Chapter 1

EQUATIONS AND INEQUALITIES

Section 1.1 Linear Equations

1. An equation is a statement that two expressions are equal.

2. To solve an equation means to find all numbers that make the equation a true statement.

3. A linear equation is a first-degree equation because the greatest degree of the variable is 1.

4. An identity is an equation satisfied by every number that is a meaningful replacement for the variable.

5. A contradiction is an equation that has no solution.

6. True. Replacing x with -8 in $2x+5=x-3$ yields a true statement. Therefore, the given statement is true.

7. True. The left side can be written as
$$5(x-8)=5[x+(-8)]=5x+5(-8)$$
$$=5x+(-40)=5x-40,$$
which is the same as the right side. Therefore, the statement is an identity.

8. False. If $x=0$, then the equation is true. A contradiction is false for all values of x.

9. False. Solving the literal equation $A=\frac{1}{2}bh$ for h gives
$$A=\frac{1}{2}bh$$
$$2A=bh$$
$$\frac{2A}{b}=h$$

10. B cannot be written in the form $ax+b=0$. A can be written as $15x-7=0$ or $15x+(-7)=0$, C can be written as $2x=0$ or $2x+0=0$, and D can be written as $-0.04x-0.4=0$ or $-0.04x+(-0.4)=0$.

11. $5x+4=3x-4$
$$2x+4=-4$$
$$2x=-8 \Rightarrow x=-4$$
Solution set: $\{-4\}$

12. $9x+11=7x+1$
$$2x+11=1$$
$$2x=-10 \Rightarrow x=-5$$
Solution set: $\{-5\}$

13. $6(3x-1)=8-(10x-14)$
$$18x-6=8-10x+14$$
$$18x-6=22-10x$$
$$28x-6=22$$
$$28x=28 \Rightarrow x=1$$
Solution set: $\{1\}$

14. $4(-2x+1)=6-(2x-4)$
$$-8x+4=6-2x+4$$
$$-8x+4=10-2x$$
$$4=10+6x$$
$$-6=6x \Rightarrow -1=x$$
Solution set: $\{-1\}$

15. $\frac{5}{6}x-2x+\frac{4}{3}=\frac{5}{3}$
$$6\cdot\left[\frac{5}{6}x-2x+\frac{4}{3}\right]=6\cdot\frac{5}{3}$$
$$5x-12x+8=10$$
$$-7x+8=10$$
$$-7x=2 \Rightarrow x=-\frac{2}{7}$$
Solution set: $\{-\frac{2}{7}\}$

16. $\frac{7}{4}+\frac{1}{5}x-\frac{3}{2}=\frac{4}{5}x$
$$20\cdot\left[\frac{7}{4}+\frac{1}{5}x-\frac{3}{2}\right]=20\cdot\frac{4}{5}x$$
$$35+4x-30=16x$$
$$4x+5=16x$$
$$5=12x \Rightarrow \frac{5}{12}=x$$
Solution set: $\{\frac{5}{12}\}$

17. $3x+5-5(x+1)=6x+7$
$3x+5-5x-5=6x+7$
$-2x=6x+7$
$-8x=7 \Rightarrow x=\frac{7}{-8}=-\frac{7}{8}$
Solution set: $\left\{-\frac{7}{8}\right\}$

18. $5(x+3)+4x-3=-(2x-4)+2$
$5x+15+4x-3=-2x+4+2$
$9x+12=-2x+6$
$11x+12=6$
$11x=-6 \Rightarrow x=\frac{-6}{11}=-\frac{6}{11}$
Solution set: $\left\{-\frac{6}{11}\right\}$

19. $2\left[x-(4+2x)+3\right]=2x+2$
$2(x-4-2x+3)=2x+2$
$2(-x-1)=2x+2$
$-2x-2=2x+2$
$-2=4x+2$
$-4=4x$
$-1=x$
Solution set: $\{-1\}$

20. $4\left[2x-(3-x)+5\right]=-6x-28$
$4(2x-3+x+5)=-6x-28$
$4(3x+2)=-6x-28$
$12x+8=-6x-28$
$18x+8=-28$
$18x=-36$
$x=-2$
Solution set: $\{-2\}$

21. $\frac{1}{14}(3x-2)=\frac{x+10}{10}$
$70\cdot\left[\frac{1}{14}(3x-2)\right]=70\cdot\left[\frac{x+10}{10}\right]$
$5(3x-2)=7(x+10)$
$15x-10=7x+70$
$8x-10=70$
$8x=80 \Rightarrow x=10$
Solution set: $\{10\}$

22. $\frac{1}{15}(2x+5)=\frac{x+2}{9}$
$45\cdot\left[\frac{1}{15}(2x+5)\right]=45\cdot\left[\frac{x+2}{9}\right]$
$3(2x+5)=5(x+2)$
$6x+15=5x+10$
$x+15=10 \Rightarrow x=-5$
Solution set: $\{-5\}$

23. $0.2x-0.5=0.1x+7$
$10(0.2x-0.5)=10(0.1x+7)$
$2x-5=x+70$
$x-5=70$
$x=75$
Solution set: $\{75\}$

24. $0.01x+3.1=2.03x-2.96$
$100(0.01x+3.1)=100(2.03x-2.96)$
$x+310=203x-296$
$310=202x-296$
$606=202x \Rightarrow 3=x$
Solution set: $\{3\}$

25. $-4(2x-6)+8x=5x+24+x$
$-8x+24+8x=6x+24$
$24=6x+24$
$0=6x \Rightarrow 0=x$
Solution set: $\{0\}$

26. $-8(3x+4)+6x=4(x-8)+4x$
$-24x-32+6x=4x-32+4x$
$-18x-32=8x-32$
$-32=26x-32$
$0=26x \Rightarrow 0=x$
Solution set: $\{0\}$

27. $0.5x+\frac{4}{3}x=x+10$
$\frac{1}{2}x+\frac{4}{3}x=x+10$
$6\left(\frac{1}{2}x+\frac{4}{3}x\right)=6(x+10)$
$3x+8x=6x+60$
$11x=6x+60$
$5x=60 \Rightarrow x=12$
Solution set: $\{12\}$

28. $\frac{2}{3}x+0.25x=x+2$
$\frac{2}{3}x+\frac{1}{4}x=x+2$
$12\left(\frac{2}{3}x+\frac{1}{4}x\right)=12(x+2)$
$8x+3x=12x+24$
$11x=12x+24$
$-x=24 \Rightarrow x=-24$
Solution set: $\{-24\}$

29.
$$\begin{aligned} 0.08x+0.06(x+12)&=7.72\\ 100\left[0.08x+0.06(x+12)\right]&=100\cdot 7.72\\ 8x+6(x+12)&=772\\ 8x+6x+72&=772\\ 14x+72&=772\\ 14x&=700\Rightarrow x=50 \end{aligned}$$
Solution set: $\{50\}$

30.
$$\begin{aligned} 0.04(x-12)+0.06x&=1.52\\ 100\left[0.04(x-12)+0.06x\right]&=100\cdot 1.52\\ 4(x-12)+6x&=152\\ 4x-48+6x&=152\\ 10x-48&=152\\ 10x&=200\Rightarrow x=20 \end{aligned}$$
Solution set: $\{20\}$

31.
$$\begin{aligned} 4(2x+7)&=2x+22+3(2x+2)\\ 8x+28&=2x+22+6x+6\\ 8x+28&=8x+28\\ 28&=28\Rightarrow 0=0 \end{aligned}$$
identity; $\{\text{all real numbers}\}$

32.
$$\begin{aligned} \frac{1}{2}(6x+20)&=x+4+2(x+3)\\ 3x+10&=x+4+2x+6\\ 3x+10&=3x+10\\ 10&=10\Rightarrow 0=0 \end{aligned}$$
identity; $\{\text{all real numbers}\}$

33.
$$\begin{aligned} 2(x-8)&=3x-16\\ 2x-16&=3x-16\\ -16&=x-16\Rightarrow 0=x \end{aligned}$$
conditional equation; $\{0\}$

34.
$$\begin{aligned} -8(x+5)&=-8x-5(x+8)\\ -8x-40&=-8x-5x-40\\ -8x-40&=-13x-40\\ 5x-40&=-40\\ 5x&=0\\ x&=0 \end{aligned}$$
conditional equation; $\{0\}$

35.
$$\begin{aligned} 4(x+7)&=2(x+12)+2(x+1)\\ 4x+28&=2x+24+2x+2\\ 4x+28&=4x+26\\ 28&=26 \end{aligned}$$
contradiction; $\varnothing$

36.
$$\begin{aligned} -6(2x+1)-3(x-4)&=-15x+1\\ -12x-6-3x+12&=-15x+1\\ -15x+6&=-15x+1\\ 6&=1 \end{aligned}$$
contradiction; $\varnothing$

37.
$$\begin{aligned} 0.3(x+2)-0.5(x+2)&=-0.2x-0.4\\ 10\left[0.3(x+2)-0.5(x+2)\right]&=10[-0.2x-0.4]\\ 3(x+2)-5(x+2)&=-2x-4\\ 3x+6-5x-10&=-2x-4\\ -2x-4&=-2x-4\\ 0&=0 \end{aligned}$$
identity; $\{\text{all real numbers}\}$

38.
$$\begin{aligned} -0.6(x-5)+0.8(x-6)&=0.2x-1.8\\ 10\left[-0.6(x-5)+0.8(x-6)\right]&=10[0.2x-1.8]\\ -6(x-5)+8(x-6)&=2x-18\\ -6x+30+8x-48&=2x-18\\ 2x-18&=2x-18\\ 0&=0 \end{aligned}$$
identity; $\{\text{all real numbers}\}$

39.
$$\begin{aligned} V&=lwh\\ \frac{V}{wh}&=\frac{lwh}{wh}\\ l&=\frac{V}{wh} \end{aligned}$$

40.
$$\begin{aligned} I&=Prt\\ \frac{I}{rt}&=\frac{Prt}{rt}\\ P&=\frac{I}{rt} \end{aligned}$$

41.
$$\begin{aligned} P&=a+b+c\\ P-a-b&=c\\ c&=P-a-b \end{aligned}$$

42.
$$\begin{aligned} P&=2l+2w\\ P-2l&=2w\\ \frac{P-2l}{2}&=\frac{2w}{2}\\ w&=\frac{P-2l}{2}=\frac{P}{2}-l \end{aligned}$$

43.
$$\begin{aligned} \mathcal{A}&=\frac{1}{2}h(B+b)\\ 2\mathcal{A}&=2\left[\frac{1}{2}h(B+b)\right]\\ 2\mathcal{A}&=h(B+b)\\ 2\mathcal{A}&=Bh+bh\\ 2\mathcal{A}-bh&=Bh\\ \frac{2\mathcal{A}-bh}{h}&=\frac{Bh}{h}\\ B&=\frac{2\mathcal{A}-bh}{h}=\frac{2\mathcal{A}}{h}-b \end{aligned}$$

44.
$$\mathcal{A} = \frac{1}{2}h(B+b)$$
$$2\mathcal{A} = 2\left[\frac{1}{2}h(B+b)\right]$$
$$2\mathcal{A} = h(B+b)$$
$$\frac{2\mathcal{A}}{B+b} = \frac{h(B+b)}{B+b}$$
$$h = \frac{2\mathcal{A}}{B+b}$$

45.
$$S = 2\pi rh + 2\pi r^2$$
$$S - 2\pi r^2 = 2\pi rh$$
$$\frac{S-2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$
$$h = \frac{S-2\pi r^2}{2\pi r} = \frac{S}{2\pi r} - r$$

46.
$$s = \frac{1}{2}gt^2$$
$$2s = 2\left[\frac{1}{2}gt^2\right]$$
$$2s = gt^2$$
$$\frac{2s}{t^2} = \frac{gt^2}{t^2}$$
$$g = \frac{2s}{t^2}$$

47.
$$S = 2lw + 2wh + 2hl$$
$$S - 2lw = 2wh + 2hl$$
$$S - 2lw = (2w+2l)h$$
$$\frac{S-2lw}{2w+2l} = \frac{(2w+2l)h}{2w+2l}$$
$$h = \frac{S-2lw}{2w+2l}$$

48.
$$z = \frac{x-\mu}{\sigma}$$
$$z\sigma = x - \mu$$
$$x = z\sigma + \mu$$

49.
$$2(x-a)+b = 3x+a$$
$$2x-2a+b = 3x+a$$
$$-3a+b = x$$
$$x = -3a+b$$

50.
$$5x-(2a+c) = 4(x+c)$$
$$5x-2a-c = 4x+4c$$
$$5x-2a = 4x+5c$$
$$x-2a = 5c$$
$$x = 2a+5c$$

51.
$$ax+b = 3(x-a)$$
$$ax+b = 3x-3a$$
$$3a+b = 3x-ax$$
$$3a+b = (3-a)x$$
$$\frac{3a+b}{3-a} = x$$
$$x = \frac{3a+b}{3-a}$$

52.
$$4a-ax = 3b+bx$$
$$4a-3b = bx+ax$$
$$4a-3b = (b+a)x$$
$$\frac{4a-3b}{b+a} = x$$
$$x = \frac{4a-3b}{b+a}$$

53.
$$\frac{x}{a-1} = ax+3$$
$$(a-1)\left[\frac{x}{a-1}\right] = (a-1)(ax+3)$$
$$x = a^2x+3a-ax-3$$
$$3-3a = a^2x-ax-x$$
$$3-3a = (a^2-a-1)x$$
$$\frac{3-3a}{a^2-a-1} = x$$
$$x = \frac{3-3a}{a^2-a-1}$$

54.
$$\frac{x-1}{2a} = 2x-a$$
$$2a\left[\frac{x-1}{2a}\right] = 2a(2x-a)$$
$$x-1 = 4ax-2a^2$$
$$x = 4ax-2a^2+1$$
$$x-4ax = -2a^2+1$$
$$x(1-4a) = -2a^2+1$$
$$x = \frac{-2a^2+1}{1-4a} = \frac{1-2a^2}{1-4a}$$

55.
$$a^2x+3x = 2a^2$$
$$(a^2+3)x = 2a^2$$
$$x = \frac{2a^2}{a^2+3}$$

56.
$$ax+b^2 = bx-a^2$$
$$a^2+b^2 = bx-ax$$
$$a^2+b^2 = (b-a)x$$
$$\frac{a^2+b^2}{b-a} = x$$
$$x = \frac{a^2+b^2}{b-a}$$

57.
$$3x = (2x-1)(m+4)$$
$$3x = 2xm+8x-m-4$$
$$m+4 = 2xm+5x$$
$$m+4 = (2m+5)x$$
$$\frac{m+4}{2m+5} = x$$
$$x = \frac{m+4}{2m+5}$$

58.
$$\begin{aligned} -x &= (5x+3)(3k+1) \\ -x &= 15xk + 5x + 9k + 3 \\ -6x - 15xk &= 9k + 3 \\ (-6-15k)x &= 9k+3 \\ x &= \frac{9k+3}{-6-15k} = \frac{3(3k+1)}{-3(2+5k)} \\ &= -\frac{3k+1}{5k+2} \end{aligned}$$

59. **(a)** Here, $r = 0.04$, $P = 3150$, and $t = \frac{6}{12} = \frac{1}{2}$ (year).

$$I = Prt = 3150(0.04)\left(\frac{1}{2}\right) = \$63$$

The interest is $63.

(b) The amount Miguel must pay Julio at the end of the six months is $3150 + $63 = $3213.

60. **(a)** Here, $r = 0.055$, $P = 30{,}900$, and $t = \frac{18}{12} = \frac{3}{2}$ (year).

$$I = Prt = 30,900(0.055)\left(\frac{3}{2}\right) = \$2549.25$$

She must pay the bank $30,900 + $2549.25 = $33,449.25.

(b) The interest is $2549.25.

61. $F = \frac{9}{5}C + 32$

$F = \frac{9}{5} \cdot 20 + 32 = 36 + 32 = 68$

Therefore, 20°C = 68°F.

62. $F = \frac{9}{5}C + 32$

$F = \frac{9}{5} \cdot 200 + 32 = 360 + 32 = 392$

Therefore, 200°C = 392°F.

63. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(50 - 32) = \frac{5}{9} \cdot 18 = 10$

Therefore, 50°F = 10°C.

64. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(77 - 32) = \frac{5}{9} \cdot 45 = 25$

Therefore, 77°F = 25°C.

65. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(100 - 32) = \frac{5}{9} \cdot 68 \approx 37.8$

Therefore, 100°F ≈ 37.8°C.

66. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(350 - 32) = \frac{5}{9} \cdot 318 \approx 176.7$

Therefore, 350°F ≈ 176.7°C.

67. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(867 - 32) = \frac{5}{9} \cdot 835 \approx 463.9$

Therefore, 865°F ≈ 463.9°C.

68. $F = \frac{9}{5}C + 32$

$F = \frac{9}{5} \cdot (-89.4) + 32 = -160.92 + 32 \approx -128.9$

Therefore, –89.4°C ≈ –128.9°F.

69. $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(113 - 32) = \frac{5}{9} \cdot (81) = 45$

Therefore, 113°F = 45°C.

70. $F = \frac{9}{5}C + 32$

$F = \frac{9}{5} \cdot (28.1) + 32 \approx 50.58 + 32 \approx 82.6$

Therefore, 28.1°C ≈ 82.6°F.

Section 1.2 Applications and Modeling with Linear Equations

1. Distance = rate × time, so time = distance ÷ rate. Divide 400 miles by 50 mph to obtain 8 hours.
2. 15 minutes is $\frac{1}{4}$ of an hour, so multiply 80 mph by $\frac{1}{4}$ to get a distance of 20 mi.
3. 2% is 0.02, so multiply $500 by 0.02 and by 4 yrs to get interest of $40
4. Multiply 40 half-dollars by $0.50 to get $20; multiply 200 quarters by $0.25 to get $50. Together, the monetary value is $70.
5. 75% is $\frac{3}{4}$, so multiply 120 L by $\frac{3}{4}$, to get 90 L acid.

6. Expression D, $x - 0.60$ does not represent the sales price. $x - 0.60$ represents x dollars discounted by 60 cents, not x dollars discounted by 60%. All of the other choices are equivalent and represent the sales price.

7. Concentration A, 24%, cannot possibly be the concentration of the mixture because it is less than both the original concentrations.

8. A **9.** D

10. B and C cannot be correct equations.
In B,
$$-2x + 7(5 - x) = 52$$
$$-2x + 35 - 7x = 52$$
$$-9x + 35 = 52$$
$$-9x = 17 \Rightarrow x = -\tfrac{17}{9}$$
but the length of a rectangle cannot be negative.
In C,
$$5(x + 2) + 5x = 10$$
$$5x + 10 + 5x = 10$$
$$10x + 10 = 10 \Rightarrow 10x = 0 \Rightarrow x = 0$$
but the length of a rectangle cannot be zero.

