_e = \frac{|V_T|^2 \cdot R_T}{2\, W_i |Z_t|^2}, \text{ where } W_i = |E|^2/2\eta$$

Defining the effective length l_e as $V_T = E \cdot l_e$ reduces A_e to

$$A_e = \frac{\eta R_T l_e^2}{|Z_t|^2} \Rightarrow l_e = \sqrt{\frac{A_e |Z_t|^2}{\eta R_T}}$$

For maximum power transfer and lossless antenna ($R_L = 0$)

$$X_A = -X_T, R_r = R_T \Rightarrow |Z_t| = 2R_r = 2R_T$$

Thus $l_e = \sqrt{\dfrac{4A_{em} \cdot {R_T}^2}{\eta R_T}} = 2\sqrt{\dfrac{A_{em} R_T}{\eta}} = 2 \cdot \sqrt{\dfrac{A_{em} R_r}{\eta}}$

2-67. $A_{em} = 2.147 = \left(\dfrac{\lambda^2}{4\pi}\right) \cdot e_{cd} \cdot (1 - |\Gamma|^2) \cdot |\hat{\rho}_w \cdot \hat{\rho}_a|^2 \cdot D_0$

$$\Gamma = \frac{75 - 50}{75 + 50} = 0.2; \lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

$$\therefore D_0 = \frac{2.147}{\frac{3^2}{4\pi}[(1 - (0.2)^2]} = 3.125$$

2-68. $d = 1$ m, $f = 3$ GHz, $\varepsilon_{ap} = 68\% \Rightarrow \lambda = \dfrac{3 \times 10^8}{3 \times 10^9} = 0.1$ m

(a) $A_p = \pi r^2 = \pi \left(\dfrac{d}{2}\right)^2 = \dfrac{\pi\, d^2}{4} = \dfrac{\pi (1)^2}{4} = \boxed{\dfrac{\pi}{4} = 0.785 \text{ m}^2}$

(b) $\varepsilon_{ap} = \dfrac{A_{em}}{A_p} \Rightarrow A_{em} = \varepsilon_{ap} A_p$

$A_{em} = \varepsilon_{ap} A_p = 0.68(0.785) = \boxed{0.534 \text{ m}^2}$

(c) $A_{em} = \dfrac{\lambda^2}{4\pi} D_0 \Rightarrow D_0 = \dfrac{4\pi}{\lambda^2} A_{em}$

$D_0 = \dfrac{4\pi}{\lambda^2} A_{em} = \dfrac{4\pi}{(0.1)^2}(0.534) = \dfrac{4\pi}{0.01}(0.534) = 671.044$

$D_0 = \boxed{671.044 = 28.268 \text{ dB}}$

(d) $P_L = A_{em} W_L = 0.534(10 \times 10^{-6})$

$$P_L = \boxed{5.34 \times 10^{-6} \text{ watts}}$$

2-69. $W_i = 10^{-3}$ W/m^2

$$A_{em} = \frac{\lambda^2}{4\pi} \cdot D_0, D_0 = 20 \text{ dB} = 10 \log_{10} x \Rightarrow x = 100$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m} = 3 \times 10^{-2} \text{ m}$$

$$A_{em} = \frac{(3 \times 10^{-2})^2}{4\pi} \cdot 100 = \frac{9 \times 10^{-4}}{4\pi} \cdot (100) = 0.716 \times 10^{-2} = 7.16 \times 10^{-3}$$

$$P_{\text{rec}} = 10^{-3} \cdot \left(\frac{9 \times 10^{-2}}{4\pi}\right) = \frac{9 \times 10^{-5}}{4\pi} = 0.716 \times 10^{-5} = 7.16 \times 10^{-6} \text{ watts}$$

$$P_{\text{rec}} = 7.16 \times 10^{-6} \text{ watts.}$$

2-70. $A_p = 10 \text{ cm}^2, f = 10 \text{ GHz} \Rightarrow \lambda = 30 \times 10^9/10 \times 10^9 = 3 \text{ cm}, W^i = 10 \times 10^{-3} \text{ W/cm}^2$