11. In the formula $P = 2l + 2w$, let $P = 294$ and $w = 57$.
$$294 = 2l + 2 \cdot 57$$
$$294 = 2l + 114$$
$$180 = 2l \Rightarrow 90 = l$$
The length is 90 cm.

12. Let w = width of the rectangular storage shed.
Then $w + 6$ = the length of the storage shed.
Use the formula for the perimeter of a rectangle.
$$P = 2l + 2w$$
$$44 = 2(w + 6) + 2w$$
$$44 = 2w + 12 + 2w$$
$$44 = 4w + 12$$
$$32 = 4w \Rightarrow 8 = w$$
The width is 8 ft and the length is $8 + 6 = 14$ ft.

13. Let x = the length of the shortest side.
Then $2x$ = the length of each of the longer sides.
The perimeter of a triangle is the sum of the measures of the three sides.
$$x + 2x + 2x = 30 \Rightarrow 5x = 30 \Rightarrow x = 6$$
The length of the shortest side is 6 cm.

14. Let w = the width of the rectangle.
Then $2w - 2.5$ = the length of the rectangle.
Use the formula for the perimeter of a rectangle.
$$P = 2l + 2w$$
$$40.6 = 2(2w - 2.5) + 2w$$
$$40.6 = 4w - 5 + 2w$$
$$40.6 = 6w - 5 \Rightarrow 45.6 = 6w \Rightarrow 7.6 = w$$
The width is 7.6 cm.

15. Let x = the length of the shortest side.
Then $2x - 200$ = the length of the longest side and the length of the middle side is $(2x - 200) - 200 = 2x - 400$.
The perimeter of a triangle is the sum of the measures of the three sides.
$$x + (2x - 200) + (2x - 400) = 2400$$
$$x + 2x - 200 + 2x - 400 = 2400$$
$$5x - 600 = 2400$$
$$5x = 3000 \Rightarrow x = 600$$
The length of the shortest side is 600 ft. The middle side is $2 \cdot 600 - 400 = 1200 - 400 = 800$ ft. The longest side is $2 \cdot 600 - 200 = 1200 - 200 = 1000$ ft.

16. Let w = the width of the cake.
Then $w + 0.39$ = the length of the cake.
Use the formula for the perimeter of a rectangle.
$$P = 2l + 2w$$
$$17.02 = 2(w + 0.39) + 2w$$
$$17.02 = 2w + 0.78 + 2w$$
$$17.02 = 4w + 0.78$$
$$16.24 = 4w \Rightarrow 4.06 = w$$
The width of the cake was 4.06 m and the length was $4.06 + 0.39 = 4.45$ m.

17. Let h = the height of box.
Use the formula for the surface area of a rectangular box.
$$S = 2lw + 2wh + 2hl$$
$$496 = 2 \cdot 18 \cdot 8 + 2 \cdot 8h + 2h \cdot 18$$
$$496 = 288 + 16h + 36h$$
$$496 = 288 + 52h$$
$$208 = 52h \Rightarrow 4 = h$$
The height of the box is 4 ft.

18. The volume of a right circular cylinder is $V = \pi r^2 h$
$$V = \pi r^2 h$$
$$144\pi = \pi 6^2 h$$
$$144\pi = 36\pi h$$
$$\frac{144\pi}{36\pi} = \frac{36\pi h}{36\pi} \Rightarrow 4 = h$$
The height of the cylinder is 4 in.

19. Let x = the time (in hours) spent on the way to the business appointment.

	r	t	d
Morning	50	x	$50x$
Afternoon	40	$x+\frac{1}{4}$	$40\left(x+\frac{1}{4}\right)$

The distance on the way to the business appointment is the same as the return trip, so

$$50x = 40\left(x+\tfrac{1}{4}\right)$$
$$50x = 40x + 10$$
$$10x = 10 \Rightarrow x = 1$$

Because she drove 1 hr, her distance traveled would be $50 \cdot 1 = 50$ mi.

20. Let x = time (in hours) on trip from Denver to Minneapolis.

	r	t	d
Going	50	x	$50x$
Returning	55	$32-x$	$55(32-x)$

The distance going and returning are the same, so we have

$$50x = 55(32-x)$$
$$50x = 1760 - 55x$$
$$105x = 1760 \Rightarrow x \approx 16.76$$

Because he traveled approximately 16.8 hr to Minneapolis, the distance would be about $50 \cdot 16.8 = 840$ mi.

21. Let x = David's speed (in mph) on bike. Then $x+4.5$ = David's speed (in mph) driving.

	r	t	d
Car	$x+4.5$	20 min = $\frac{1}{3}$ hr	$\frac{1}{3}(x+4.5)$
Bike	x	45 min = $\frac{3}{4}$ hr	$\frac{3}{4}x$

The distance by bike and car are the same, so

$$\tfrac{1}{3}(x+4.5) = \tfrac{3}{4}x$$
$$12\left[\tfrac{1}{3}(x+4.5)\right] = 12\left[\tfrac{3}{4}x\right]$$
$$4(x+4.5) = 9x$$
$$4x + 18 = 9x$$
$$18 = 5x \Rightarrow \frac{18}{5} = x$$

Because his rate is $\frac{18}{5}$ (or 3.6) mph, David travels $\frac{3}{4}\left(\frac{18}{5}\right) = \frac{27}{10} = 2.7$ mi to work.

22. Let x = rate (in mph) the San Diego bound plane travels. Then $x + 50$ = rate (in mph) the San Francisco bound plane travels.

	r	t	d
San Diego	x	$\frac{1}{2}$	$\frac{1}{2}x$
San Francisco	$x+50$	$\frac{1}{2}$	$\frac{1}{2}(x+50)$

The distance traveled by the two planes is 275 miles. The rate of the San Diego bound plane can be found by solving $\frac{1}{2}x+\frac{1}{2}(x+50) = 275.$

$$\tfrac{1}{2}x+\tfrac{1}{2}(x+50) = 275$$
$$2\left[\tfrac{1}{2}x+\tfrac{1}{2}(x+50)\right] = 2[275]$$
$$x+(x+50) = 550 \Rightarrow 2x+50 = 550$$
$$2x = 500 \Rightarrow x = 250$$

The San Diego bound plane travels at 250 mph, and the San Francisco bound plane travels at $250+50 = 300$ mph.

23. Let x = time (in hours) it takes for Russ and Janet to be 1.5 mi apart.

	r	t	d
Mary	7	x	$7x$
Janet	5	x	$5x$

Because Mary's rate is faster than Janet's, she travels farther than Janet in the same amount of time. To have the difference between Mary and Janet to be 1.5 mi, solve the following equation.

$$7x - 5x = 1.5 \Rightarrow 2x = 1.5 \Rightarrow x = 0.75$$

It will take 0.75 hr = 45 min for Mary and Janet to be 1.5 mi apart.

24. Let x = time (in hours) Mary runs. Because Janet has a ten-minute start and 10 minutes is $\frac{1}{6}$ hr, Janet's time running is $x+\frac{1}{6}$ hr.

	r	t	d
Mary	7	x	$7x$
Janet	5	$x+\frac{1}{6}$	$5\left(x+\frac{1}{6}\right)$

Because Mary must travel the same distance as Janet, we must solve the following equation.

$$7x = 5\left(x+\tfrac{1}{6}\right)$$
$$7x = 5x+\tfrac{5}{6}$$
$$2x = \tfrac{5}{6}$$
$$x = \tfrac{5}{12}$$

It will take $\frac{5}{12}$ hr $= \frac{5}{12}\cdot 60$ min $=$ 25 min for Mary to catch up with Janet.

25. We need to determine how many meters are in 26 miles.

$$26 \text{ mi} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ m}}{3.281 \text{ ft}} \approx 41,840.9 \text{ m}$$

Usain Bolt's rate in the 100-m dash would be

$r = \frac{d}{t} = \frac{100}{9.69}$ meters per second.

Thus, the time it would take for Usain to run the 26-mi marathon would be

$$t = \frac{d}{r} = \frac{41,840.9}{\frac{100}{9.69}} = 41,840.9 \cdot \frac{9.69}{100} \approx 4054 \text{ sec.}$$

Because there are 60 seconds in one minute and $60 \cdot 60 = 3,600$ seconds in one hour,

$4054 \text{ sec} = 1 \cdot 3600 + 7 \cdot 60 + 34 \text{ sec}$

or 1 hr, 7 min, 34 sec. This is about $\frac{1}{2}$ the world record time.

26. We know $26 \text{ mi} \approx 41,840.9 \text{ m}$ from exercise 25. Usain Bolt's rate in the 100-m dash would be $r = \frac{d}{t} = \frac{100}{9.58}$ meters per second. Thus, the time it would take for Usain to run the 26-mi marathon would be

$$t = \frac{d}{r} = \frac{41,840.9}{\frac{100}{9.58}} = 41,840.9 \cdot \frac{9.58}{100} \approx 4008 \text{ sec}$$

$4008 \text{ sec} = 1 \cdot 3600 + 6 \cdot 60 + 48 \text{ sec}$ or 1 hr, 6 min, 48 sec. This is about $\frac{1}{2}$ the world record time.

27. Let x = speed (in km/hr) of Callie's boat. When Callie is traveling upstream, the current slows her down, so we subtract the speed of the current from the speed of the boat. When she is traveling downstream, the current speeds her up, so we add the speed of the current to the speed of the boat.

	r	t	d
Upstream	$x-5$	20 min = $\frac{1}{3}$ hr	$\frac{1}{3}(x-5)$
Downstream	$x+5$	15 min = $\frac{1}{4}$ hr	$\frac{1}{4}(x+5)$

Because the distance upstream and downstream are the same, we must solve the following equation.

$$\frac{1}{3}(x-5) = \frac{1}{4}(x+5)$$
$$12\left[\frac{1}{3}(x-5)\right] = 12\left[\frac{1}{4}(x+5)\right]$$
$$4(x-5) = 3(x+5)$$
$$4x - 20 = 3x + 15$$
$$x - 20 = 15 \Rightarrow x = 35$$

The speed of Callie's boat is 35 km per hour.

28. Let x = speed (in mph) of the wind. When Joe is traveling against the wind, the wind slows him down, so we subtract the speed of the wind from the speed of the plane. When he is traveling with the wind, the wind speeds him up, so we add the speed of the wind to the speed of the plane.

	r	t	d
Against wind	$180-x$	3	$3(180-x)$
With wind	$180+x$	2.8	$2.8(180+x)$

Because the distance going and coming are the same, we must solve the following equation.

$$2.8(180+x) = 3(180-x)$$
$$504 + 2.8x = 540 - 3x$$
$$504 + 5.8x = 540$$
$$5.8x = 36 \Rightarrow x \approx 6.2$$

The speed of the wind is about 6.2 mph.

29. Let x = the amount of 5% acid solution (in gallons).

Strength	Gallons of Solution	Gallons of Pure Acid
5%	x	$0.05x$
10%	5	$0.10 \cdot 5 = 0.5$
7%	$x+5$	$0.07(x+5)$

The number of gallons of pure acid in the 5% solution plus the number of gallons of pure acid in the 10% solution must equal the number of gallons of pure acid in the 7% solution.

$$0.05x + 0.5 = 0.07(x+5)$$
$$0.05x + 0.5 = 0.07x + 0.35$$
$$0.5 = 0.02x + 0.35$$
$$0.15 = 0.02x$$
$$\frac{0.15}{0.02} = x \Rightarrow x = 7.5 = 7\tfrac{1}{2} \text{ gal}$$

$7\frac{1}{2}$ gallons of the 5% solution should be added.

30. Let x = the amount of 5% hydrochloric acid solution (in mL).

Strength	Milliliters of Solution	Milliliters of Hydrochloric Acid
5%	x	$0.05x$
20%	60	$0.20 \cdot 60 = 12$
10%	$x + 60$	$0.10(x+60)$

The number of milliliters of hydrochloric acid in the 5% solution plus the number of milliliters of hydrochloric acid in the 20% solution must equal the number of milliliters of hydrochloric acid in the 10% solution.

$$0.05x + 12 = 0.10(x+60)$$
$$0.05x + 12 = 0.10x + 6$$
$$12 = 0.05x + 6$$
$$6 = 0.05x$$
$$\frac{6}{0.05} = \frac{600}{5} = 120 \text{ mL}$$

120 mL of 5% hydrochloric acid solution should be added.

31. Let x = the amount of 100% alcohol solution (in liters).

Strength	Liters of Solution	Liters of Pure Alcohol
100%	x	$1x = x$
10%	7	$0.10 \cdot 7 = 0.7$
30%	$x + 7$	$0.30(x+7)$

The number of liters of pure alcohol in the 100% solution plus the number of liters of pure alcohol in the 10% solution must equal the number of liters of pure alcohol in the 30% solution.

$$x + 0.7 = 0.30(x+7)$$
$$x + 0.7 = 0.30x + 2.1$$
$$0.7x + 0.7 = 2.1$$
$$0.7x = 1.4$$
$$x = \frac{1.4}{0.7} = \frac{14}{7} = 2 \text{ L}$$

2 L of the 100% solution should be added.

32. Let x = the amount of 100% alcohol solution (in gallons).

Strength	Gallons of Solution	Gallons of Pure Alcohol
100%	x	$1x = x$
15%	20	$0.15 \cdot 20 = 3$
25%	$x + 20$	$0.25(x+20)$

The number of gallons of pure alcohol in the 100% solution plus the number of gallons of pure alcohol in the 15% solution must equal the number of gallons of pure alcohol in the 25% solution.

$$x + 3 = 0.25(x+20)$$
$$x + 3 = 0.25x + 5$$
$$0.75x + 3 = 5$$
$$0.75x = 2$$
$$x = \frac{2}{0.75} = \frac{200}{75} = \frac{8}{3} = 2\tfrac{2}{3} \text{ gal}$$

$2\frac{2}{3}$ gallons of the 100% solution should be added.

33. Let x = the amount of water (in mL).

Strength	Milliliters of Solution	Milliliters of Salt
6%	8	$0.06(8) = 0.48$
0%	x	$0(x) = 0$
4%	$8 + x$	$0.04(8+x)$

The number of milliliters of salt in the 6% solution plus the number of milliliters of salt in the water (0% solution) must equal the number of milliliters in the 4% solution.

$$0.48 + 0 = 0.04(8+x)$$
$$0.48 = 0.32 + 0.04x$$
$$0.16 = 0.04x$$
$$\frac{0.16}{0.04} = x \Rightarrow x = \frac{16}{4} = 4 \text{ mL}$$

To reduce the saline concentration to 4%, 4 mL of water should be added.

34. Let x = the amount of 100% acid (in liters).

Strength	Liters of Solution	Liters of Pure Acid
100%	x	$1x = x$
30%	18	$0.30 \cdot 18 = 5.4$
50%	$x + 18$	$0.50(x+18)$

The number of liters of acid in the pure acid (100%) plus the number of liters of acid in the 30% solution must equal the number of liters of acid in the 50% solution.

$$x + 5.4 = 0.50(x+18)$$
$$x + 5.4 = 0.50x + 9$$
$$0.5x + 5.4 = 9$$
$$0.5x = 3.6$$
$$x = \frac{3.6}{0.5} = \frac{36}{5} = 7.2 \text{ L}$$

7.2 L pure acid should be added.

35. Let x = amount of the short-term note. Then 240,000 – x = amount of the long-term note.

Note Amount	Interest Rate	Time (years)	Interest Paid
x	2%	1	$x(0.02)(1)$
$240,000 - x$	2.5%	1	$(240,000 - x)(0.025)(1)$
			5500

The amount of interest from the 2% note plus the amount of interest from the 2.5% note must equal the total amount of interest.

$$\begin{aligned} 0.02x + 0.025(240,000 - x) &= 5500 \\ 0.02x + 6000 - 0.025x &= 5500 \\ -0.005x + 6000 &= 5500 \\ -0.005x &= -500 \\ x &= 100,000 \end{aligned}$$

The amount of the short-term note is \$100,000 and the amount of the long-term note is \$240,000 – \$100,000 = \$140,000.

36. Let x = amount paid for the first plot. Then 120,000 – x = amount paid for the second plot.

Land Price	Rate of Return	Profit or Loss
x	15%	$0.15x$
$120,000 - x$	−10%	$-0.10(120,000 - x)$
120,000		5,500

$$\begin{aligned} 0.15x - 0.10(120,000 - x) &= 5500 \\ 0.15x - 12,000 + 0.10x &= 5500 \\ 0.25x - 12,000 &= 5500 \\ 0.25x &= 17,500 \\ x &= \$70,000 \end{aligned}$$

Roger paid \$70,000 for the first plot and 120,000 − 70,000 = \$50,000 for the second plot.

37. Let x = amount invested at 2.5%. Then $2x$ = amount invested at 3%.

Amount in Account	Interest Rate	Interest
x	2.5%	$0.025x$
$2x$	3%	$0.03(2x) = 0.06x$
		850

The amount of interest from the 2.5% account plus the amount of interest from the 3% account must equal the total amount of interest.

$$\begin{aligned} 0.025x + 0.03(2x) &= 850 \\ 0.025x + 0.06x &= 850 \\ 0.085x &= 850 \Rightarrow x = \$10,000 \end{aligned}$$

Janet deposited \$10,000 at 2.5% and 2(\$10,000) = \$20,000 at 3%.

38. Let x = amount invested at 3%. Then $4x$ = amount invested at 2.75%.

Amount in Account	Interest Rate	Interest
x	3%	$0.03x$
$4x$	2.75%	$0.0275(4x) = 0.11x$
		2800

The amount of interest from the 3% account plus the amount of interest from the 2.75% account must equal the total amount of interest.

$$\begin{aligned} 0.03x + 0.0275(4x) &= 2800 \\ 0.03x + 0.11x &= 2800 \\ 0.14x &= 2800 \Rightarrow x = \$20,000 \end{aligned}$$

The church invested \$20,000 at 3% and $4 \cdot 20,000 = \$80,000$ at 2.75%.

39. 30% of \$200,000 is \$60,000, so after paying her income tax, Linda had \$140,000 left to invest. Let x = amount invested at 1.5%. Then 140,000 – x = amount invested at 4%.

Amount Invested	Interest Rate	Interest
x	1.5%	$0.015x$
$140,000 - x$	4%	$0.04(140,000 - x)$
140,000		4350

$$\begin{aligned} 0.015x + 0.04(140,000 - x) &= 4350 \\ 0.015x + 5600 - 0.04x &= 4350 \\ -0.025x + 5600 &= 4350 \\ -0.025x &= -1250 \\ x &= \$50,000 \end{aligned}$$

Linda invested \$50,000 at 1.5% and \$140,000 – \$50,000 = \$90,000 at 4%.

40. 28% of \$48,000 is \$13,440, so after paying her income tax, Becky had \$34,560 left to invest. Let x = amount invested at 3.25%. Then 34,560 – x = amount invested at 1.75%.

Amount Invested	Interest Rate	Interest
x	3.25%	$0.0325x$
$34,560 - x$	1.75%	$0.0175(34,560 - x)$
34,560		904.80

$$\begin{aligned} 0.0325x + 0.0175(34,560 - x) &= 904.80 \\ 0.0325x + 604.80 - 0.0175x &= 904.80 \\ 0.015x + 604.80 &= 904.80 \\ 0.015x &= 300 \\ x &= \$20,000 \end{aligned}$$

Becky invested \$20,000 at 3.25% and \$34,560 – \$20,000 = \$14,560 at 1.75%.

41. **(a)** $y = 100 - 0.02x$
$y = 100 - 0.02(2400) = 100 - 48 = 52$
The annual cost is \$52.

(b) $50 = 100 - 0.02x$
$-50 = -0.02x$
$2500 = x$
The annual cost of membership will be \$50 if the club purchases are \$2500.

(c) $0 = 100 - 0.02x$
$-100 = -0.02x$
$5000 = x$
The annual cost of membership will be \$0 if the club purchases are \$5000.

42. **(a)** $y = 50 - 0.016x$
$y = 50 - 0.016(1500) = 50 - 24 = 26$
The annual cost is \$26.

(b) $0 = 50 - 0.016x$
$-50 = -0.016x$
$3125 = x$
The annual cost of membership will be \$0 if the club purchases are \$3125.

(c) If the annual club purchases are more than \$3125, then the model would yield a negative value for y, the actual annual cost of membership. Essentially, the cash-back reward exceeds the initial fee of \$50, creating a positive gain for the member.

43. **(a)** Let x = the number of hours. Then $F = 100(140)x = 14{,}000x$.

(b) Because $33\ \mu g/ft^3$ causes irritation, the room would need $33 \cdot 800 = 26{,}400\ \mu g$ to cause irritation.
$F = 14{,}000x$
$26{,}400 = 14{,}000x$
$\frac{26{,}400}{14{,}000} = x$
$x \approx 1.9$ hr
It will take about 1.9 hours for concentrations to reach $33\ \mu g/ft^3$.

44. **(a)** Because each student needs 15 ft^3 each minute and there are 60 minutes in an hour, the ventilation required by x students per hour would be $V = 60(15x) = 900x$.

(b) The number of air exchanges per hour is
$A = \frac{900x}{15{,}000} = 0.06x.$

(c) If $x = 40$, then $A = 0.06 \cdot 40 = 2.4$ ach.

(d) The ventilation should be increased by $\frac{50}{15} = \frac{10}{3} = 3\frac{1}{3}$ times. (Smoking areas require more than triple the ventilation.)

45. **(a)** In 2018, $x = 4$.
$y = 0.3143x + 21.95$
$y = 0.3143 \cdot 4 + 21.95 = 23.2072$
The projected enrollment for fall 2018 is approximately 23.2 million.

(b) $y = 0.3143x + 21.95$
$24 = 0.3143x + 21.95$
$2.05 = 0.3143x$
$x = \frac{2.05}{0.3143} \approx 6.52$
$2014 + 6 = 2020$
Enrollment is projected to reach 24 million during the year 2020.

(c) They are quite close.

(d) The year 2000 is represented by $x = -14$.
$y = 0.3143x + 21.95$
$y = 0.3143(-14) + 21.95 \approx 17.5$
According to the model, the enrollment was approximately 17.5 million

(e) Answers will vary. Sample answer: When using the model for predictions, it is best to stay within the scope of the sample data.

46. **(a)** The year 1952 corresponds to $x = 6$.
$y = 2.8370x + 140.83$
$y = 2.8370(6) + 140.83 = 157.852$
According to the model, the U.S. population on July 1, 1952 was 157.852 million or 157,852,000.

(b) $y = 2.8370x + 140.83$
$150 = 2.8370x + 140.83$
$9.17 = 2.8370x$
$x = \frac{9.17}{2.8370} \approx 3.2$
According to the model, the U.S. population reached 150 million during 1949.

Section 1.3 Complex Numbers

1. By definition, $i = \underline{\sqrt{-1}}$, and therefore, $i^2 = \underline{-1}$.

2. If a and b are real numbers, then any number of the form $a + bi$ is a <u>complex number</u>.

3. The numbers 6 + 5i and 6 – 5i, which differ only in the sign of their imaginary parts, are <u>complex conjugates</u>.

4. The product of a complex number and its conjugate is always a <u>real number</u>.

5. To find the quotient of two complex numbers in standard form, multiply both the numerator and the denominator by the complex conjugate of the <u>denominator</u>.

6. True. $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1} = 5i$

7. True. $\sqrt{-4} \cdot \sqrt{-9} = 2i \cdot 3i = 6i^2 = -6$

8. True. $i^{12} = \left(i^4\right)^3 = 1^3 = 1$

9. False.
$$\begin{aligned}(-2+7i)-(10-6i) &= -2+7i-10+6i \\ &= -12+13i\end{aligned}$$

10. False.
$$\begin{aligned}(5+3i)^2 &= 5^2 + 2\cdot 5\cdot 3i + (3i)^2 \\ &= 25+30i+9i^2 \\ &= 25+30i-9 \\ &= 16+30i\end{aligned}$$

11. −4 is real and complex.

12. 0 is real and complex.

13. 13i is complex, pure imaginary, and nonreal complex.

14. −7i is complex, pure imaginary, and nonreal complex.

15. $5+i$ is complex and nonreal complex.

16. $-6-2i$ is complex and nonreal complex.

17. π is real and complex.

18. $\sqrt{24}$ is real and complex.

19. $\sqrt{-25} = 5i$ is complex, pure imaginary, and nonreal complex.

20. $\sqrt{-36} = 6i$ is complex, pure imaginary, and nonreal complex.

21. $\sqrt{-25} = i\sqrt{25} = 5i$

22. $\sqrt{-36} = i\sqrt{36} = 6i$

23. $\sqrt{-10} = i\sqrt{10}$

24. $\sqrt{-15} = i\sqrt{15}$

25. $\sqrt{-288} = i\sqrt{288} = i\sqrt{144\cdot 2} = 12i\sqrt{2}$

26. $\sqrt{-500} = i\sqrt{500} = i\sqrt{100\cdot 5} = 10i\sqrt{5}$

27. $-\sqrt{-18} = -i\sqrt{18} = -i\sqrt{9\cdot 2} = -3i\sqrt{2}$

28. $-\sqrt{-80} = -i\sqrt{80} = -i\sqrt{16\cdot 5} = -4i\sqrt{5}$

29.
$$\begin{aligned}\sqrt{-13}\cdot\sqrt{-13} &= i\sqrt{13}\cdot i\sqrt{13} \\ &= i^2\left(\sqrt{13}\right)^2 = -1\cdot 13 = -13\end{aligned}$$

30.
$$\begin{aligned}\sqrt{-17}\cdot\sqrt{-17} &= i\sqrt{17}\cdot i\sqrt{17} \\ &= i^2\left(\sqrt{17}\right)^2 = -1\cdot 17 = -17\end{aligned}$$

31.
$$\begin{aligned}\sqrt{-3}\cdot\sqrt{-8} &= i\sqrt{3}\cdot i\sqrt{8} = i^2\sqrt{3\cdot 8} \\ &= -1\cdot\sqrt{24} = -\sqrt{4\cdot 6} = -2\sqrt{6}\end{aligned}$$

32.
$$\begin{aligned}\sqrt{-5}\cdot\sqrt{-15} &= i\sqrt{5}\cdot i\sqrt{15} = i^2\sqrt{5\cdot 15} \\ &= -1\cdot\sqrt{75} = -\sqrt{25\cdot 3} = -5\sqrt{3}\end{aligned}$$

33. $\dfrac{\sqrt{-30}}{\sqrt{-10}} = \dfrac{i\sqrt{30}}{i\sqrt{10}} = \sqrt{\dfrac{30}{10}} = \sqrt{3}$

34. $\dfrac{\sqrt{-70}}{\sqrt{-7}} = \dfrac{i\sqrt{70}}{i\sqrt{7}} = \sqrt{\dfrac{70}{7}} = \sqrt{10}$

35. $\dfrac{\sqrt{-24}}{\sqrt{8}} = \dfrac{i\sqrt{24}}{\sqrt{8}} = i\sqrt{\dfrac{24}{8}} = i\sqrt{3}$

36. $\dfrac{\sqrt{-54}}{\sqrt{27}} = \dfrac{i\sqrt{54}}{\sqrt{27}} = i\sqrt{\dfrac{54}{27}} = i\sqrt{2}$

37. $\dfrac{\sqrt{-10}}{\sqrt{-40}} = \dfrac{i\sqrt{10}}{i\sqrt{40}} = \sqrt{\dfrac{10}{40}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$

38. $\dfrac{\sqrt{-8}}{\sqrt{-72}} = \dfrac{i\sqrt{8}}{i\sqrt{72}} = \sqrt{\dfrac{8}{72}} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3}$

39.
$$\begin{aligned}\frac{\sqrt{-6}\cdot\sqrt{-2}}{\sqrt{3}} &= \frac{i\sqrt{6}\cdot i\sqrt{2}}{\sqrt{3}} = i^2\sqrt{\frac{6\cdot 2}{3}} \\ &= -1\cdot\sqrt{\frac{12}{3}} = -\sqrt{4} = -2\end{aligned}$$

40.
$$\begin{aligned}\frac{\sqrt{-12}\cdot\sqrt{-6}}{\sqrt{8}} &= \frac{i\sqrt{12}\cdot i\sqrt{6}}{\sqrt{8}} = i^2\sqrt{\frac{12\cdot 6}{8}} \\ &= -1\cdot\sqrt{\frac{72}{8}} = -\sqrt{9} = -3\end{aligned}$$

41. $\frac{-6-\sqrt{-24}}{2}=\frac{-6-\sqrt{-4\cdot 6}}{2}=\frac{-6-2i\sqrt{6}}{2}$
$=\frac{2\left(-3-i\sqrt{6}\right)}{2}=-3-i\sqrt{6}$

42. $\frac{-9-\sqrt{-18}}{3}=\frac{-9-\sqrt{-9\cdot 2}}{3}=\frac{-9-3i\sqrt{2}}{3}$
$=\frac{3\left(-3-i\sqrt{2}\right)}{3}=-3-i\sqrt{2}$

43. $\frac{10+\sqrt{-200}}{5}=\frac{10+\sqrt{-100\cdot 2}}{5}$
$=\frac{10+10i\sqrt{2}}{5}=\frac{5\left(2+2i\sqrt{2}\right)}{5}$
$=2+2i\sqrt{2}$

44. $\frac{20+\sqrt{-8}}{2}=\frac{20+\sqrt{-4\cdot 2}}{2}=\frac{20+2i\sqrt{2}}{2}$
$=\frac{2\left(10+i\sqrt{2}\right)}{2}=10+i\sqrt{2}$

45. $\frac{-3+\sqrt{-18}}{24}=\frac{-3+\sqrt{-9\cdot 2}}{24}=\frac{-3+3i\sqrt{2}}{24}$
$=\frac{3\left(-1+i\sqrt{2}\right)}{24}=\frac{-1+i\sqrt{2}}{8}$
$=-\frac{1}{8}+\frac{\sqrt{2}}{8}i$

46. $\frac{-5+\sqrt{-50}}{10}=\frac{-5+\sqrt{-25\cdot 2}}{10}=\frac{-5+5i\sqrt{2}}{10}$
$=\frac{5\left(-1+i\sqrt{2}\right)}{10}=\frac{-1+i\sqrt{2}}{2}$
$=-\frac{1}{2}+\frac{\sqrt{2}}{2}i$

47. $(3+2i)+(9-3i)=(3+9)+[2+(-3)]i$
$=12+(-1)i=12-i$

48. $(4-i)+(8+5i)=(4+8)+(-1+5)i$
$=12+4i$

49. $(-2+4i)-(-4+4i)$
$=[-2-(-4)]+(4-4)i$
$=2+0i=2$

50. $(-3+2i)-(-4+2i)$
$=[-3-(-4)]+(2-2)i=1+0i=1$

51. $(2-5i)-(3+4i)-(-2+i)$
$=[2-3-(-2)]+(-5-4-1)i$
$=1+(-10)i=1-10i$

52. $(-4-i)-(2+3i)+(-4+5i)$
$=[-4-2+(-4)]+(-1-3+5)i=-10+i$

53. $-i\sqrt{2}-2-\left(6-4i\sqrt{2}\right)-\left(5-i\sqrt{2}\right)$
$=(-2-6-5)+\left[-\sqrt{2}-\left(-4\sqrt{2}\right)-\left(-\sqrt{2}\right)\right]i$
$=-13+4i\sqrt{2}$

54. $3\sqrt{7}-\left(4\sqrt{7}-i\right)-4i+\left(-2\sqrt{7}+5i\right)$
$=\left[3\sqrt{7}-4\sqrt{7}+\left(-2\sqrt{7}\right)\right]+[-(-1)-4+5]i$
$=-3\sqrt{7}+2i$

55. $(2+i)(3-2i)$
$=2(3)+2(-2i)+i(3)+i(-2i)$
$=6-4i+3i-2i^2=6-i-2(-1)$
$=6-i+2=8-i$

56. $(-2+3i)(4-2i)$
$=-2(4)-2(-2i)+3i(4)+3i(-2i)$
$=-8+4i+12i-6i^2=-8+16i-6(-1)$
$=-8+16i+6=-2+16i$

57. $(2+4i)(-1+3i)$
$=2(-1)+2(3i)+4i(-1)+4i(3i)$
$=-2+6i-4i+12i^2=-2+2i+12(-1)$
$=-2+2i-12=-14+2i$

58. $(1+3i)(2-5i)$
$=1(2)+1(-5i)+3i(2)+3i(-5i)$
$=2-5i+6i-15i^2=2+i-15(-1)$
$=2+i+15=17+i$

59. $(3-2i)^2=3^2-2(3)(2i)+(2i)^2$
$=9-12i-4=5-12i$

60. $(2+i)^2=2^2+2(2)(i)+i^2=4+4i+i^2$
$=4+4i+(-1)=3+4i$

61. $(3+i)(3-i)=3^2-i^2=9-(-1)=10$

62. $(5+i)(5-i)=5^2-i^2=25-(-1)=26$

63. $(-2-3i)(-2+3i)=(-2)^2-(3i)^2=4-9i^2$
$=4-9(-1)=13$

64. $(6-4i)(6+4i)=6^2-(4i)^2$
$=36-16i^2=36-16(-1)$
$=36+16=52$

65. $\left(\sqrt{6}+i\right)\left(\sqrt{6}-i\right)=\left(\sqrt{6}\right)^2-i^2$
$=6-(-1)=6+1=7$

66. $\left(\sqrt{2}-4i\right)\left(\sqrt{2}+4i\right)=\left(\sqrt{2}\right)^2-(4i)^2=2-16i^2$
$=2-16(-1)=2+16=18$

67. $i(3-4i)(3+4i)=i\left[(3-4i)(3+4i)\right]$
$=i\left[3^2-(4i)^2\right]$
$=i\left[9-16i^2\right]$
$=i\left[9-16(-1)\right]$
$=i(9+16)=25i$

68. $i(2+7i)(2-7i)=i\left[(2+7i)(2-7i)\right]$
$=i\left[2^2-(7i)^2\right]$
$=i\left[4-49i^2\right]$
$=i\left[4-49(-1)\right]$
$=i(4+49)=53i$

69. $3i(2-i)^2=3i\left(2^2-2(2i)+i^2\right)$
$=3i(4-4i-1)=3i(3-4i)$
$=9i-12i^2=9i-12(-1)$
$=12+9i$

70. $-5i(4-3i)^2=-5i\left[4^2-2(4)(3i)+(3i)^2\right]$
$=-5i\left[16-24i+9i^2\right]$
$=-5i\left[16-24i+9(-1)\right]$
$=-5i(16-24i-9)$
$=-5i(7-24i)$
$=-35i+120i^2=-35i+120(-1)$
$=-35i-120=-120-35i$

71. $(2+i)(2-i)(4+3i)=\left[(2+i)(2-i)\right](4+3i)$
$=\left[2^2-i^2\right](4+3i)$
$=\left[4-(-1)\right](4+3i)$
$=5(4+3i)=20+15i$

72. $(3-i)(3+i)(2-6i)=\left[(3-i)(3+i)\right](2-6i)$
$=\left[3^2-i^2\right](2-6i)$
$=\left[9-(-1)\right](2-6i)$
$=10(2-6i)=20-60i$

73. $\dfrac{6+2i}{1+2i}=\dfrac{(6+2i)(1-2i)}{(1+2i)(1-2i)}$
$=\dfrac{6-12i+2i-4i^2}{1^2-(2i)^2}=\dfrac{6-10i-4(-1)}{1-4i^2}$
$=\dfrac{6-10i+4}{1-4(-1)}=\dfrac{10-10i}{1+4}=\dfrac{10-10i}{5}$
$=\dfrac{10}{5}-\dfrac{10}{5}i=2-2i$

74. $\dfrac{14+5i}{3+2i}=\dfrac{(14+5i)(3-2i)}{(3+2i)(3-2i)}$
$=\dfrac{42-28i+15i-10i^2}{3^2-(2i)^2}$
$=\dfrac{42-13i-10(-1)}{9-4i^2}=\dfrac{42-13i+10}{9-4(-1)}$
$=\dfrac{52-13i}{9+4}=\dfrac{52-13i}{13}$
$=\dfrac{52}{13}-\dfrac{13}{13}i=4-i$

75. $\dfrac{2-i}{2+i}=\dfrac{(2-i)(2-i)}{(2+i)(2-i)}=\dfrac{2^2-2(2i)+i^2}{2^2-i^2}$
$=\dfrac{4-4i+(-1)}{4-(-1)}=\dfrac{3-4i}{5}=\dfrac{3}{5}-\dfrac{4}{5}i$

76. $\dfrac{4-3i}{4+3i}=\dfrac{(4-3i)(4-3i)}{(4+3i)(4-3i)}=\dfrac{4^2-2(4)(3i)+(3i)^2}{4^2-(3i)^2}$
$=\dfrac{16-24i+9i^2}{16-9i^2}=\dfrac{16-24i+9(-1)}{16-9(-1)}$
$=\dfrac{16-24i-9}{16+9}=\dfrac{7-24i}{25}=\dfrac{7}{25}-\dfrac{24}{25}i$

77. $\dfrac{1-3i}{1+i}=\dfrac{(1-3i)(1-i)}{(1+i)(1-i)}=\dfrac{1-i-3i+3i^2}{1^2-i^2}$
$=\dfrac{1-4i+3(-1)}{1-(-1)}=\dfrac{1-4i-3}{2}$
$=\dfrac{-2-4i}{2}=\dfrac{-2}{2}-\dfrac{4}{2}i=-1-2i$

78. $\frac{-3+4i}{2-i}=\frac{(-3+4i)(2+i)}{(2-i)(2+i)}=\frac{-6-3i+8i+4i^2}{2^2-i^2}$
$=\frac{-6+5i+4(-1)}{4-(-1)}=\frac{-6+5i-4}{5}$
$=\frac{-10+5i}{5}=\frac{-10}{5}+\frac{5}{5}i=-2+i$

79. $\frac{-5}{i}=\frac{-5(-i)}{i(-i)}=\frac{5i}{-i^2}$
$=\frac{5i}{-(-1)}=\frac{5i}{1}=5i$ or $0+5i$

80. $\frac{-6}{i}=\frac{-6(-i)}{i(-i)}=\frac{6i}{-i^2}$
$=\frac{6i}{-(-1)}=\frac{6i}{1}=6i$ or $0+6i$

81. $\frac{8}{-i}=\frac{8\cdot i}{-i\cdot i}=\frac{8i}{-i^2}$
$=\frac{8i}{-(-1)}=\frac{8i}{1}=8i$ or $0+8i$

82. $\frac{12}{-i}=\frac{12\cdot i}{-i\cdot i}=\frac{12i}{-i^2}$
$=\frac{12i}{-(-1)}=\frac{12i}{1}=12i$ or $0+12i$

83. $\frac{2}{3i}=\frac{2(-3i)}{3i\cdot(-3i)}=\frac{-6i}{-9i^2}=\frac{-6i}{-9(-1)}$
$=\frac{-6i}{9}=-\frac{2}{3}i$ or $0-\frac{2}{3}i$

Note: In the above solution, we multiplied the numerator and denominator by the complex conjugate of $3i$, namely $-3i$. Because there is a reduction in the end, the same results can be achieved by multiplying the numerator and denominator by $-i$.