(a) $A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} G_0 = A_p = 10$

$$\Rightarrow G_0 = \frac{4\pi(10)}{\lambda^2} = \frac{4\pi(10)}{(3)^2} = 13.96 = 11.45 \text{ dB}$$

(b) $P_r = A_{em} W^i(\text{PLF}) = \frac{1}{2}(10)(10 \times 10^{-3}) = 100 \times 10^{-3}/2 = 0.05$ Watts

$$P_r = 0.05 \text{ Watts}$$

$$\text{PLF} = \left| \hat{a}_x \cdot \left(\frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}} \right) \right|^2 = \tfrac{1}{2}$$

2-71. $\underline{W}_{\text{rad}} = \underline{W}_{\text{ave}} \simeq C_0 \frac{1}{r^2} \cos^4(\theta) \hat{a}_r \quad (0 \leqslant \theta \leqslant \pi/2, 0 \leqslant \phi \leqslant 2\pi)$

a. $P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi/2} \underline{W}_{\text{rad}} \cdot d\underline{s} = \int_0^{2\pi} \int_0^{\pi/2} \hat{a}_r W_{\text{rad}} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$

$$= C_0 \int_0^{2\pi} \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta \, d\phi = 2\pi C_0 \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta$$

$$= 2\pi C_0 \left(-\frac{\cos^5\theta}{5} \right)_0^{\pi/2}$$

$$P_{\text{rad}} = 2\pi C_0 \left(0 + \frac{1}{5} \right) = \frac{2\pi}{5} C_0 = 1.2566 C_0$$

b. $D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \Rightarrow U_{\text{max}} = r^2 \left. W_{\text{rad}} \right|_{\text{max}} = C_0 \left. \cos^4\theta \right|_{\text{max}} = C_0$

$$D_0 = \frac{4\pi C_0}{2\pi C_0/5} = 10 = 10 \log_{10}(10) = 10 \text{ dB}$$

c. $D_0 = 10$ toward $\theta = 0°$

d. $A_{em} = \frac{\lambda^2}{4\pi} D_0 \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1 \times 10^9} = 0.3 \text{ m}$

$$A_{em} = \frac{(0.3)^2}{4\pi}(10) = \frac{0.09}{4\pi}(10) = \frac{0.225}{\pi} = 0.0716 \text{ m}^2$$

e. $P_L = A_{em} W^i = 0.0716 \times (10 \times 10^{-3}) = 0.716 \times 10^{-3}$ Watts

2-72. $A_{em} = \frac{\lambda^2}{4\pi} e_t D_0 = \frac{\lambda^2}{4\pi} G_0$

a. $G_0 = 14.8 \text{ dB} \Rightarrow G_0(\text{power ratio}) = 10^{1.48} = 30.2$

$f = 8.2 \text{ GHZ} \Rightarrow \lambda = 3.6585 \text{ cm}$

$$A_{em} = \frac{(3.6585)^2}{4\pi}(30.2) = 32.167 \text{ cm}^2$$

The physical aperture is equal to $A_p = 5.5(7.4) = 40.7 \text{ cm}^2$

b. $G_0 = 16.5 \text{ dB} \Rightarrow G_0(\text{power ratio}) = 10^{1.65} = 44.668$

$f = 10.3 \text{ GHZ} \Rightarrow \lambda = 2.912 \text{ cm}$

$$A_{em} = \frac{(2.912)^2}{4\pi}(44.668) = 30.142 \text{ cm}^2$$

c. $G_0 = 18.0 \text{ dB} \Rightarrow G_0(\text{power ratio}) = 10^{1.8} = 63.096$

$f = 12.4 \text{ GHZ} \Rightarrow \lambda = 2.419 \text{ cm}$

$$A_{em} = \frac{(2.419)^2}{4\pi}(63.096) = 29.389 \text{ cm}^2$$

2-73. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-54:

Computer Program **Directivity**: $D_0 = 80.2511 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(80.2511) = 6.386\lambda^2$

Table 12.1: $D_0 = 75.398 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(75.398) = 6.0\lambda^2$

2-74. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-55:

Computer Program **Directivity**: $D_0 = 62.4635 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(62.4635) = 4.971\lambda^2$

Table 12.1: $D_0 = 61.072 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(61.072) = 4.86\lambda^2$

2-75. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-56:

Computer Program **Directivity**: $D_0 = 4.2443 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(4.2443) = 0.3378\lambda^2$

Table 12.1: $D_0 = 2.627 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(2.627) = 0.20905\lambda^2$

2-76. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-57:

Computer Program **Directivity**: $D_0 = 92.947 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(92.947) = 7.396\lambda^2$

Table 12.2: $D_0 = 88.826 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(88.826) = 7.068\lambda^2$

2-77. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-58:

Computer Program **Directivity**: $D_0 = 75.1735 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(75.1735) = 5.982\lambda^2$

Table 12.2: $D_0 = 74.2589 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(74.2589) = 5.909\lambda^2$

2-78. $A_{em} = \frac{\lambda^2}{4\pi} D_0$

From Problem 2-59:

Computer Program **Directivity**: $D_0 = 8.0236 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(8.0236) = 0.638\lambda^2$

Table 12.2: $D_0 = 4.791 \Rightarrow A_{em} = \frac{\lambda^2}{4\pi}(4.791) = 0.3813\lambda^2$

2-79. Gain = 30 dB, $f = 2$ GHZ, $P_{\text{rad}} = 5$ W

Receiving antenna VSWR = 2, efficiency = 95%

$$\underline{E}_R = (2\hat{a}_x + j\hat{a}_y)F_R(\theta, \phi), \text{ Use Friis transmission formula (2-118)}$$

$$P_r = P_t e_{cdt} e_{cdr}(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) \cdot \text{PLF}$$

$$P_r = 10^{-14} \text{ W}, e_{cdt} = 1 \text{ (we assume that)}, e_{cdr} = 0.95, 1 - |\Gamma_t|^2 = 1$$

Since VSWR = 2 $\Rightarrow |\Gamma_r| = \left|\frac{\text{VSWR} - 1}{\text{VSWR}+1}\right| = \frac{2-1}{2+1} = \frac{1}{3}, (1 - |\Gamma_r|^2) = 8/9$

$$\lambda = \frac{3 \times 10^8}{2 \times 10^9} = 0.15 \text{ m}, R = 4000 \times 10^3 \text{ m},$$

$$\text{Hence } \left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{0.15}{4\pi 4000 \times 10^3}\right)^2 = 8.9 \times 10^{-18}$$

$$D_t = 30 \text{ dB} = 10^3, \text{PLF} \Rightarrow \begin{cases} \rho_t = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y) \Rightarrow |\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 0.1 \\ \rho_r = \frac{1}{\sqrt{5}}(2\hat{a}_x + j\hat{a}_y) \end{cases}$$

$$\Rightarrow 10^{-14} = 5(1)(0.95)(1)\left(\frac{8}{9}\right)(8.9 \times 10^{-18})(10^3)D_r(0.1)$$

$$D_r = 2.661$$

$$\text{Hence } A_{em} = \frac{\lambda^2}{4\pi}2.661 = 0.00476 \text{ m}^2$$

2-80. $U(\theta, \phi) = \left\{\begin{matrix} \cos^4(\theta), & 0° \leqslant \theta \leqslant 90° \\ 0, & 90° \leqslant \theta \leqslant 180° \end{matrix}\right\} 0° \leqslant \phi \leqslant 360°$

$$A_{em} = \frac{\lambda^2}{4\pi}D_0$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = 2\pi\int_0^{\pi/2}\cos^4(\theta)\sin\theta\, d\theta = 2\pi\left[-\frac{\cos^5\theta}{5}\right]_0^{\pi/2}$$

$$P_{\text{rad}} = 2\pi\left(-0 + \frac{1}{5}\right) = \frac{2\pi}{5},$$

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi(1)}{2\pi/5} = 10$$

$$A_{em} = \frac{\lambda^2}{4\pi}D_0 = \frac{\lambda^2}{4\pi}\cdot 10 = \frac{10\lambda^2}{4\pi}, \lambda = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2} = 0.03 \text{ m}$$

$$A_{em} = \frac{10(0.03)^2}{4\pi} = \frac{10\cdot(3 \times 10^{-2})^2}{4\pi} = \frac{10\cdot(9 \times 10^{-4})}{4\pi} = 7.16197 \times 10^{-4}$$

$$A_{em} = 7.16197 \times 10^{-4}$$

2-81. 1 status mile = 1609.3 meters, 22,300(status miles) = 3.588739×10^7 m

a. $P_i = \frac{P_{\text{rad}}}{4\pi R^2} = \frac{8 \times 10^{-14}}{4\pi \times (3.58874)} = 4.943 \times 10^{-16}$ watts/m^2.

b. $A_{em} = \frac{\lambda^2}{4\pi}D_0$, $(\leftarrow D_0 = 60 \text{ dB} = 10^6)$

$(\leftarrow \lambda = 0.15 \text{ m})$

$$A_{em} = \frac{(0.15)^2}{4\pi} \cdot 10^6 = 1790.493 \text{ m}^2$$

$$P_{\text{received}} = A_{em} \cdot P_i = (1790.493) \cdot (4.943 \times 10^{-16})$$

$$= 8.85 \times 10^{-13} \text{ watts.}$$

2-82. $A_{em} = 0.7162 \text{ m}^2$

$$A_{em} = \left(\frac{\lambda}{4\pi}\right)^2 \cdot e_{cd}(1 - |\Gamma|^2)|\hat{\rho}_w \cdot \hat{\rho}_a|^2 \cdot D_0$$

$$D_0 = \frac{A_{em}}{\left(\frac{\lambda}{4\pi}\right)^2 (1 - 1\Gamma|^2)}, \Gamma = \frac{75-50}{75+50} = 0.2, \lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

$$D_0 = \frac{0.7162}{\frac{3^2}{4\pi}(1 - |0.2|^2)}$$

$$D_0 = 1.0417$$

2-83. $P_r = W_i A_{em} = W_i e_{cd}(1 - |\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$

$$W_i = 5 \text{ W/m}^2, e_{cd} = 1(\text{lossless}), \quad \Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{73-50}{73+50} = 0.187$$

$$\lambda = \frac{3 \times 10^8}{10 \times 10^6} = 30 \text{ m}, D_0 = 2.156 \text{ dB} = 1.643, \text{PLF} = 1$$

$$P_r = (5)(1)(1 - (0.187)^2)\left(\frac{30^2}{4\pi}\right)(1.643)(1) = 567.78 \text{ watts}$$

$$P_r = 567.78 \text{ watts.}$$

2-84. $\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0r}G_{0t}, G_{0r} = G_{0t} = 16.3 \Rightarrow G_0(\text{power ratio}) = 42.66$

$f = 10 \text{ GHz} \Rightarrow \lambda = 0.03$ meters.

$P_t = 200$ m watts $= 0.2$ watts

a. $R = 5$ m: $P_r = \left[\frac{0.03}{4\pi(5)}\right]^2 (42.66)^2(0.2) = 82.9\ \mu$watts

b. $R = 50$ m: $P_r = 0.829\ \mu$watts

c. $R = 500$ m: $P_r = 8.29\ n$watts

The VSWR was not needed because the gain was given.