84. $\frac{5}{9i}=\frac{5(-9i)}{9i\cdot(-9i)}=\frac{-45i}{-81i^2}=\frac{-45i}{-81(-1)}$
$=\frac{-45i}{81}=-\frac{5}{9}i$ or $0-\frac{5}{9}i$

85. $I=5+7i, Z=6+4i$
$E=IZ$
$E=(5+7i)(6+4i)$
$=5(6)+5(4i)+7i(6)+7i(4i)$
$=30+20i+42i+28i^2$
$=30+62i-28$
$=2+62i$

86. $I=20+12i, Z=10-5i$
$E=IZ$
$E=(20+12i)(10-5i)$
$=20(10)+20(-5i)+12i(10)+12i(-5i)$
$=200-100i+120i-60i^2$
$=200+20i+60$
$=260+20i$

87. $I=10+4i,\ E=88+128i$
$E=IZ$
$88+128i=(10+4i)Z$
$Z=\frac{88+128i}{10+4i}$
$Z=\frac{(88+128i)(10-4i)}{(10+4i)(10-4i)}$
$Z=\frac{88(10)+88(-4i)+128i(10)+128i(-4i)}{10^2-(4i)^2}$
$Z=\frac{880-352i+1280i-512i^2}{100-(-16)}$
$Z=\frac{880-352i+1280i+512}{116}$
$Z=\frac{1392+928i}{116}=12+8i$

88. $E=57+67i,\ Z=9+5i$
$E=IZ$
$57+67i=I(9+5i)$
$I=\frac{57+67i}{9+5i}$
$I=\frac{(57+67i)(9-5i)}{(9+5i)(9-5i)}$
$I=\frac{57(9)-57(5i)+67i(9)-67i(5i)}{9^2-(5i)^2}$
$I=\frac{513-285i+603i-335i^2}{81-(-25)}$
$I=\frac{513+318i+335}{106}$
$I=\frac{848+318i}{106}=8+3i$

89. $i^{25}=i^{24}\cdot i=(i^4)^6\cdot i=1^6\cdot i=i$

90. $i^{29}=i^{28}\cdot i=(i^4)^7\cdot i=1^7\cdot i=i$

91. $i^{22}=i^{20}\cdot i^2=(i^4)^5\cdot(-1)=1^5\cdot(-1)=-1$

92. $i^{26}=i^{24}\cdot i^2=(i^4)^6\cdot(-1)=1^6\cdot(-1)=-1$

93. $i^{23} = i^{20} \cdot i^3 = \left(i^4\right)^5 \cdot i^3 = 1^5 \cdot (-i) = -i$

94. $i^{27} = i^{24} \cdot i^3 = \left(i^4\right)^6 \cdot i^3 = 1^6 \cdot (-i) = -i$

95. $i^{32} = \left(i^4\right)^8 = 1^8 = 1$

96. $i^{40} = \left(i^4\right)^{10} = 1^{10} = 1$

97. $i^{-13} = i^{-16} \cdot i^3 = \left(i^4\right)^{-4} \cdot i^3 = 1^{-4} \cdot (-i) = -i$

98. $i^{-14} = i^{-16} \cdot i^2 = \left(i^4\right)^{-4} \cdot i^2 = 1^{-4} \cdot (-1) = -1$

99. $\frac{1}{i^{-11}} = i^{11} = i^8 \cdot i^3 = \left(i^4\right)^2 \cdot i^3 = 1^2 \cdot (-i) = -i$

100. $\frac{1}{i^{-12}} = i^{12} = \left(i^4\right)^3 = 1^3 = 1$

101. We need to show that $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 = i.$

$$\begin{aligned}
&\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2 \\
&= \left(\frac{\sqrt{2}}{2}\right)^2 + 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}i + \left(\frac{\sqrt{2}}{2}i\right)^2 \\
&= \frac{2}{4} + 2 \cdot \frac{2}{4}i + \frac{2}{4}i^2 = \frac{1}{2} + i + \frac{1}{2}i^2 \\
&= \frac{1}{2} + i + \frac{1}{2}(-1) = \frac{1}{2} + i - \frac{1}{2} = i
\end{aligned}$$

Thus, $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ is a square root of *i*.

102. We need to show that $\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^2 = i.$

$$\begin{aligned}
&\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^2 \\
&= \left(-\frac{\sqrt{2}}{2}\right)^2 + 2\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}i\right) + \left(-\frac{\sqrt{2}}{2}i\right)^2 \\
&= \frac{2}{4} + 2 \cdot \frac{2}{4}i + \frac{2}{4}i^2 = \frac{1}{2} + i + \frac{1}{2}i^2 \\
&= \frac{1}{2} + i + \frac{1}{2}(-1) = \frac{1}{2} + i - \frac{1}{2} = i
\end{aligned}$$

Thus, $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ is a square root of *i*.

103. We need to show that $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3 = i.$

$$\begin{aligned}
&\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3 \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left[\left(\frac{\sqrt{3}}{2}\right)^2 + 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}i + \left(\frac{1}{2}i\right)^2\right] \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left[\frac{3}{4} + \frac{\sqrt{3}}{2}i + \frac{1}{4}i^2\right] \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left[\frac{3}{4} + \frac{\sqrt{3}}{2}i + \frac{1}{4}(-1)\right] \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left[\frac{3}{4} + \frac{\sqrt{3}}{2}i - \frac{1}{4}\right] \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(\frac{2}{4} + \frac{\sqrt{3}}{2}i\right) \\
&= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
&= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}i + \frac{1}{2} \cdot \frac{1}{2}i + \frac{1}{2}\frac{\sqrt{3}}{2}i^2 \\
&= \frac{\sqrt{3}}{4} + \frac{3}{4}i + \frac{1}{4}i + \frac{\sqrt{3}}{4}i^2 \\
&= \frac{\sqrt{3}}{4} + \frac{4}{4}i + \frac{\sqrt{3}}{4}(-1) = \frac{\sqrt{3}}{4} + i + \left(-\frac{\sqrt{3}}{4}\right) = i
\end{aligned}$$

104. We need to show that $\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3 = i.$

$$\begin{aligned}
&\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3 \\
&= \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 \\
&= \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \cdot \\
&\qquad \left[\left(-\frac{\sqrt{3}}{2}\right)^2 + 2\left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}i + \left(\frac{1}{2}i\right)^2\right] \\
&= \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left[\frac{3}{4} - \frac{\sqrt{3}}{2}i + \frac{1}{4}i^2\right]
\end{aligned}$$

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(*continued*)

$$= \left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left[\frac{3}{4}-\frac{\sqrt{3}}{2}i+\frac{1}{4}(-1)\right]$$
$$= \left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left[\frac{3}{4}-\frac{\sqrt{3}}{2}i-\frac{1}{4}\right]$$
$$= \left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{2}{4}-\frac{\sqrt{3}}{2}i\right)$$
$$= \left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$$
$$= -\frac{\sqrt{3}}{2}\cdot\frac{1}{2}-\frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}i\right)+\frac{1}{2}\cdot\frac{1}{2}i-\frac{1}{2}\frac{\sqrt{3}}{2}i^2$$
$$= -\frac{\sqrt{3}}{4}+\frac{3}{4}i+\frac{1}{4}i-\frac{\sqrt{3}}{4}i^2$$
$$= -\frac{\sqrt{3}}{4}+\frac{4}{4}i-\frac{\sqrt{3}}{4}(-1)$$
$$= -\frac{\sqrt{3}}{4}+i+\frac{\sqrt{3}}{4}=i$$

105. If $-2+i$ is a solution of the equation, then substituting that value for x makes a true statement.

$$\begin{aligned} x^2+4x+5&=0\\ (-2+i)^2+4(-2+i)+5&=0\\ 4-4i+i^2-8+4i+5&=0\\ 4-4i-1-8+4i+5&=0\\ 0&=0 \end{aligned}$$

106. If $-2-i$ is a solution of the equation, then substituting that value for x makes a true statement.

$$\begin{aligned} x^2+4x+5&=0\\ (-2-i)^2+4(-2-i)+5&=0\\ 4+4i+i^2-8-4i+5&=0\\ 4+4i-1-8-4i+5&=0\\ 0&=0 \end{aligned}$$

107. If $-3+4i$ is a solution of the equation, then substituting that value for x makes a true statement.

$$\begin{aligned} x^2+6x+25&=0\\ (-3+4i)^2+6(-3+4i)+25&=0\\ 9-24i+16i^2-18+24i+25&=0\\ 9-24i-16-18+24i+25&=0\\ 0&=0 \end{aligned}$$

108. If $-3-4i$ is a solution of the equation, then substituting that value for x makes a true statement.

$$\begin{aligned} x^2+6x+25&=0\\ (-3-4i)^2+6(-3-4i)+25&=0\\ 9+24i+16i^2-18-24i+25&=0\\ 9+24i-16-18-24i+25&=0\\ 0&=0 \end{aligned}$$

Section 1.4 Quadratic Equations

1. G; $x^2=25 \Rightarrow x=\pm\sqrt{25}=\pm 5$
2. A; $x^2=-25 \Rightarrow x=\pm\sqrt{-25}=\pm 5i$
3. C; $x^2+5=0 \Rightarrow x^2=-5 \Rightarrow x=\pm\sqrt{-5}=\pm i\sqrt{5}$
4. E; $x^2-5=0 \Rightarrow x^2=5 \Rightarrow x=\pm\sqrt{5}$
5. H; $x^2=-20 \Rightarrow x=\pm\sqrt{-20}=\pm 2i\sqrt{5}$
6. B; $x^2=20 \Rightarrow x=\pm\sqrt{20}=\pm 2\sqrt{5}$
7. D**;** $x-5=0 \Rightarrow x=5$
8. F; $x+5=0 \Rightarrow x=-5$
9. D is the only one set up for direct use of the zero-factor property.

$$(3x-1)(x-7)=0$$
$$3x-1=0 \quad \text{or} \quad x-7=0$$
$$x=\tfrac{1}{3} \quad \text{or} \quad x=7$$

Solution set: $\left\{\frac{1}{3},7\right\}$

10. B is the only one set up for direct use of the square root property.

$$(2x+5)^2=7 \Rightarrow 2x+5=\pm\sqrt{7} \Rightarrow$$
$$2x=-5\pm\sqrt{7} \Rightarrow x=\frac{-5\pm\sqrt{7}}{2}$$

Solution set: $\left\{\frac{-5\pm\sqrt{7}}{2}\right\}$

11. C is the only one that does not require Step 1 of the method of completing the square.

$$x^2+x=12 \qquad \text{Note:}$$
$$x^2+x+\tfrac{1}{4}=12+\tfrac{1}{4} \qquad \left[\tfrac{1}{2}\cdot 1\right]^2=\left(\tfrac{1}{2}\right)^2=\tfrac{1}{4}$$
$$\left(x+\tfrac{1}{2}\right)^2=\tfrac{49}{4}$$
$$x+\tfrac{1}{2}=\pm\sqrt{\tfrac{49}{4}}$$
$$x+\tfrac{1}{2}=\pm\tfrac{7}{2} \Rightarrow x=-\tfrac{1}{2}\pm\tfrac{7}{2}$$
$$-\tfrac{1}{2}-\tfrac{7}{2}=\tfrac{-8}{2}=-4 \text{ and } -\tfrac{1}{2}+\tfrac{7}{2}=\tfrac{6}{2}=3$$

Solution set: $\{-4,3\}$

12. A is the only one set up so that the values of a, b, and c can be determined immediately.

$3x^2 - 17x - 6 = 0$ yields $a = 3$, $b = -17$, and $c = -6$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(3)(-6)}}{2(3)} = \frac{17 \pm \sqrt{289 - (-72)}}{6} = \frac{17 \pm \sqrt{361}}{6} = \frac{17 \pm 19}{6}$$

$\frac{17+19}{6} = \frac{36}{6} = 6$ and $\frac{17-19}{6} = \frac{-2}{6} = -\frac{1}{3}$

Solution set: $\left\{-\frac{1}{3}, 6\right\}$

13. $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x - 2 = 0 \Rightarrow x = 2$ or $x - 3 = 0 \Rightarrow x = 3$

Solution set: $\{2, 3\}$

14. $x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x + 4 = 0 \Rightarrow x = -4$ or $x - 2 = 0 \Rightarrow x = 2$

Solution set: $\{-4, 2\}$

15. $5x^2 - 3x - 2 = 0$

$(5x+2)(x-1) = 0$

$5x + 2 = 0 \Rightarrow x = -\frac{2}{5}$ or $x - 1 = 0 \Rightarrow x = 1$

Solution set: $\left\{-\frac{2}{5}, 1\right\}$

16. $2x^2 - x - 15 = 0$

$(2x+5)(x-3) = 0$

$2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$ or $x - 3 = 0 \Rightarrow x = 3$

Solution set: $\left\{-\frac{5}{2}, 3\right\}$

17. $-4x^2 + x = -3$

$0 = 4x^2 - x - 3$

$0 = (4x+3)(x-1)$

$4x + 3 = 0 \Rightarrow x = -\frac{3}{4}$ or $x - 1 = 0 \Rightarrow x = 1$

Solution set: $\left\{-\frac{3}{4}, 1\right\}$

18. $-6x^2 + 7x = -10$

$0 = 6x^2 - 7x - 10 = 0$

$0 = (6x+5)(x-2) = 0$

$6x + 5 = 0 \Rightarrow x = -\frac{5}{6}$ or $x - 2 = 0 \Rightarrow x = 2$

Solution set: $\left\{-\frac{5}{6}, 2\right\}$

19. $x^2 - 100 = 0$

$(x+10)(x-10) = 0$

$x + 10 = 0 \Rightarrow x = -10$ or $x - 10 = 0 \Rightarrow x = 10$

Solution set: $\{-10, 10\}$

20. $x^2 - 64 = 0$

$(x+8)(x-8) = 0$

$x + 8 = 0 \Rightarrow x = -8$ or $x - 8 = 0 \Rightarrow x = 8$

Solution set: $\{-8, 8\}$

21. $4x^2 - 4x + 1 = 0$

$(2x-1)^2 = 0$

$2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Solution set: $\left\{\frac{1}{2}\right\}$

22. $9x^2 - 12x + 4 = 0$

$(3x-2)^2 = 0$

$3x - 2 = 0 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

Solution set: $\left\{\frac{2}{3}\right\}$

23. $25x^2 + 30x + 9 = 0$

$(5x+3)^2 = 0$

$5x + 3 = 0 \Rightarrow 5x = -3 \Rightarrow x = -\frac{3}{5}$

Solution set: $\left\{-\frac{3}{5}\right\}$

24. $36x^2 + 60x + 25 = 0$

$(6x+5)^2 = 0$

$6x + 5 = 0 \Rightarrow 6x = -5 \Rightarrow x = -\frac{5}{6}$

Solution set: $\left\{-\frac{5}{6}\right\}$

25. $x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

Solution set: $\{\pm 4\}$

26. $x^2 = 121$

$x = \pm\sqrt{121} = \pm 11$

Solution set: $\{\pm 11\}$

27. $27 - x^2 = 0$

$27 = x^2$

$x = \pm\sqrt{27} = \pm 3\sqrt{3}$

Solution set: $\{\pm 3\sqrt{3}\}$

28. $48 - x^2 = 0$

$48 = x^2$

$x = \pm\sqrt{48} = \pm 4\sqrt{3}$

Solution set: $\{\pm 4\sqrt{3}\}$

29. $x^2 = -81$

$x = \pm\sqrt{-81} = \pm 9i$

Solution set: $\{\pm 9i\}$

30. $x^2 = -400$

$x = \pm\sqrt{-400} = \pm 20i$

Solution set: $\{\pm 20i\}$

31. $(3x-1)^2 = 12$

$3x - 1 = \pm\sqrt{12}$

$3x = 1 \pm 2\sqrt{3} \Rightarrow x = \dfrac{1 \pm 2\sqrt{3}}{3}$

Solution set: $\left\{\frac{1\pm 2\sqrt{3}}{3}\right\}$

32. $(4x+1)^2 = 20$

$4x + 1 = \pm\sqrt{20}$

$4x = -1 \pm 2\sqrt{5} \Rightarrow x = \dfrac{-1 \pm 2\sqrt{5}}{4}$

Solution set: $\left\{\frac{-1\pm 2\sqrt{5}}{4}\right\}$

33. $(x+5)^2 = -3$

$x + 5 = \pm\sqrt{-3}$

$x + 5 = \pm i\sqrt{3}$

$x = -5 \pm i\sqrt{3}$

Solution set: $\{-5 \pm i\sqrt{3}\}$

34. $(x-4)^2 = -5$

$x - 4 = \pm\sqrt{-5}$

$x - 4 = \pm i\sqrt{5}$

$x = 4 \pm i\sqrt{5}$

Solution set: $\{4 \pm i\sqrt{5}\}$

35. $(5x-3)^2 = -3$

$5x - 3 = \pm\sqrt{-3}$

$5x - 3 = \pm i\sqrt{3}$

$5x = 3 \pm i\sqrt{3}$

$x = \dfrac{3 \pm i\sqrt{3}}{5} = \dfrac{3}{5} \pm \dfrac{\sqrt{3}}{5}i$

Solution set: $\left\{\frac{3}{5} \pm \frac{\sqrt{3}}{5}i\right\}$

36. $(-2x+5)^2 = -8$

$-2x + 5 = \pm\sqrt{-8}$

$-2x + 5 = \pm 2i\sqrt{2}$

$-2x = -5 \pm 2i\sqrt{2}$

$x = \dfrac{-5 \pm 2i\sqrt{2}}{-2} = \dfrac{5}{2} \pm i\sqrt{2}$

Solution set: $\left\{\frac{5}{2} \pm i\sqrt{2}\right\}$

37. $x^2 - 4x + 3 = 0$

$x^2 - 4x + 4 = -3 + 4$ Note: $\left[\frac{1}{2}\cdot(-4)\right]^2 = (-2)^2 = 4$

$(x-2)^2 = 1$

$x - 2 = \pm\sqrt{1}$

$x - 2 = \pm 1$

$x = 2 \pm 1$

$2 - 1 = 1$ and $2 + 1 = 3$

Solution set: $\{1, 3\}$

38. $x^2 - 7x + 12 = 0$

$x^2 - 7x + \frac{49}{4} = -12 + \frac{49}{4}$ Note: $\left[\frac{1}{2}\cdot(-7)\right]^2 = \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2 = \frac{1}{4}$

$x - \frac{7}{2} = \pm\sqrt{\frac{1}{4}}$

$x - \frac{7}{2} = \pm\frac{1}{2}$

$x = \frac{7}{2} \pm \frac{1}{2}$

$\frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$ and $\frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

Solution set: $\{3, 4\}$

39. $2x^2 - x - 28 = 0$

$x^2 - \frac{1}{2}x - 14 = 0$ Multiply by $\frac{1}{2}$.