2-85. $\dfrac{P_r}{P_t} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\dfrac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r}$

$G_{0t} = 20 \text{ dB} \Rightarrow G_{0t}(\text{power ratio}) = 10^2 = 100$

$G_{0r} = 15 \text{ dB} \Rightarrow G_{0r}(\text{power ratio}) = 10^{1.5} = 31.623$

$f = 1 \text{ GHz} \Rightarrow \lambda = 0.3 \text{ meters}$

$R = 1 \times 10^3 \text{meters}$

a. For $|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1$

$$P_r = \left(\frac{0.3}{4\pi \times 10^3}\right)^2 (100)(31.623)\ (150 \times 10^{-3}) = 270.344\ \mu\text{watts}$$

b. When transmitting antennas is circularly polarized and receiving antenna is linearly polarized, the PLF is equal to

$$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = \left|\left(\frac{\hat{a}_x \pm j\hat{a}_y}{\sqrt{2}}\right) \cdot \hat{a}_x\right|^2 = \tfrac{1}{2}$$

Thus

$$P_r = \tfrac{1}{2}(270.344 \times 10^{-6}) = 135.172 \times 10^{-6} = 135.172\ \mu\text{watts}$$

2-86. Lossless: $e_{cd} = 1$, polarization matched: $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$, line matched: $(1 - |\Gamma|^2) = 1$

$D_0 = 20 \text{ dB} = 10^2 = 100 = D_{0r} = D_{0t}$

$P_r = P_t \left(\dfrac{\lambda}{4\pi R}\right)^2 D_{0t} D_{0r} = 10 \left(\dfrac{\lambda}{4\pi \cdot 50\lambda}\right)^2 (100)(100) = 0.253 \text{ watts}$

$P_r = 0.253 \text{ watts}$

2-87. Lossless: $e_{cd} = 1, \text{PLF} = 1$. Line matched: $(1 - |\Gamma|^2) = 1$.

$D_0 = 30 \text{ dB} = 10^3 = 1000 = D_{0r} = D_{0t}$

$P_r = P_t \left(\dfrac{\lambda}{4\pi \cdot 100\lambda}\right)^2 \cdot (1000)^2 = 20 \cdot \left(\dfrac{1}{4\pi}\right)^2 \cdot 100 = 12.665 \text{ watts}$

2-88. $G_{0r} = 20 \text{ dB} = 100, G_{0t} = 25 \text{ dB} = 316.23. \lambda = \dfrac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m.}$

$P_r = P_t \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \cdot \left(\dfrac{\lambda}{4\pi R}\right)^2 \cdot G_{0r} \cdot G_{0t}$

$= 100 \cdot (1) \cdot \left(\dfrac{0.1}{4\pi \times 500}\right)^2 (100)(316.23)$

$P_r = 8 \times 10^{-4} \text{ watts}$

2-89. $f = 10 \text{ GHZ}, \rightarrow \lambda = \dfrac{3 \times 10^8}{10^{10}} = 0.03 \text{ m}$

$G_{0t} = G_{0r} = 15 \text{ dB} = 10^{1.5} = 31.62$

$R = 10 \text{ km} = 10^4 \text{ m}$

$P_r \geq 10 \text{ nw} = 10^{-8} \text{ w}$

$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = -3 \text{ dB} = \frac{1}{2}$

Friis Transmission Equation:

$$\frac{P_r}{P_t} = G_{0t} G_{0r} \cdot \left(\frac{\lambda}{4\pi R}\right)^2 \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

$$= (10^{1.5})^2 \cdot \left(\frac{0.03}{4\pi \times 10^4}\right)^2 \cdot \left(\tfrac{1}{2}\right) = 2.85 \times 10^{-11}$$

$$P_t = \frac{P_r}{2.85 \times 10^{-11}}$$

$$P_r \geq 10^{-8} \text{ W} \rightarrow (P_t)_{\min} = 351 \text{ W}$$

2-90.

$$\frac{P_r}{P_t} = (\text{PLF}) e_t e_r D_{0t} D_{0r} \left(\frac{\lambda}{4\pi R}\right)^2$$

$$= (\text{PLF})(e_{rt} \cdot e_{cdt})(e_{rr} \cdot e_{cdr}) \left(\frac{\lambda}{4\pi R}\right)^2 \cdot D_{0t} \cdot D_{0r}$$

$$\frac{P_r}{P_t} = (1)(e_{rt} \cdot (1))(e_{rr} \cdot (1)) \left(\frac{\lambda}{4\pi R}\right)^2 \cdot D_{0t} \cdot D_{0r}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^8} = 3 \text{ m}, R = 10 \times 10^3 = 10^4.$$

$$\left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{3}{4\pi \times 10^4}\right)^2 = \left(\frac{3}{4\pi} \times 10^{-4}\right)^2$$

$$= (0.2387 \times 10^{-4})^2 = 5.699 \times 10^{-2} \times 10^{-8}$$

$$\left(\frac{\lambda}{4\pi R}\right)^2 = 5.699 \times 10^{-10}$$

$$e_{rt} = e_{rr} = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73.3 - 50}{73.3 + 50}\right|^2\right) = \left(1 - \left|\frac{23.3}{123.3}\right|^2\right)$$

$$= (1 - (0.18897)^2) = (1 - 0.0357) = 0.9643$$

$$e_{cdt} = e_{cdr} = 1$$

$$D_{0t} = D_{0r} = 1.643$$

$$\frac{P_r}{P_t} = (0.9643)^2(1.643)^2(5.699 \times 10^{-10})$$
$$= (0.92987)(2.699)\ (5.699 \times 10^{-10})$$
$$= 2.51 \cdot (5.699 \times 10^{-10}) = 14.305 \times 10^{-10}$$
$$P_t = \frac{P_r}{14.305 \times 10^{-10}} = 6.99 \times 10^{-2} \times 10^{10}(1 \times 10^{-6})$$
$$= 6.99 \times 10^2 = 699.$$
$$P_t = 699 \text{ watts}$$

2-91. $\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 \cdot G_{0t} \cdot G_{0r},\ \lambda = \frac{3 \times 10^8}{9 \times 10^9} = \frac{3 \times 10^8}{90 \times 10^8} = \frac{1}{30}$

$$R = 10{,}000 \text{ meter} = \frac{10{,}000}{1/30}\lambda = 3 \times 10^5 \lambda$$
$$\frac{P_r}{P_t} = \left[\frac{\lambda}{4\pi(3 \times 10^5 \lambda)}\right]^2 \cdot {G_0}^2 = \frac{10 \times 10^{-6}}{10} = 10^{-6}$$
$${G_0}^2 = 10^{-6}(4\pi \times 3 \times 10^5)^2$$
$$G_0 = 10^{-3}(4\pi \times 3 \times 10^5) = 12\pi \times 10^2 = 1200\pi$$
$$G_0 = 1200\pi = 3{,}769.91 = 10\log_{10}(3{,}769.91) \text{ dB}$$
$$G_0 = 3{,}769.91 = 35.76 \text{ dB}$$

2-92. $R = 16 \times 10^3$ m, $f = 2$ GHz, $G_{0t} = 20$ dB, $P_t = 100$ watts,

$P_r = 5 \times 10^{-9}$ watts $\quad G_{0r} = ?$

$$G_{0t} = 20 \text{ dB} = 10\log_{10}[G_{0t}(\text{dim})] \Rightarrow G_{0t}(\text{dimensionless}) = 10^2 = 100$$
$$G_{0t}(\text{dimensionless}) = 100$$
$$f = 2 \text{ GHz} \Rightarrow \lambda = \frac{3 \times 10^8}{2 \times 10^9} = 0.15 \text{ meters}$$

Friis Transmission Equation (2-119):

$$\frac{P_r}{P_t} = G_{0t}G_{0r}\left(\frac{\lambda}{4\pi R}\right)^2 \text{PLF} \Rightarrow G_{0r} = \frac{P_r}{P_t}\left(\frac{1}{G_{0t}}\right)\left(\frac{4\pi R}{\lambda}\right)^2\left(\frac{1}{\text{PLF}}\right)$$
$$G_{0r} = \frac{5 \times 10^{-9}}{100}\left(\frac{1}{100}\right)\left[\frac{4\pi(16 \times 10^3)}{0.15}\right]^2\left(\frac{2}{1}\right)$$
$$= \frac{10 \times 10^{-9} \times 10^6}{10^4}\left[\frac{4\pi(16)}{0.15}\right]^2 = 10^{-6}(1{,}340.413)^2$$
$$G_{0r} = 1{,}796{,}706.65 \times 10^{-6} = 1.7967 = 2.545 \text{ dB}$$
$$\boxed{G_{0r} = 1.7967 = 2.545 \text{ dB}}$$