$x^2 - \frac{1}{2}x + \frac{1}{16} = 14 + \frac{1}{16}$

Note: $\left[\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\right]^2 = \left(-\frac{1}{4}\right)^2 = \frac{1}{16}$

$\left(x - \frac{1}{4}\right)^2 = \frac{225}{16}$

$x - \frac{1}{4} = \pm\sqrt{\frac{225}{16}}$

$x - \frac{1}{4} = \pm\frac{15}{4}$

$x = \frac{1}{4} \pm \frac{15}{4}$

$\frac{1}{4} - \frac{15}{4} = \frac{-14}{4} = -\frac{7}{2}$ and $\frac{1}{4} + \frac{15}{4} = \frac{16}{4} = 4$

Solution set: $\left\{-\frac{7}{2}, 4\right\}$

40. $4x^2 - 3x - 10 = 0$

$x^2 - \frac{3}{4}x - \frac{10}{4} = 0$

$x^2 - \frac{3}{4}x - \frac{5}{2} = 0$

$x^2 - \frac{3}{4}x + \frac{9}{64} = \frac{5}{2} + \frac{9}{64}$

Note: $\left[\frac{1}{2}\cdot\left(-\frac{3}{4}\right)\right]^2 = \left(-\frac{3}{8}\right)^2 = \frac{9}{64}$

$\left(x - \frac{3}{8}\right)^2 = \frac{169}{64}$

$x - \frac{3}{8} = \pm\sqrt{\frac{169}{64}} = \pm\frac{13}{8}$

$x = \frac{3}{8} \pm \frac{13}{8}$

$\frac{3}{8} - \frac{13}{8} = \frac{-10}{8} = -\frac{5}{4}$ and $\frac{3}{8} + \frac{13}{8} = \frac{16}{8} = 2$

Solution set: $\left\{-\frac{5}{4}, 2\right\}$

41. $x^2 - 2x - 2 = 0$

$x^2 - 2x + 1 = 2 + 1$

Note: $\left[\frac{1}{2}\cdot(-2)\right]^2 = (-1)^2 = 1$

$(x-1)^2 = 3$

$x - 1 = \pm\sqrt{3}$

$x = 1 \pm \sqrt{3}$

Solution set: $\left\{1 \pm \sqrt{3}\right\}$

42. $x^2 - 10x + 18 = 0$

$x^2 - 10x + 25 = -18 + 25$

Note: $\left[\frac{1}{2}\cdot(-10)\right]^2 = (-5)^2 = 25$

$(x-5)^2 = 7$

$x - 5 = \pm\sqrt{7}$

$x = 5 \pm \sqrt{7}$

Solution set: $\left\{5 \pm \sqrt{7}\right\}$

43. $2x^2 + x = 10$

$x^2 + \frac{1}{2}x = 5$

$x^2 + \frac{1}{2}x + \frac{1}{16} = 5 + \frac{1}{16}$

Note: $\left[\frac{1}{2}\cdot\frac{1}{2}\right]^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\left(x + \frac{1}{4}\right)^2 = \frac{81}{16}$

$x + \frac{1}{4} = \pm\sqrt{\frac{81}{16}}$

$x + \frac{1}{4} = \pm\frac{9}{4}$

$x = -\frac{1}{4} \pm \frac{9}{4}$

$-\frac{1}{4} - \frac{9}{4} = \frac{-10}{4} = -\frac{5}{2}$ and $-\frac{1}{4} + \frac{9}{4} = \frac{8}{4} = 2$

Solution set: $\left\{-\frac{5}{2}, 2\right\}$

44. $3x^2 + 2x = 5$

$x^2 + \frac{2}{3}x = \frac{5}{3}$

$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{5}{3} + \frac{1}{9}$

Note: $\left[\frac{1}{2}\cdot\frac{2}{3}\right]^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\left(x + \frac{1}{3}\right)^2 = \frac{16}{9}$

$x + \frac{1}{3} = \pm\sqrt{\frac{16}{9}}$

$x + \frac{1}{3} = \pm\frac{4}{3}$

$x = -\frac{1}{3} \pm \frac{4}{3}$

$-\frac{1}{3} - \frac{4}{3} = -\frac{5}{3}$ and $-\frac{1}{3} + \frac{4}{3} = \frac{3}{3} = 1$

Solution set: $\left\{-\frac{5}{3}, 1\right\}$

45. $-2x^2 + 4x + 3 = 0$

$x^2 - 2x - \frac{3}{2} = 0$

$x^2 - 2x + 1 = \frac{3}{2} + 1$

Note: $\left[\frac{1}{2}\cdot(-2)\right]^2 = (-1)^2 = 1$

$(x-1)^2 = \frac{5}{2}$

$x - 1 = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{10}}{2}$

$x = 1 \pm \frac{\sqrt{10}}{2} = \frac{2\pm\sqrt{10}}{2}$

Solution set: $\left\{\frac{2\pm\sqrt{10}}{2}\right\}$

46. $-3x^2 + 6x + 5 = 0$

$x^2 - 2x - \frac{5}{3} = 0$

$x^2 - 2x + 1 = \frac{5}{3} + 1$

Note: $\left[\frac{1}{2}\cdot(-2)\right]^2 = (-1)^2 = 1$

$(x-1)^2 = \frac{8}{3}$

$x - 1 = \pm\sqrt{\frac{8}{3}} = \pm\frac{\sqrt{24}}{3} = \pm\frac{2\sqrt{6}}{3}$

$x = 1 \pm \frac{2\sqrt{6}}{3} = \frac{3\pm2\sqrt{6}}{3}$

Solution set: $\left\{\frac{3\pm2\sqrt{6}}{3}\right\}$

47. $-4x^2+8x=7$

$x^2-2x=-\frac{7}{4}$

$x^2-2x+1=-\frac{7}{4}+1$ Note: $\left[\frac{1}{2}\cdot(-2)\right]^2=(-1)^2=1$

$(x-1)^2=\frac{-3}{4}$

$x-1=\pm\sqrt{\frac{-3}{4}}=\pm\frac{i\sqrt{3}}{2}$

$x=1\pm\frac{\sqrt{3}}{2}i$

Solution set: $\left\{1\pm\frac{\sqrt{3}}{2}i\right\}$

48. $-3x^2+9x=7$

$x^2-3x=-\frac{7}{3}$

$x^2-3x+\frac{9}{4}=-\frac{7}{3}+\frac{9}{4}=\frac{-28}{12}+\frac{27}{12}$

Note: $\left[\frac{1}{2}\cdot(-3)\right]^2=\left(-\frac{3}{2}\right)^2=\frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2=\frac{-1}{12}$

$x-\frac{3}{2}=\pm\sqrt{\frac{-1}{12}}=\pm\frac{i\sqrt{12}}{12}=\pm\frac{2\sqrt{3}}{12}i=\pm\frac{\sqrt{3}}{6}i$

$x=\frac{3}{2}\pm\frac{\sqrt{3}}{6}i$

Solution set: $\left\{\frac{3}{2}\pm\frac{\sqrt{3}}{6}i\right\}$

49. Francisco is incorrect because $c=0$ and the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, can be evaluated with $a=1,\ b=-8,$ and $c=0.$

50. Francesca is incorrect because $b=0$ and the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, can be evaluated with $a=1,\ b=0,$ and $c=-19.$

51. $x^2-x-1=0$

Let $a=1, b=-1,$ and $c=-1.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}$$

Solution set: $\left\{\frac{1\pm\sqrt{5}}{2}\right\}$

52. $x^2-3x-2=0$

Let $a=1, b=-3,$ and $c=-2.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-2)}}{2(1)}=\frac{3\pm\sqrt{9+8}}{2}=\frac{3\pm\sqrt{17}}{2}$$

Solution set: $\left\{\frac{3\pm\sqrt{17}}{2}\right\}$

53. $x^2-6x=-7$

$x^2-6x+7=0$

Let $a=1, b=-6,$ and $c=7.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}=\frac{6\pm\sqrt{36-28}}{2}=\frac{6\pm\sqrt{8}}{2}=\frac{6\pm2\sqrt{2}}{2}=3\pm\sqrt{2}$$

Solution set: $\left\{3\pm\sqrt{2}\right\}$

54. $x^2-4x=-1$

$x^2-4x+1=0$

Let $a=1, b=-4,$ and $c=1.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

Solution set: $\left\{2\pm\sqrt{3}\right\}$

55. $x^2=2x-5$

$x^2-2x+5=0$

Let $a=1, b=-2,$ and $c=5.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}=\frac{2\pm\sqrt{4-20}}{2}=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm4i}{2}=1\pm2i$$

Solution set: $\left\{1\pm2i\right\}$

56. $x^2 = 2x - 10$

$x^2 - 2x + 10 = 0$

Let $a = 1, b = -2,$ and $c = 10.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$$

Solution set: $\{1 \pm 3i\}$

57. $-4x^2 = -12x + 11$

$0 = 4x^2 - 12x + 11$

Let $a = 4, b = -12,$ and $c = 11.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(11)}}{2(4)}$$
$$= \frac{12 \pm \sqrt{144-176}}{8} = \frac{12 \pm \sqrt{-32}}{8}$$
$$= \frac{12 \pm 4i\sqrt{2}}{8} = \frac{12}{8} \pm \frac{4\sqrt{2}}{8}i = \frac{3}{2} \pm \frac{\sqrt{2}}{2}i$$

Solution set: $\left\{\frac{3}{2} \pm \frac{\sqrt{2}}{2}i\right\}$

58. $-6x^2 = 3x + 2$

$0 = 6x^2 + 3x + 2$

Let $a = 6, b = 3,$ and $c = 2.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(6)(2)}}{2(6)}$$
$$= \frac{-3 \pm \sqrt{9-48}}{12} = \frac{-3 \pm \sqrt{-39}}{12} = \frac{-3 \pm i\sqrt{39}}{12}$$
$$x = -\frac{3}{12} \pm \frac{\sqrt{39}}{12}i = -\frac{1}{4} \pm \frac{\sqrt{39}}{12}i$$

Solution set: $\left\{-\frac{1}{4} \pm \frac{\sqrt{39}}{12}i\right\}$

59. $\frac{1}{2}x^2 + \frac{1}{4}x - 3 = 0$

$4\left(\frac{1}{2}x^2 + \frac{1}{4}x - 3\right) = 4 \cdot 0$

$2x^2 + x - 12 = 0$

Let $a = 2, b = 1,$ and $c = -12.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-12)}}{2(2)}$$
$$= \frac{-1 \pm \sqrt{1+96}}{4} = \frac{-1 \pm \sqrt{97}}{4}$$

Solution set: $\left\{\frac{-1 \pm \sqrt{97}}{4}\right\}$

60. $\frac{2}{3}x^2 + \frac{1}{4}x = 3$

$12\left(\frac{2}{3}x^2 + \frac{1}{4}x\right) = 12 \cdot 3$

$8x^2 + 3x = 36$

$8x^2 + 3x - 36 = 0$

Let $a = 8, b = 3,$ and $c = -36.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(8)(-36)}}{2(8)} = \frac{-3 \pm \sqrt{9+1152}}{16}$$
$$= \frac{-3 \pm \sqrt{1161}}{16} = \frac{-3 \pm 3\sqrt{129}}{16}$$

Solution set: $\left\{\frac{-3 \pm 3\sqrt{129}}{16}\right\}$

61. $0.2x^2 + 0.4x - 0.3 = 0$

$10(0.2x^2 + 0.4x - 0.3) = 10 \cdot 0$

$2x^2 + 4x - 3 = 0$

Let $a = 2, b = 4,$ and $c = -3.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$
$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

Solution set: $\left\{\frac{-2 \pm \sqrt{10}}{2}\right\}$

62.
$$0.1x^2 - 0.1x = 0.3$$
$$10\left(0.1x^2 - 0.1x\right) = 10 \cdot 0.3$$
$$x^2 - x = 3$$
$$x^2 - x - 3 = 0$$
Let $a = 1, b = -1,$ and $c = -3.$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$$
Solution set: $\left\{\frac{1 \pm \sqrt{13}}{2}\right\}$

63. $(4x-1)(x+2) = 4x$
$$4x^2 + 7x - 2 = 4x \Rightarrow 4x^2 + 3x - 2 = 0$$
Let $a = 4, b = 3,$ and $c = -2.$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(4)(-2)}}{2(4)} = \frac{-3 \pm \sqrt{9+32}}{8} = \frac{-3 \pm \sqrt{41}}{8}$$
Solution set: $\left\{\frac{-3 \pm \sqrt{41}}{8}\right\}$

64. $(3x+2)(x-1) = 3x$
$$3x^2 - x - 2 = 3x$$
$$3x^2 - 4x - 2 = 0$$
Let $a = 3, b = -4,$ and $c = -2.$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)} = \frac{4 \pm \sqrt{16+24}}{6} = \frac{4 \pm \sqrt{40}}{6} = \frac{4 \pm 2\sqrt{10}}{6} = \frac{2 \pm \sqrt{10}}{3}$$
Solution set: $\left\{\frac{2 \pm \sqrt{10}}{3}\right\}$

65. $(x-9)(x-1) = -16$
$$x^2 - 10x + 9 = -16$$
$$x^2 - 10x + 25 = 0$$
Let $a = 1$, $b = -10$, and $c = 25$.
$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - (4)(1)(25)}}{2(1)} = \frac{10 \pm \sqrt{100-100}}{2} = \frac{10 \pm 0}{2} = 5$$
Solution set: $\{5\}$

66. Answers will vary. Multiplying the first equation by –1 produces the second equation, and thus, the equations are equivalent.
$$-2x^2 + 3x - 6 = 0 \Rightarrow -1\left(2x^2 - 3x + 6\right) = 0 \Rightarrow$$
$$2x^2 - 3x + 6 = 0$$
Therefore, the two equations have the same solution set.

67.
$$x^3 - 8 = 0$$
$$x^3 - 2^3 = 0$$
$$(x-2)\left(x^2 + 2x + 4\right) = 0$$
$x - 2 = 0 \Rightarrow x = 2$ or
$$x^2 + 2x + 4 = 0$$
$a = 1$, $b = 2$, and $c = 4$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm \sqrt{3}i$$
Solution set: $\left\{2, -1 \pm \sqrt{3}i\right\}$

68.
$$x^3 - 27 = 0$$
$$x^3 - 3^3 = 0$$
$$(x-3)\left(x^2 + 3x + 9\right) = 0$$
$x - 3 = 0 \Rightarrow x = 3$ or
$$x^2 + 3x + 9 = 0$$
$a = 1$, $b = 3$, and $c = 9$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)} = \frac{-3 \pm \sqrt{9-36}}{2} = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3i\sqrt{3}}{2} = -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$$
Solution set: $\left\{3, -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i\right\}$

69.
$$x^3+27=0$$
$$x^3+3^3=0$$
$$(x+3)(x^2-3x+9)=0$$
$x+3=0 \Rightarrow x=-3$ or
$x^2-3x+9=0$
$a=1$, $b=-3$, and $c=9$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}=\frac{3\pm\sqrt{9-36}}{2}$$
$$=\frac{3\pm\sqrt{-27}}{2}=\frac{3\pm 3i\sqrt{3}}{2}=\frac{3}{2}\pm\frac{3\sqrt{3}}{2}i$$
Solution set: $\left\{-3, \frac{3}{2}\pm\frac{3\sqrt{3}}{2}i\right\}$

70.
$$x^3+64=0$$
$$x^3+4^3=0$$
$$(x+4)(x^2-4x+16)=0$$
$x+4=0 \Rightarrow x=-4$
$x^2-4x+16=0$
$a=1$, $b=-4$, and $c=16$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(16)}}{2(1)}=\frac{4\pm\sqrt{16-64}}{2}$$
$$=\frac{4\pm\sqrt{-48}}{2}=\frac{4\pm 4i\sqrt{3}}{2}=2\pm 2i\sqrt{3}$$
Solution set: $\left\{-4, 2\pm 2i\sqrt{3}\right\}$

71. $s=\frac{1}{2}gt^2$
$$2s=2\left[\frac{1}{2}gt^2\right]\Rightarrow 2s=gt^2\Rightarrow\frac{2s}{g}=\frac{gt^2}{g}\Rightarrow$$
$$t^2=\frac{2s}{g}\Rightarrow t=\pm\sqrt{\frac{2s}{g}}=\frac{\pm\sqrt{2s}}{\sqrt{g}}\cdot\frac{\sqrt{g}}{\sqrt{g}}=\frac{\pm\sqrt{2sg}}{g}$$

72. $\mathcal{A}=\pi r^2$
$$\frac{\mathcal{A}}{\pi}=\frac{\pi r^2}{\pi}\Rightarrow r^2=\frac{\mathcal{A}}{\pi}\Rightarrow r=\pm\sqrt{\frac{\mathcal{A}}{\pi}}\Rightarrow$$
$$r=\frac{\pm\sqrt{\mathcal{A}}}{\sqrt{\pi}}\cdot\frac{\sqrt{\pi}}{\sqrt{\pi}}=\frac{\pm\sqrt{\mathcal{A}\pi}}{\pi}$$