2-93. $\sigma = \pi a^2 = 25\pi\lambda^2$

Got = Gor = 16.3 dB $\Rightarrow$ Got (power ratio) = $10^{1.63} = 42.66$

$f = 10$ GHz $\Rightarrow \lambda = 0.03$ m

$$\frac{P_r}{P_t} = \sigma \frac{\text{Got} \cdot \text{Gor}}{4\pi} \left(\frac{\lambda}{4\pi R_1 \cdot R_2} \right)^2$$

a. $R_1 = R_2 = 200\lambda = 6$ meters;

$$P_r = 25 \cdot \pi\lambda^2 \frac{(42.66)^2}{4\pi} \cdot \left[\frac{\lambda}{4\pi(200\lambda)^2} \right]^2 \cdot (0.2) = 9.00\ n\text{watts}$$

b. $R_1 = R_2 = 500\lambda = 15$ meters;

$P_r = 0.23\ n$watts

2-94. $$P_r = P_t\sigma \cdot \frac{\text{Got} \cdot \text{Gor}}{4\pi} \cdot \left[\frac{\lambda}{4\pi R_1 \cdot R_2} \right]^2, \lambda = \frac{3 \times 10^8}{5 \times 10^9} = 0.06 \text{ m}$$

$$P_r = 10^5 \cdot (3) \cdot \frac{150^2}{4\pi} \cdot \left[\frac{0.06}{4\pi(10^6)} \right]^2$$

$P_r = 1.22 \times 10^{-8}$ watts

2-95. $$\frac{P_r}{P_t} = \sigma \frac{\text{Gor} \cdot \text{Got}}{4\pi} \cdot \left[\frac{\lambda}{4\pi R_1 R_2} \right]^2 \Rightarrow \sigma = \frac{P_r \cdot 4\pi}{P_t \cdot \text{Gor} \cdot \text{Got}} \left[\frac{4\pi R_1 \cdot R_2}{\lambda} \right]^2$$

$$\lambda = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ m}$$

$$\therefore \sigma = \frac{0.1425 \times 10^{-3}(4\pi)}{1000(75)(75)} \left[\frac{4\pi(500)(500)}{1} \right]^2 = 3142 \text{ m}^2$$

2-96. $$\sigma = \frac{P_r \cdot 4\pi}{P_t \cdot \text{Gor} \cdot \text{Got}} \left[\frac{4\pi R_1 R_2}{\lambda} \right]^2$$

$$\lambda = \frac{3 \times 10^8}{1 \times 10^8} = 3 \text{ m}$$

$$\sigma = \frac{0.01 \cdot (4\pi)}{1000(75)(75)} \left[\frac{4\pi(700)(700)}{3} \right]^2 = 94{,}114.5 \text{ m}^2$$

$\sigma = 94{,}114.5$ m^2

2-97. $\sigma = 0.85\lambda^2$

$$\frac{P_r}{P_t} = \sigma \cdot \frac{\text{Got} \cdot \text{Gor}}{4\pi}\left(\frac{\lambda}{4\pi R_1 \cdot R_2}\right)^2 |\hat{P}_w \cdot \hat{P}_r|^2$$

$\sigma = 0.85\lambda^2, \text{Got} = \text{Gor} = 15 \text{ dB} \Rightarrow \text{Got} = \text{Gor} = 31.6228$ (dimensionless)

$R_1 = R_2 = 100 \text{ meter} \Rightarrow R_1 = R_2 = 1{,}000\lambda$

$$f = 3 \text{ GHz} \Rightarrow \lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ meters}$$

$|\hat{\rho}_w \cdot \hat{\rho}_r|^2 = 1 \text{ dB} \Rightarrow |\hat{\rho}_w \cdot \hat{\rho}_r|^2 = 0.7943$

$$\frac{P_r}{P_t} = 0.85\lambda^2 \cdot \frac{(31.6228)^2}{4\pi} \cdot \left(\frac{\lambda}{4\pi \times 10^6\lambda^2}\right)^2 \cdot (0.7943)$$

$$= \frac{0.85(31.6228)^2(0.7943)}{(4\pi)^3(10^{12})} = 0.3402 \times 10^{-12}$$

$P_r = 0.3402 \times 10^{-12}(10^2) = 0.3402 \times 10^{-10} = 34.02 \times 10^{-12}$ watts

$P_r = 34.02$ $p$watts

2-98. $T_a = T_A e^{-2\alpha l} + T_0(1 - e^{-2\alpha l})$

$T_A = 5°\text{K}$

$$T_0 = 72°\text{F} = \frac{5}{9}(72 - 32) + 273 = 295.2°\text{K}$$

$-4 \text{ dB} = 20 \log_{10} e^{-\alpha} = -\alpha(20)\log_{10} e = -\alpha(20)(0.434)$

$$\alpha = \frac{4}{8.68} = 0.460 \text{ Nepers/100 ft} = 0.0046 \text{ Nepers/ft.}$$

a. $l = 2$ feet;

$$T_a = 5e^{-2(0.0046)2} + 295.2[1 - e^{-2(0.0046)2]} = 4.91 + 5.38 = 10.29°\text{K}$$

b. $l = 100$ feet;

$$T_a = 5e^{-2(0.0046)100} + 295.2[1 - e^{-2(0.0046)100]} = 179.72°\text{K}$$

2-99. $T_a = T_A e^{-\int_0^d 2\alpha(z)\,dz} + \int_0^d \epsilon(z) T_m(z) e^{-\int_z^d 2\alpha(z')\,dz'}\,dz$

If $\alpha(z) = \alpha_0 = \text{Constant}$

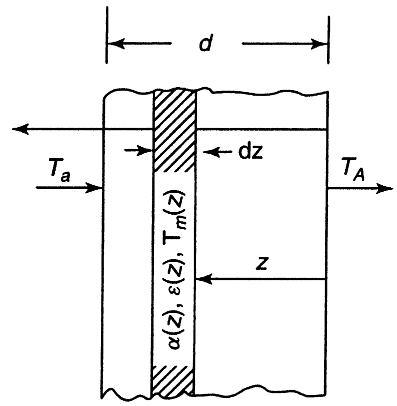
$$T_a = T_A e^{-2\alpha_0 d} + \int_0^d \epsilon(z) T_m(z) e^{-2\alpha_0(d-z)}\,dz$$

$$T_a = T_A e^{-2\alpha_0 d} + e^{-2\alpha_0 d} + \int_0^d \epsilon(z) T_m(z) e^{+2\alpha_0 z}\,dz$$

If $T_m(z) = T_0 =$ Constant and $\epsilon(z) = \epsilon_0 =$ constant

$$T_a = T_A e^{-2\alpha_0\, d} + \epsilon_0 T_0 e^{-2\alpha_0\, d} \int_0^d e^{2\alpha_0 z}\, dz$$

$$T_a = T_A e^{-2\alpha_0\, d} + \frac{\epsilon_0}{2\alpha_0} T_0 e^{-2\alpha_0\, d}(e^{2\alpha_0\, d} - 1)$$



For $\epsilon_0 = 2\alpha_0$

$$\begin{aligned} T_a &= T_A e^{-2\alpha_0\, d} + T_0 e^{-2\alpha_0\, d}(e^{2\alpha_0\, d} - 1) \\ &= T_A e^{-2\alpha_0\, d} + T_0(1 - e^{-2\alpha_0\, d}) \end{aligned}$$