73. $F=\frac{kMv^2}{r}$
$$rF=r\left[\frac{kMv^2}{r}\right]\Rightarrow Fr=kMv^2\Rightarrow$$
$$\frac{Fr}{kM}=\frac{kMv^2}{kM}\Rightarrow v^2=\frac{Fr}{kM}\Rightarrow v=\pm\sqrt{\frac{Fr}{kM}}\Rightarrow$$
$$v=\frac{\pm\sqrt{Fr}}{\sqrt{kM}}\cdot\frac{\sqrt{kM}}{\sqrt{kM}}=\frac{\pm\sqrt{FrkM}}{kM}$$

74. $E=\frac{e^2k}{2r}$
$$2Er=e^2k$$
$$\frac{2Er}{k}=e^2\Rightarrow\pm\sqrt{\frac{2Er}{k}}=\pm\frac{\sqrt{2Erk}}{k}=e$$

75.
$$r=r_0+\frac{1}{2}at^2$$
$$r-r_0=\frac{1}{2}at^2$$
$$2(r-r_0)=at^2$$
$$\frac{2(r-r_0)}{a}=t^2$$
$$\pm\sqrt{\frac{2(r-r_0)}{a}}=\frac{\pm\sqrt{2a(r-r_0)}}{a}=t$$

76.
$$s=s_0+gt^2+k$$
$$s-s_0-k=gt^2$$
$$\frac{s-s_0-k}{g}=\frac{gt^2}{g}\Rightarrow t^2=\frac{s-s_0-k}{g}$$
$$t=\pm\sqrt{\frac{s-s_0-k}{g}}=\frac{\pm\sqrt{s-s_0-k}}{\sqrt{g}}\cdot\frac{\sqrt{g}}{\sqrt{g}}$$
$$t=\frac{\pm\sqrt{(s-s_0-k)g}}{g}$$

77. $h=-16t^2+v_0t+s_0$
$$16t^2-v_0t+h-s_0=0$$
$$16t^2-v_0t+(h-s_0)=0 \qquad a=16,\ b=-v_0,\ c=h-s_0$$
$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-v_0)\pm\sqrt{(-v_0)^2-4(16)(h-s_0)}}{2(16)}$$
$$=\frac{v_0\pm\sqrt{v_0^2-64(h-s_0)}}{32}$$
$$=\frac{v_0\pm\sqrt{v_0^2-64h+64s_0}}{32}$$

78. $S = 2\pi rh + 2\pi r^2$

$0 = 2\pi r^2 + 2\pi rh - S$

$0 = (2\pi)r^2 + (2\pi h)r - S \quad a = 2\pi,\ b = 2\pi h,\ c = -S$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)} = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S}}{4\pi} = \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi S}}{4\pi} = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}$$

79. $4x^2 - 2xy + 3y^2 = 2$

$4x^2 - 2xy + 3y^2 - 2 = 0$

(a) Solve for *x* in terms of *y*.

$4x^2 - (2y)x + (3y^2 - 2) = 0$

$a = 4$, $b = -2y$, and $c = 3y^2 - 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2y) \pm \sqrt{(-2y)^2 - 4(4)(3y^2 - 2)}}{2(4)} = \frac{2y \pm \sqrt{4y^2 - 16(3y^2 - 2)}}{8} = \frac{2y \pm \sqrt{4y^2 - 48y^2 + 32}}{8}$$

$$= \frac{2y \pm \sqrt{32 - 44y^2}}{8} = \frac{2y \pm \sqrt{4(8 - 11y^2)}}{8} = \frac{2y \pm 2\sqrt{8 - 11y^2}}{8} = \frac{y \pm \sqrt{8 - 11y^2}}{4}$$

(b) Solve for *y* in terms of *x*.

$3y^2 - (2x)y + (4x^2 - 2) = 0$

$a = 3$, $b = -2x$, and $c = 4x^2 - 2$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(3)(4x^2 - 2)}}{2(3)} = \frac{2x \pm \sqrt{4x^2 - 12(4x^2 - 2)}}{6} = \frac{2x \pm \sqrt{4x^2 - 48x^2 + 24}}{6}$$

$$= \frac{2x \pm \sqrt{24 - 44x^2}}{6} = \frac{2x \pm \sqrt{4(6 - 11x^2)}}{6} = \frac{2x \pm 2\sqrt{6 - 11x^2}}{6} = \frac{x \pm \sqrt{6 - 11x^2}}{3}$$

80. $3y^2 + 4xy - 9x^2 = -1$

$-9x^2 + 4xy + 3y^2 + 1 = 0$

(a) Solve for *x* in terms of *y*.

$-9x^2 + (4y)x + (3y^2 + 1) = 0$

$a = -9$, $b = 4y$, and $c = 3y^2 + 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4y \pm \sqrt{(4y)^2 - 4(-9)(3y^2 + 1)}}{2(-9)} = \frac{-4y \pm \sqrt{16y^2 + 36(3y^2 + 1)}}{-18} = \frac{-4y \pm \sqrt{16y^2 + 108y^2 + 36}}{-18}$$

$$= \frac{-4y \pm \sqrt{124y^2 + 36}}{-18} = \frac{-4y \pm \sqrt{4(31y^2 + 9)}}{-18} = \frac{-4y \pm 2\sqrt{31y^2 + 9}}{-18} = \frac{-2y \pm \sqrt{31y^2 + 9}}{-9} = \frac{2y \pm \sqrt{31y^2 + 9}}{9}$$

(b) Solve for y in terms of x.

$$3y^2+(4x)y+(1-9x^2)=0$$

$a=3$, $b=4x$, and $c=1-9x^2$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-4x\pm\sqrt{(4x)^2-4(3)(1-9x^2)}}{2(3)}$$
$$=\frac{-4x\pm\sqrt{16x^2-12(1-9x^2)}}{6}$$
$$=\frac{-4x\pm\sqrt{16x^2-12+108x^2}}{6}$$
$$=\frac{-4x\pm\sqrt{124x^2-12}}{6}$$
$$=\frac{-4x\pm\sqrt{4(31x^2-3)}}{6}$$
$$=\frac{-4x\pm2\sqrt{31x^2-3}}{6}=\frac{-2x\pm\sqrt{31x^2-3}}{3}$$

81. $2x^2+4xy-3y^2=2$

$2x^2+4xy-3y^2-2=0$

a. Solve for x in terms of y.

$$2x^2+(4y)x+(-3y^2-2)=0$$

$a=2$, $b=4y$, and $c=-3y^2-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(4y)\pm\sqrt{(4y)^2-4(2)(-3y^2-2)}}{2(2)}$$
$$=\frac{-4y\pm\sqrt{16y^2-8(-3y^2-2)}}{4}$$
$$=\frac{-4y\pm\sqrt{16y^2+24y^2+16}}{4}$$
$$=\frac{-4y\pm\sqrt{40y^2+16}}{4}$$
$$=\frac{-4y\pm\sqrt{4(10y^2+4)}}{4}$$
$$=\frac{-4y\pm2\sqrt{10y^2+4}}{4}=\frac{-2y\pm\sqrt{10y^2+4}}{2}$$

b. Solve for y in terms of x.

$$-3y^2+(4x)y+(2x^2-2)=0$$

$a=-3$, $b=4x$, $c=2x^2-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(4x)\pm\sqrt{(4x)^2-4(-3)(2x^2-2)}}{2(-3)}$$
$$=\frac{-4x\pm\sqrt{16x^2+12(2x^2-2)}}{-6}$$
$$=\frac{-4x\pm\sqrt{16x^2+24x^2-24}}{-6}$$
$$=\frac{-4x\pm\sqrt{40x^2-24}}{-6}$$
$$=\frac{-4x\pm\sqrt{4(10x^2-6)}}{-6}$$
$$=\frac{-4x\pm2\sqrt{10x^2-6}}{-6}$$
$$=\frac{2x\pm\sqrt{10x^2-6}}{3}$$

82. $5x^2-6xy+2y^2=1$

$5x^2-6xy+2y^2-1=0$

a. Solve for x in terms of y.

$$5x^2-(6y)x+(2y^2-1)=0$$

$a=5$, $b=-6y$, and $c=2y^2-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6y)\pm\sqrt{(-6y)^2-4(5)(2y^2-1)}}{2(5)}$$
$$=\frac{6y\pm\sqrt{36y^2-20(2y^2-1)}}{10}$$
$$=\frac{6y\pm\sqrt{36y^2-40y^2+20}}{10}$$
$$=\frac{6y\pm\sqrt{20-4y^2}}{10}=\frac{6y\pm\sqrt{4(5-y^2)}}{10}$$
$$=\frac{6y\pm2\sqrt{5-y^2}}{10}=\frac{3y\pm\sqrt{5-y^2}}{5}$$

b. Solve for y in terms of x.

$2y^2-(6x)y+(5x^2-1)=0$

$a=2,\ b=-6x,\ c=5x^2-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-6x)\pm\sqrt{(-6x)^2-4(2)(5x^2-1)}}{2(2)}$$
$$=\frac{6x\pm\sqrt{36x^2-8(5x^2-1)}}{4}$$
$$=\frac{6x\pm\sqrt{36x^2-40x^2+8}}{4}$$
$$=\frac{6x\pm\sqrt{8-4x^2}}{4}=\frac{6x\pm\sqrt{4(2-x^2)}}{4}$$
$$=\frac{6x\pm2\sqrt{2-x^2}}{4}=\frac{3x\pm\sqrt{2-x^2}}{2}$$

83. $x^2-8x+16=0$
$a=1, b=-8$, and $c=16$
$b^2-4ac=(-8)^2-4(1)(16)=64-64=0$
One rational solution (a double solution)

84. $x^2+4x+4=0$
$a=1, b=4$, and $c=4$
$b^2-4ac=4^2-4(1)(4)=16-16=0$
One rational solution (a double solution)

85. $3x^2+5x+2=0$
$a=3, b=5$, and $c=2$
$b^2-4ac=5^2-4(3)(2)=25-24=1=1^2$
Two distinct rational solutions

86. $8x^2=-14x-3$
$8x^2+14x+3=0$
$a=8, b=14$, and $c=3$
$b^2-4ac=14^2-4(8)(3)$
$=196-96=100=10^2$
Two distinct rational solutions

87. $4x^2=-6x+3$
$4x^2+6x-3=0$
$a=4, b=6$, and $c=-3$
$b^2-4ac=6^2-4(4)(-3)=36+48=84$
Two distinct irrational solutions

88. $2x^2+4x+1=0$
$a=2, b=4$, and $c=1$
$b^2-4ac=4^2-4(2)(1)=16-8=8$
Two distinct irrational solutions

89. $9x^2+11x+4=0$
$a=9, b=11$, and $c=4$
$b^2-4ac=11^2-4(9)(4)=121-144=-23$
Two distinct nonreal complex solutions

90. $3x^2=4x-5$
$3x^2-4x+5=0$
$a=3, b=-4$, and $c=5$
$b^2-4ac=(-4)^2-4(3)(5)=16-60=-44$
Two distinct nonreal complex solutions

91. $8x^2-72=0$
$a=8, b=0$, and $c=-72$
$b^2-4ac=0^2-4(8)(-72)=2304=48^2$
Two distinct rational solutions

92. Answers will vary.
$\sqrt{2}x^2+5x-3\sqrt{2}=0$
$a=\sqrt{2}, b=5$, and $c=-3\sqrt{2}$
$b^2-4ac=5^2-4(\sqrt{2})(-3\sqrt{2})$
$=25+12\cdot2=25+24=49$
No, this does not contradict the discussion in this section because a condition that is placed on the quadratic equation is that it has integer coefficients in order to investigate the discriminant.

93. No, it is not possible for the solution set of a quadratic equation with integer coefficients to consist of a single irrational number. Additional responses will vary.

94. No, it is not possible for the solution set of a quadratic equation with real coefficients to consist of one real number and one nonreal complex number. Answers will vary.

In exercises 95−98, there are other possible answers.

95. $x=4$ or $x=5$
$x-4=0$ or $x-5=0$
$(x-4)(x-5)=0$
$x^2-5x-4x+20=0$
$x^2-9x+20=0$
$a=1, b=-9$, and $c=20$

96.
$$x=-3 \quad \text{or} \quad x=2$$
$$x+3=0 \quad \text{or} \quad x-2=0$$
$$(x+3)(x-2)=0$$
$$x^2-2x+3x-6=0$$
$$x^2+x-6=0$$
$a=1, b=1,$ and $c=-6$

97.
$$x=1+\sqrt{2} \quad \text{or} \quad x=1-\sqrt{2}$$
$$x-\left(1+\sqrt{2}\right)=0 \quad \text{or} \quad x-\left(1-\sqrt{2}\right)=0$$
$$\left[x-\left(1+\sqrt{2}\right)\right]\left[x-\left(1-\sqrt{2}\right)\right]=0$$
$$x^2-x\left(1-\sqrt{2}\right)-x\left(1+\sqrt{2}\right)+\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)=0$$
$$x^2-x+x\sqrt{2}-x-x\sqrt{2}+\left[1^2-\left(\sqrt{2}\right)^2\right]=0$$
$$x^2-2x+(1-2)=0$$
$$x^2-2x-1=0$$
$a=1, b=-2,$ and $c=-1$

98.
$$x=i \quad \text{or} \quad x=-i$$
$$x-i=0 \quad \text{or} \quad x+i=0$$
$$(x-i)(x+i)=0$$
$$x^2-i^2=0$$
$$x^2-(-1)=0 \Rightarrow x^2+1=0$$
$a=1, b=0,$ and $c=1$

Chapter 1 Quiz
(Sections 1.1−1.4)

1.
$$3(x-5)+2=1-(4+2x)$$
$$3x-15+2=1-4-2x$$
$$3x-13=-3-2x$$
$$5x-13=-3$$
$$5x=10 \Rightarrow x=2$$
Solution set $\{2\}$

2. **(a)**
$$4x-5=-2(3-2x)+3$$
$$4x-5=-6+4x+3$$
$$4x-5=4x-3$$
$$-5=-3$$
contradiction; solution set: $\varnothing$

(b)
$$5x-9=5(-2+x)+1$$
$$5x-9=-10+5x+1$$
$$5x-9=5x-9$$
identity; solution set: {all real numbers} or $(-\infty,\infty)$

(c)
$$5x-4=3(6-x)$$
$$5x-4=18-3x$$
$$8x-4=18$$
$$8x=22 \Rightarrow x=\tfrac{22}{8}=\tfrac{11}{4}$$
conditional equation; solution set: $\left\{\frac{11}{4}\right\}$

3.
$$ay+2x=y+5x$$
$$ay-3x=y$$
$$-3x=y-ay=y(1-a)$$
$$3x=y(a-1)$$
$$\frac{3x}{a-1}=y$$

4. Let x = the amount deposited at 2.5% interest. Then $2x$ = the amount depositied at 3.0% interest. The interest earned on x dollars at 2.5% is $0.025x$, and the interest earned on $2x$ at 3.0% is $(2x)(0.03)=0.06x$. The total earned is \$850, so we have
$$0.025x+0.06x=850$$
$$0.085x=850 \Rightarrow x=10{,}000$$
\$10,000 was invested at 2.5%, and \$20,000 was invested at 3.0%.

5. Substitute 2008 for x in the equation:
$$y=0.128(2008)-250.43\approx 6.59$$
So, the model predicts that the minimum hourly wage for 2008 was \$6.59. The model predicts a wage that is \$0.04 greater than the actual wage.

6.
$$\frac{-4+\sqrt{-24}}{8}=\frac{-4+\sqrt{-4\cdot 6}}{8}=\frac{-4}{8}+\frac{2i\sqrt{6}}{8}=-\frac{1}{2}+\frac{\sqrt{6}}{4}i$$

7.
$$\frac{7-2i}{2+4i}=\frac{7-2i}{2+4i}\cdot\frac{2-4i}{2-4i}=\frac{14-28i-4i+(-8)}{4-(-16)}=\frac{6-32i}{20}=\frac{6}{20}-\frac{32}{20}i=\frac{3}{10}-\frac{8}{5}i$$

8. $3x^2-x=-1 \Rightarrow 3x^2-x+1=0$
Use the quadratic formula. $a=3, b=-1, c=1$
$$\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(1)}}{2(3)}=\frac{1\pm\sqrt{-11}}{6}=\frac{1}{6}\pm\frac{\sqrt{11}}{6}i$$
Solution set: $\left\{\frac{1}{6}\pm\frac{\sqrt{11}}{6}i\right\}$

9. $x^2-29=0 \Rightarrow x^2=29 \Rightarrow x=\pm\sqrt{29}$
Solution set: $\{\pm 29\}$

10. $\mathcal{A} = \frac{1}{2}r^2\theta \Rightarrow 2\mathcal{A} = r^2\theta \Rightarrow \frac{2\mathcal{A}}{\theta} = r^2 \Rightarrow$
$$r = \pm\sqrt{\frac{2\mathcal{A}}{\theta}} = \pm\frac{\sqrt{2\mathcal{A}}\cdot\sqrt{\theta}}{\sqrt{\theta}\cdot\sqrt{\theta}} = \pm\frac{\sqrt{2\mathcal{A}\theta}}{\theta}$$

Section 1.5 Applications and Modeling with Quadratic Equations

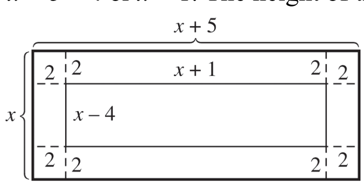
1. A. The length of the parking area is $2x + 200$, while the width is x, so the area is $(2x + 200)x$. Set the area equal to 40,000 to obtain $x(2x + 200) = 40{,}000$.

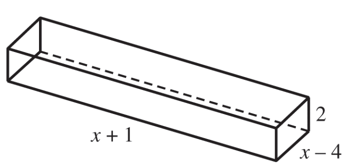
2. C. The diagonal of this rectangle is the hypotenuse of a right triangle with legs r feet and s feet. By the Pythagorean theorem, the length of the diagonal is $\sqrt{r^2 + s^2}$.

3. D. Use the Pythagorean theorem with $a = x$, $b = 2x - 2$, and $c = x + 4$.
$$x^2 + (2x-2)^2 = (x+4)^2$$

4. B. The length of the picture is $34 - 2x$, while the width is $21 - 2x$, giving an area of $(34 - 2x)(21 - 2x)$. Use the formula for the area of a rectangle, $A = lw$, and set the area equal to 600 to obtain
$$(34-2x)(21-2x) = 600.$$

5. A. Let x = the width, so $x + 5$ = the length. If 2 in. are cut from each corner, then the width of the box is $x - 4$ and the length of the box is $x + 5 - 4$ or $x + 1$. The height of the box is 2.





Then, the volume of the box is represented by $V = (x+1)(x-4)2 = 64$.

6. C. The height is given to be 40 ft and we are seeking t. Thus, $40 = -16t^2 + 60t$ is the correct equation.

7. B. We are seeking the height given the time $t = 2$ seconds. Therefore, the correct equation is $s = -16(2)^2 + 45(2)$.

8. C. The year 2010 corresponds to $x = 10$. We are seeking the value of the model for $x = 10$, so the correct equation is
$$S = 0.0538(10)^2 - 0.807(10) + 8.84.$$

9. Let x = the first integer. Then $x + 1$ = the next consecutive integer.
$$x(x+1) = 56 \Rightarrow x^2 + x = 56$$
$$x^2 + x - 56 = 0 \Rightarrow (x+8)(x-7) = 0$$
$$x + 8 = 0 \Rightarrow x = -8 \text{ or } x - 7 = 0 \Rightarrow x = 7$$
If $x = -8$, then $x + 1 = -7$. If $x = 7$, then $x + 1 = 8$. So the two integers are -8 and -7, or 7 and 8.

10. Let x = the first integer. Then $x + 1$ = the next consecutive integer.
$$x(x+1) = 110 \Rightarrow x^2 + x = 110 \Rightarrow$$
$$x^2 + x - 110 = 0 \Rightarrow (x+11)(x-10) = 0$$
$$x + 11 = 0 \Rightarrow x = -11 \text{ or}$$
$$x - 10 = 0 \Rightarrow x = 10$$
If $x = -11$, then $x + 1 = -10$. If $x = 10$, then $x + 1 = 11$. So the two integers are -11 and -10, or 10 and 11.

11. Let x = the first even integer. Then $x + 2$ = the next consecutive even integer.
$$x(x+2) = 168 \Rightarrow x^2 + 2x = 168$$
$$x^2 + 2x - 168 = 0 \Rightarrow (x+14)(x-12) = 0$$
$$x + 14 = 0 \Rightarrow x = -14 \text{ or}$$
$$x - 12 = 0 \Rightarrow x = 12$$
If $x = -14$, then $x + 2 = -12$. If $x = 12$, then $x + 2 = 14$. so, the two even integers are -14 and -12, or 12 and 14.

12. Let x = the first even integer. Then $x + 2$ = the next consecutive even integer.
$$x(x+2) = 224 \Rightarrow x^2 + 2x = 224 \Rightarrow$$
$$x^2 + 2x - 224 = 0 \Rightarrow (x+16)(x-14) = 0$$
$$x + 16 = 0 \Rightarrow x = -16 \text{ or}$$
$$x - 14 = 0 \Rightarrow x = 14$$
If $x = -16$, then $x + 2 = -14$. If $x = 14$, then $x + 2 = 16$. So, the two even integers are -16 and -14, or 14 and 16.

13. Let x = the first odd integer. Then $x + 2$ = the next consecutive odd integer.
$$x(x+2) = 63 \Rightarrow x^2 + 2x = 63 \Rightarrow$$
$$x^2 + 2x - 63 = 0 \Rightarrow (x+9)(x-7) = 0$$
$$x + 9 = 0 \Rightarrow x = -9 \text{ or}$$
$$x - 7 = 0 \Rightarrow x = 7$$
If $x = -9$, then $x + 2 = -7$. If $x = 7$, then $x + 2 = 9$. so, the two odd integers are -9 and -7, or 7 and 9.

14. Let x = the first odd integer. Then $x + 2$ = the next consecutive odd integer.

$$x(x+2)=143 \Rightarrow x^2+2x=143 \Rightarrow$$
$$x^2+2x-143=0 \Rightarrow (x+13)(x-11)=0$$
$$x+13=0 \Rightarrow x=-13 \text{ or}$$
$$x-11=0 \Rightarrow x=11$$

If $x = -13$, then $x + 2 = -11$. If $x = 11$, then $x + 2 = 13$. so, the two odd integers are -13 and -11, or 11 and 13.

15. Let x = the first odd integer. Then $x + 2$ = the next consecutive odd integer.

$$x^2+(x+2)^2=202$$
$$x^2+x^2+4x+4=202$$
$$2x^2+4x+4=202 \Rightarrow 2x^2+4x-198=0$$
$$2(x^2+2x-99)=0 \Rightarrow x^2+2x-99=0$$
$$(x+11)(x-9)=0$$
$$x+11=0 \Rightarrow x=-11 \text{ or}$$
$$x-9=0 \Rightarrow x=9$$

If $x = -11$, then $x + 2 = -9$. If $x = 9$, then $x + 2 = 11$. So the two integers are -11 and -9, or 9 and 11.

16. Let x = the first even integer. Then $x + 2$ = the next consecutive even integer.

$$x^2+(x+2)^2=52$$
$$x^2+x^2+4x+4=52 \Rightarrow 2x^2+4x+4=52 \Rightarrow$$
$$2x^2+4x-48=0 \Rightarrow 2(x^2+2x-24)=0 \Rightarrow$$
$$x^2+2x-24=0 \Rightarrow (x+6)(x-4)=0$$
$$x+6=0 \Rightarrow x=-6 \text{ or}$$
$$x-4=0 \Rightarrow x=4$$

If $x = -6$, then $x + 2 = -4$. If $x = 4$, then $x + 2 = 6$. So the two even integers are -6 and -4, or 4 and 6.

17. Let x = the first even integer. Then $x + 2$ = the next consecutive even integer.

$$(x+2)^2-x^2=84$$
$$x^2+4x+4-x^2=84 \Rightarrow 4x+4=84 \Rightarrow$$
$$4x=80 \Rightarrow x=20$$

If $x = 20$, then $x + 2 = 22$. The two integers are 20 and 22.

18. Let x = the first odd integer. Then $x + 2$ = the next consecutive odd integer.

$$(x+2)^2-x^2=32$$
$$x^2+4x+4-x^2=32 \Rightarrow 4x+4=32$$
$$4x=28 \Rightarrow x=7$$

If $x = 7$, then $x + 2 = 9$. So the two integers are 7 and 9.

19. Let x = the length of one leg, $x + 2$ = the length of the other leg, and $x + 4$ = the length of the hypotenuse. (Remember that the hypotenuse is the longest side in a right triangle.) The Pythagorean theorem gives

$$x^2+(x+2)^2=(x+4)^2$$
$$x^2+x^2+4x+4=x^2+8x+16$$
$$x^2-4x-12=0 \Rightarrow (x-6)(x+2)=0$$
$$x-6=0 \Rightarrow x=6 \text{ or}$$
$$x+2=0 \Rightarrow x=-2$$

Length cannot be negative, so reject that solution. If $x = 6$, then $x + 2 = 8$ and $x + 4 = 10$. The sides of the right triangle are 6, 8, and 10.

20. Let x = the length of one leg, $x + 1$ = the length of the other leg, and $x + 2$ = the length of the hypotenuse. (Remember that the hypotenuse is the longest side in a right triangle.) The Pythagorean theorem gives

$$x^2+(x+1)^2=(x+2)^2$$
$$x^2+x^2+2x+1=x^2+4x+4$$
$$2x^2+2x+1=x^2+4x+4$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x-3=0 \Rightarrow x=3 \text{ or}$$
$$x+1=0 \Rightarrow x=-1$$

Length must be a positive number, so reject $x = -1$. If $x = 3$, then $x + 1 = 4$ and $x + 2 = 5$. The sides of the right triangle are 3, 4, and 5.

21. Let x = the length of the side of the smaller square. Then $x + 3$ = the length of the side of the larger square.

$$(x+3)^2+x^2=149$$
$$x^2+6x+9+x^2=149 \Rightarrow 2x^2+6x-140=0$$
$$x^2+3x-70=0 \Rightarrow (x-7)(x+10)=0$$
$$x-7=0 \Rightarrow x=7 \text{ or}$$
$$x+10=0 \Rightarrow x=-10$$

Length cannot be negative, so reject that solution. If $x = 7$, then $x + 3 = 10$. The length of the side of smaller square is 7 in., and the length of the side of the larger square is 10 in.

22. Let x = the length of the side of the smaller square. Then $x + 5$ = the length of the side of the larger square.

$$(x+5)^2-x^2=95$$
$$x^2+10x+25-x^2=95$$
$$10x+25=95 \Rightarrow 10x=70 \Rightarrow x=7$$

If $x = 7$, then $x + 5 = 12$. The length of the side of the smaller square is 7 in., and the length of the side of the larger square is 12 in.

23. Use the figure and equation A from Exercise 1.

$$x(2x+200)=40,000$$
$$2x^2+200x=40,000$$
$$2x^2+200x-40,000=0$$
$$x^2+100x-20,000=0$$
$$(x-100)(x+200)=0$$
$$x=100 \text{ or } x=-200$$

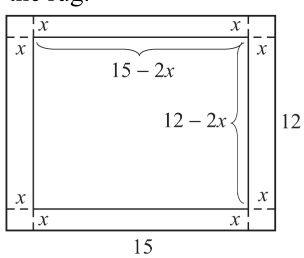
The negative solution is not meaningful. If $x = 100$, then $2x + 200 = 400$. The dimensions of the lot are 100 yd by 400 yd.

24. Use the formula for the area of a rectangle.

$$A=lw$$
$$5000=(150-x)x$$
$$5000=150x-x^2$$
$$x^2-150x+5000=0 \Rightarrow (x-100)(x-50)=0$$
$$x-100=0 \Rightarrow x=100 \text{ or}$$
$$x-50=0 \Rightarrow x=50$$

If $x = 100$, then $150 - x = 50$. If $x = 50$, then $150 - x = 100$. The dimensions of the garden are 50 m by 100 m.

25. Let x = the width of the strip of floor around the rug.

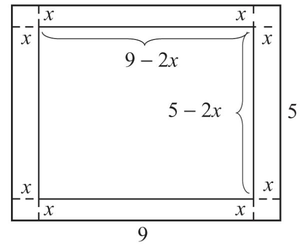


The dimensions of the carpet are $15 - 2x$ by $12 - 2x$. Because $A = lw$, the equation for the carpet area is $(15 - 2x)(12 - 2x) = 108$. Put this equation in standard form and solve by factoring.

$$(15-2x)(12-2x)=108$$
$$180-30x-24x+4x^2=108$$
$$180-54x+4x^2=108$$
$$4x^2-54x+72=0$$
$$2x^2-27x+36=0$$
$$(2x-3)(x-12)=0$$
$$2x-3=0 \Rightarrow x=\frac{3}{2}$$
$$x-12=0 \Rightarrow x=12$$

The solutions of the quadratic equation are $\frac{3}{2}$ and 12. We eliminate 12 as meaningless in this problem. If $x = \frac{3}{2}$, then $15 - 2x = 12$ and $12 - 2x = 9$. The dimensions of the carpet are 9 ft by 12 ft.

26. Let x = the width of the border.

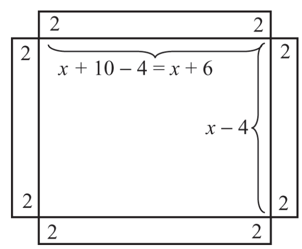


The dimensions of the center plot are $9 - 2x$ by $5 - 2x$ The total area is $5 \cdot 9 = 45$ sq ft. The border area is 24 sq ft, so the area of the center plot is $45 - 24 = 21$ sq ft. Apply the formula for the area of a rectangle to the center plot.

$$A=lw$$
$$(9-2x)(5-2x)=21$$
$$45-18x-10x+4x^2=21$$
$$45-28x+4x^2=21$$
$$4x^2-28x+24=0 \Rightarrow x^2-7x+6=0$$
$$(x-6)(x-1)=0 \Rightarrow x-6=0 \Rightarrow x=6 \text{ or}$$
$$x-1=0 \Rightarrow x=1$$

The solutions are 1 and 6. We eliminate 6 as meaningless in this problem. The border can be 1 ft wide.

27. Let x = the width of the metal. The dimensions of the base of the box are $x - 4$ by $x + 6$.

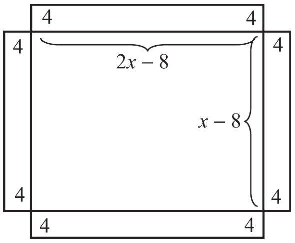


Because the formula for the volume of a box is $V = lwh$, we have

$$(x+6)(x-4)(2)=832$$
$$(x+6)(x-4)=416$$
$$x^2-4x+6x-24=416$$
$$x^2+2x-24=416$$
$$x^2+2x-440=0 \Rightarrow (x+22)(x-20)=0$$
$$x+22=0 \Rightarrow x=-22 \text{ or}$$
$$x-20=0 \Rightarrow x=20$$

The negative solution is not meaningful. If $x = 20$, then $x + 10 = 30$. The dimensions of the sheet of metal are 20 in by 30 in.

28. Let x = the width of the metal. The dimensions of the base of the box are $x-8$ by $2x-8$.



Because the formula for the volume of a box is $V = lwh$, we have

$$(2x-8)(x-8)(4) = 1536$$
$$(2x-8)(x-8) = 384$$
$$2x^2 - 16x - 8x + 64 = 384$$
$$2x^2 - 24x + 64 = 384$$
$$2x^2 - 24x - 320 = 0$$
$$x^2 - 12x - 160 = 0 \Rightarrow (x-20)(x+8) = 0$$
$$x - 20 = 0 \Rightarrow x = 20 \text{ or}$$
$$x + 8 = 0 \Rightarrow x = -8$$

The negative solution is not meaningful. If $x = 20$, then $2x = 40$. The dimensions of the sheet of metal are 20 in by 40 in.

29. Let h = height and r = radius.
Surface area = $2\pi rh + 2\pi r^2$
$8\pi = 2\pi r(3) + 2\pi r^2$

$$8\pi = 6\pi r + 2\pi r^2$$
$$0 = 2\pi r^2 + 6\pi r - 8\pi$$
$$0 = 2\pi\left(r^2 + 3r - 4\right) \Rightarrow 0 = (r+4)(r-1)$$

$r + 4 = 0 \Rightarrow r = -4$ or $r - 1 = 0 \Rightarrow r = 1$
The r represents the radius of a cylinder, so −4 is not reasonable. The radius of the circular top is 1 ft.

30. Let h = height and r = radius. Volume = $\pi r^2 h$

$$\pi r = \pi r^2(3) \Rightarrow r = 3r^2 \Rightarrow 0 = 3r^2 - r \Rightarrow$$
$$0 = r(3r-1) \Rightarrow r = 0 \text{ or } 3r - 1 = 0 \Rightarrow r = \tfrac{1}{3}$$

A circle must have a radius greater than 0. The radius of the circular top is $\frac{1}{3}$ ft or 4 in.

31. Let x = length of side of square. Area = x^2 and perimeter = $4x$

$$x^2 = 4x \Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow$$

$x = 0$ or $x = 4$
We reject 0 because x must be greater than 0. The side of the square measures 4 units.

32. Let x = width of rectangle.
Then $2x$ = length of rectangle.
Area = lw and Perimeter = $2l + 2w$

$$(2x)(x) = 2\left[2(2x) + 2x\right]$$
$$2x^2 = 2(4x + 2x) \Rightarrow 2x^2 = 2(6x)$$
$$2x^2 = 12x \Rightarrow 2x^2 - 12x = 0$$
$$2x(x-6) = 0 \Rightarrow 2x = 0 \Rightarrow x = 0 \text{ or}$$
$$x - 6 = 0 \Rightarrow x = 6$$

We reject 0 because x must be greater than 0. The width of the rectangle measures 6 units. The length of the rectangle measures 12 units.

33. Let h = height and r = radius.
Area of side = $2\pi rh$ and Area of circle = πr^2
Surface area = area of side + area of top + area of bottom
Surface area = $2\pi rh + \pi r^2 + \pi r^2 = 2\pi rh + 2\pi r^2$
$371 = 2\pi r(12) + 2\pi r^2$

$$371 = 24\pi r + 2\pi r^2$$
$$0 = 2\pi r^2 + 24\pi r - 371$$

$a = 2\pi$, $b = 24\pi$, and $c = -371$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} r$$
$$= \frac{-24\pi \pm \sqrt{(24\pi)^2 - 4(2\pi)(-371)}}{2(2\pi)}$$
$$= \frac{-24\pi \pm \sqrt{576\pi^2 + 2968\pi}}{4\pi}$$

$r \approx -15.75$ *or* $r \approx 3.75$
The negative solution is not meaningful. The radius of the circular top is approximately 3.75 cm.

34. Let x = height, then $x - 3.2$ = length, and 2.3 = width. $V = lwx$

$$180.4 = (x - 3.2)(2.3)x$$
$$180.4 = 2.3x^2 - 7.36x$$
$$0 = 2.3x^2 - 7.36x - 180.4$$

$a = 2.3$, $b = -7.36$, and $c = -180.4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7.36) \pm \sqrt{(-7.36)^2 - 4(2.3)(-180.4)}}{2(2.3)}$$
$$= \frac{7.36 \pm \sqrt{1713.8496}}{4.6} \approx \frac{7.36 \pm 41.40}{4.6}$$
$$\approx 10.6 \text{ or } -7.4$$

A box cannot have a negative height, so reject –7.4 as a solution. The height is about 10.6 in. and the length is 10.6 – 3.2 = 7.4 in.

35. Let h = the height of the dock.
Then $2h + 3$ = the length of the rope from the boat to the top of the dock.
Apply the Pythagorean theorem to the triangle shown in the text.

$$h^2 + 12^2 = (2h+3)^2$$
$$h^2 + 144 = (2h)^2 + 2(6h) + 3^2$$
$$h^2 + 144 = 4h^2 + 12h + 9$$
$$0 = 3h^2 + 12h - 135$$
$$0 = h^2 + 4h - 45 \Rightarrow 0 = (h+9)(h-5)$$
$$h + 9 = 0 \Rightarrow h = -9 \text{ or } h - 5 = 0 \Rightarrow h = 5$$

The negative solution is not meaningful. The height of the dock is 5 ft.

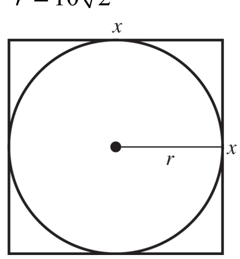
36. Let x = the horizontal distance
Apply the Pythagorean theorem to the right triangle shown in the text.

$$a^2 + b^2 = c^2$$
$$x^2 + (x+10)^2 = 50^2$$
$$x^2 + x^2 + 2(10x) + 10^2 = 2500$$
$$x^2 + x^2 + 20x + 100 = 2500$$
$$2x^2 + 20x - 2400 = 0$$
$$x^2 + 10x - 1200 = 0$$
$$(x+40)(x-30) = 0$$
$$x + 40 = 0 \Rightarrow x = -40 \text{ or}$$
$$x - 30 = 0 \Rightarrow x = 30$$

The negative solution is not meaningful.
The kite's horizontal distance is 30 ft and the vertical distance from the ground is 40 ft + 5 ft = 45 ft.

37. Let r = radius of circle and x = length of side of square. The radius is $\frac{1}{2}$ the length of the side of the square. Area = x^2

$$800 = x^2 \Rightarrow x = \sqrt{800} = 20\sqrt{2} \Rightarrow$$
$$r = 10\sqrt{2}$$



The radius is $10\sqrt{2}$ feet.

38. Let x = length of short leg.
Then $2x$ = length of long leg.
Apply the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$26^2 = x^2 + (2x)^2$$
$$676 = x^2 + 4x^2$$
$$676 = 5x^2$$
$$135.2 = x^2$$
$$\pm\sqrt{135.2} = x$$

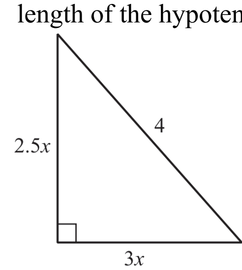
The negative solution is not meaningful. The short leg should be $\sqrt{135.2} \approx 11.6$ in. and the long leg should be $2\sqrt{135.2} \approx 23.3$ in.

39. Let x = length of ladder
Distance from building to ladder = 8 + 2 = 10.
Distance from ground to window = 13
Apply the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$10^2 + 13^2 = x^2 \Rightarrow 100 + 169 = x^2 \Rightarrow$$
$$269 = x^2 \Rightarrow \pm\sqrt{269} = x$$
$$x \approx -16.4 \text{ or } x \approx 16.4$$

The negative solution is not meaningful. The worker will need a 16.4-ft ladder.

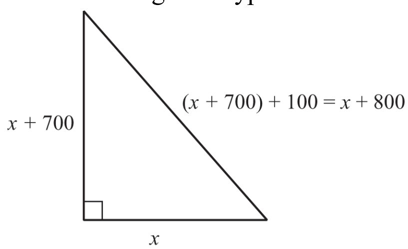
40. Let x = the number of hours they can talk to each other on the walkie-talkies.
Use $d = rt$ to determine how far each boy walks in x hours. Then $2.5x$ = the number of miles Tanner walks north and $3x$ = the number of miles Sheldon walks east. This forms a right triangle with legs of length $2.5x$ and $3x$, and length of the hypotenuse is the distance between the boys. We want to find x when the length of the hypotenuse is 4 mi.



$$a^2 + b^2 = c^2$$
$$(2.5x)^2 + (3x)^2 = 4^2 \Rightarrow 6.25x^2 + 9x^2 = 16 \Rightarrow$$
$$15.25x^2 = 16 \Rightarrow x^2 \approx 1.049 \Rightarrow x \approx \pm 1.02$$

The negative solution is not meaningful.
$1.02 \text{ hr} = 1.02\,(60 \text{ min}) \approx 61 \text{ min}$
They will be able to talk for about 61 min.

41. Let x = length of short leg, $x + 700$ = length of long leg, and $x + 700 + 100$ or $x + 800$ = length of hypotenuse.

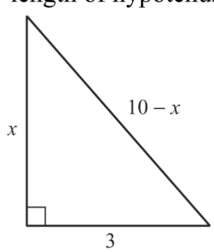


Apply the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$(x+800)^2 = x^2 + (x+700)^2$$
$$x^2 + 1600x + 640,000 = x^2 + x^2 + 1400x + 490,000$$
$$0 = x^2 - 200x - 150,000$$
$$0 = (x+300)(x-500)$$
$$x + 300 = 0 \Rightarrow x = -300 \text{ or}$$
$$x - 500 = 0 \Rightarrow x = 500$$

The negative solution is not meaningful.
500 = length of short leg
500 + 700 = 1200 = length of long leg
1200 + 100 = 1300 = length of hypotenuse
500 + 1200 + 1300 = 3000 = length of walkway. The total length is 3000 yd.

42. Let x = height of the break. Then $10 - x$ = the length of hypotenuse.



Apply the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$(10-x)^2 = x^2 + 3^2$$
$$100 - 20x + x^2 = x^2 + 9$$
$$100 - 20x = 9 \Rightarrow -20x = -91 \Rightarrow x = 4.55$$

The height of the break is 4.55 ft.

43. **(a)** $s = -16t^2 + v_0 t$
$$s = -16t^2 + 96t$$
$$80 = -16t^2 + 96t$$
$$16t^2 - 96t + 80 = 0$$
$a = 16$, $b = -96$ and $c = 80$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-96) \pm \sqrt{(-96)^2 - 4(16)(80)}}{2(16)}$$
$$t = \frac{96 \pm \sqrt{9216 - 5120}}{32} = \frac{96 \pm \sqrt{4096}}{32} = \frac{96 \pm 64}{32}$$
$$t = \frac{96 - 64}{32} = 1 \text{ or } t = \frac{96 + 64}{32} = 5$$

The projectile will reach 80 ft at 1 sec and 5 sec.

(b) $s = -16t^2 + 96t$
$$0 = -16t^2 + 96t$$
$$0 = -16t(t - 6)$$
$$-16t = 0 \Rightarrow t = 0 \quad \text{or} \quad t - 6 = 0 \Rightarrow t = 6$$

The projectile will return to the ground after 6 sec.

44. **(a)** $s = -16t^2 + v_0 t$
$$s = -16t^2 + 128t$$
$$80 = -16t^2 + 128t$$
$$16t^2 - 128t + 80 = 0$$
$$t^2 - 8t + 5 = 0$$
$a = 1$, $b = -8$ and $c = 5$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{64 - 20}}{2} = \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{2}$$
$$= 4 \pm \sqrt{11}$$
$$t = 4 - \sqrt{11} \approx 0.68 \text{ or}$$
$$t = 4 + \sqrt{11} \approx 7.32$$

The projectile will reach 80 ft at 0.68 sec and 7.32 sec.

(b) $s = -16t^2 + 128t$
$$0 = -16t^2 + 128t$$
$$0 = -16t(t - 8)$$
$$-16t = 0 \Rightarrow t = 0 \quad \text{or} \quad t - 8 = 0 \Rightarrow t = 8$$

The projectile will return to the ground after 8 sec.

45. **(a)** $s = -16t^2 + v_0 t$
$s = -16t^2 + 32t$
$80 = -16t^2 + 32t$
$16t^2 - 32t + 80 = 0$
$t^2 - 2t + 5 = 0$
$a = 1, b = -2$ and $c = 5$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$
The projectile will not reach 80 ft.

(b) $s = -16t^2 + 32t$
$0 = -16t^2 + 32t \Rightarrow 0 = -16t(t-2) \Rightarrow$
$-16t = 0 \Rightarrow t = 0$ or $t - 2 = 0 \Rightarrow t = 2$
The projectile will return to the ground after 2 sec.

46. **(a)** $s = -16t^2 + v_0 t$
$s = -16t^2 + 16t$
$80 = -16t^2 + 16t$
$16t^2 - 16t + 80 = 0 \Rightarrow t^2 - t + 5 = 0$
$a = 1, b = -1$ and $c = 5$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(5)}}{2(1)} = \frac{1 \pm \sqrt{1-20}}{2} = \frac{1 \pm \sqrt{-19}}{2} = \frac{1 \pm i\sqrt{19}}{2}$$
The projectile will not reach 80 ft.

(b) $s = -16t^2 + 16t$
$0 = -16t^2 + 16t \Rightarrow 0 = -16t(t-1) \Rightarrow$
$-16t = 0 \Rightarrow t = 0$ or $t - 1 = 0 \Rightarrow t = 1$
The projectile will return to the ground after 1 sec.

47. The height of the ball is given by
$s = -2.7t^2 + 30t + 6.5.$

(a) When the ball is 12 ft above the moon's surface, $s = 12$. Set $s = 12$ and solve for t.
$12 = -2.7t^2 + 30t + 6.5$
$2.7t^2 - 30t + 5.5 = 0$
Use the quadratic formula with $a = 2.7$, $b = -30$, and $c = 5.5$.
$$t = \frac{30 \pm \sqrt{900 - 4(2.7)(5.5)}}{2(2.7)} = \frac{30 \pm \sqrt{840.6}}{5.4}$$
$$\frac{30 + \sqrt{840.6}}{5.4} \approx 10.92 \text{ or } \frac{30 - \sqrt{840.6}}{5.4} \approx 0.19$$
Therefore, the ball reaches 12 ft first after 0.19 sec (on the way up), then again after 10.92 sec (on the way down).

(b) When the ball returns to the surface, $s = 0$.
$0 = -2.7t^2 + 30t + 6.5$
Use the quadratic formula with $a = -2.7$, $b = 30$, and $c = 6.5$.
$$t = \frac{-30 \pm \sqrt{900 - 4(-2.7)(6.5)}}{2(-2.7)} = \frac{-30 \pm \sqrt{970.2}}{-5.4}$$
$$\frac{-30 + \sqrt{970.2}}{-5.4} \approx -0.21 \text{ or}$$
$$\frac{-30 - \sqrt{970.2}}{-5.4} \approx 11.32$$
The negative solution is not meaningful. Therefore, the ball hits the moon's surface after 11.32 sec.

48. When the quadratic formula is applied to the equation $-2.7t^2 + 30t + 6.5 = 100 \Rightarrow$
$-2.7t^2 + 30t - 93.5 = 0,$ the discriminant,
$b^2 - 4ac = 30^2 - 4(-2.7)(-93.5)$
$= 900 - 1009.8 = -109.8$
is negative. Because this equation has no real solution, the ball will never reach a height of 100 ft.

49. **(a)** The year 2007 corresponds to $x = 13$.
$y = 0.2313x^2 + 2.600x + 35.17$
$y = 0.2313(13)^2 + 2.600(13) + 35.17$
≈ 108.0597
In 2007, the NFL salary cap was approximately \$108.1 million.

(b) We must solve for x when $y = 90$.
$90 = 0.2313x^2 + 2.600x + 35.17$
$0 = 0.2313x^2 + 2.600x - 54.83$
Use the quadratic formula with $a = 0.2313$, $b = 2.600$, and $c = -54.83$.

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(*continued*)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2.600 \pm \sqrt{2.600^2 - 4(0.2313)(-54.83)}}{2(0.2313)}$$

$$= \frac{-2.600 \pm \sqrt{57.4887}}{0.4626}$$

$$\approx 10.8,\ -22.0$$

The negative solution is not meaningful. Therefore, the salary cap reached 90 million dollars during 2004.

50. (a) $y = 0.0258x^2 - 1.30x + 23.3$

$$y = 0.0258(1)^2 - 1.30(1) + 23.3$$

$$\approx 22.0258$$

A player chosen first will earn about $22.0 million.

(b) $y = 0.0258x^2 - 1.30x + 23.3$

$$y = 0.0258(10)^2 - 1.30(10) + 23.3$$

$$\approx 12.88$$

A player chosen tenth will earn about $12.9 million.

51. (a) Let $x = 50$.

$$T = 0.00787(50)^2 - 1.528(50) + 75.89 \approx 19.2$$

The exposure time when $x = 50$ ppm is approximately 19.2 hr.

(b) Let $T = 3$ and solve for x.

$$3 = 0.00787x^2 - 1.528x + 75.89$$

$$0.00787x^2 - 1.528x + 72.89 = 0$$

Use the quadratic formula with $a = 0.00787$, $b = -1.528$, and $c = 72.89$.

$$x = \frac{-(-1.528) \pm \sqrt{(-1.528)^2 - 4(0.00787)(72.89)}}{2(0.00787)}$$

$$= \frac{1.528 \pm \sqrt{2.334784 - 2.2945772}}{0.01574}$$

$$= \frac{1.528 \pm \sqrt{0.0402068}}{0.01574}$$

$$\frac{1.528 + \sqrt{0.0402068}}{0.01574} \approx 109.8 \text{ or}$$

$$\frac{1.528 - \sqrt{0.0402068}}{0.01574} \approx 84.3$$

We reject the potential solution 109.8 because it is not in the interval [50, 100]. So, 84.3 ppm carbon monoxide concentration is necessary for a person to reach the 4% to 6% CoHb level in 3 hr.

52. (a) Let $x = 600$ and solve for T.

$$T = 0.0002x^2 - 0.316x + 127.9$$

$$= 0.0002(600)^2 - 0.316(600) + 127.9$$

$$= 10.3$$

The exposure time when $x = 600$ ppm is 10.3 hr.

(b) Let $T = 4$ and solve for x.

$$4 = 0.0002x^2 - 0.316x + 127.9$$

$$0.0002x^2 - 0.316x + 123.9 = 0$$

Use the quadratic formula with $a = 0.0002$, $b = -0.316$, and $c = 123.9$.

$$x = \frac{-(-0.316) \pm \sqrt{(-0.316)^2 - 4(0.0002)(123.9)}}{2(0.0002)}$$

$$= \frac{0.316 \pm \sqrt{0.099856 - 0.09912}}{0.0004}$$

$$= \frac{0.316 \pm \sqrt{0.000736}}{0.0004} \approx 857.8 \text{ or } 722.2$$

857.8 is not in the interval [500, 800]. A concentration of 722.2 ppm is required.

53. (a) 2014 is represented by $x = 6$. Substitute $x = 6$ into the equation to find y:

$$y = 0.0429x^2 - 9.73x + 606$$

$$y = 0.0429(6)^2 - 9.73(6) + 606$$

$$\approx 549.2 \text{ million tons}$$

In 2014, emissions were about 549.2 million tons.

(b) Let $y = 500$ and solve for x.

$$500 = 0.0429x^2 - 9.73x + 606$$

$$0.0429x^2 - 9.73x + 106 = 0$$

Use the quadratic formula with $a = 0.0429$, $b = -9.73$, and $c = 106$.

$$x = \frac{-(-9.73) \pm \sqrt{(-9.73)^2 - 4(0.0429)(106)}}{2(0.0429)}$$

$$= \frac{9.73 \pm \sqrt{76.4833}}{0.0858} \approx 11.5 \text{ or } 215.3$$

The model predicts that the emissions will reach 500 million tons about 11.5 years after 2008, which is during 2019.

54. Let $y = 8605$ and solve for x.

$8605 = 4.065x^2 + 370.1x + 3450 \Rightarrow$
$4.065x^2 + 370.1x - 5155 = 0$

Use the quadratic formula with $a = 4.065$, $b = 370.1$, and $c = -5155$.

$$\begin{aligned} x &= \frac{-370.1 \pm \sqrt{370.1^2 - 4(4.065)(-5155)}}{2(4.065)} \\ &= \frac{-370.1 \pm \sqrt{220,794.31}}{8.130} \\ &\approx \frac{-370.1 \pm 469.89}{8.130} \\ &\approx -103.3 \text{ or } 12.3 \end{aligned}$$

Reject −103.3 because it gives a year before 2000. Based on this model, the cost was \$8605 about 12.3 years after 2000 or in 2012.

55. The year 2010 is represented by $x = 3$.

$$\begin{aligned} y &= 710.55x^2 + 1333.7x + 32399 \\ y &= 710.55(3)^2 + 1333.7(3) + 32399 \\ &\approx 42,795 \end{aligned}$$

In 2010, the revenue from Internet publishing and broadcasting was about \$42,795 million.

56. The year 2012 is represented by $x = 5$.

$$\begin{aligned} y &= 23.09x^2 - 62.12x + 32.78 \\ &= 23.09(5)^2 - 62.12(5) + 32.78 \\ &\approx 299.43 \end{aligned}$$

According to the model, cable TV's top internet speed in 2012 was about 299.43 MBS.

57. For each \$20 increase in rent over \$300, one unit will remain vacant. Therefore, for x \$20 increases, x units will remain vacant. Therefore, the number of rented units will be $80 - x$.

58. x represents the number of \$20 increases in rent. Therefore, the rent will be $300 + 20x$ dollars.

59. $300 + 20x$ is the rent for each apartment, and $80 - x$ is the number of apartments that will be rented at that cost. The revenue generated will then be the product of $80 - x$ and $300 + 20x$, so the correct expression is

$$\begin{aligned} R &= (80 - x)(300 + 20x) \\ &= 24,000 + 1600x - 300x - 20x^2 \\ &= 24,000 + 1300x - 20x^2. \end{aligned}$$

60. Set the revenue equal to \$35,000. This gives the equation

$35,000 = 24,000 + 1300x - 20x^2$, where x represents the number of vacant apartments. Rewrite this equation in standard form and then solve.

$$\begin{aligned} 20x^2 - 1300x + 11,000 &= 0 \\ x^2 - 65x + 550 &= 0 \\ (x - 55)(x - 10) &= 0 \Rightarrow x = 55, 10 \end{aligned}$$

If $x = 55$, then only 25 apartments will be rented. This does not meet the restriction ,so we disregard this solution. If $x = 10$, then 70 apartments will be rented. This meets the restriction.

61. Let x = number of passengers in excess of 75. Then $225 – 5x$ = the cost per passenger (in dollars) and $75 + x$ = the number of passengers.

(Cost per passenger)(Number of passengers) = Revenue

$$\begin{aligned} (225 - 5x)(75 + x) &= 16,000 \\ 16,875 + 225x - 375x - 5x^2 &= 16,000 \\ 16,875 - 150x - 5x^2 &= 16,000 \\ 0 &= 5x^2 + 150x - 875 \\ 0 &= x^2 + 30x - 175 \Rightarrow 0 = (x + 35)(x - 5) \end{aligned}$$

$x + 35 = 0 \Rightarrow x = -35$ or $x - 5 = 0 \Rightarrow x = 5$

The negative solution is not meaningful. Because there are 5 passengers in excess of 75, the total number of passengers is 80.

62. Let x = the number of unsold seats. Then the number of passengers is $100 - x$. The revenue is given by $y = (40 + 2x)(100 - x)$.

$$\begin{aligned} 5950 &= (40 + 2x)(100 - x) \\ 5950 &= 4000 - 40x + 200x - 2x^2 \\ 0 &= -1950 + 160x - 2x^2 \\ 0 &= -2(x^2 - 80x + 975) \\ 0 &= (x - 15)(x - 65) \end{aligned}$$

$x - 15 = 0 \Rightarrow x = 15$
$x - 65 = 0 \Rightarrow x = 65$

There must be no more than 50 unsold seats, so only $x = 15$ is valid. There are $100 - 15 = 85$ passengers.

63. Let x = number of weeks the manager should wait. Then $100 + 5x$ =number of pounds and $0.40 - 0.02x$ = cost per pound
(Cost per pound)(Number of pounds) = Revenue

$$\begin{aligned}(0.40-0.02x)(100+5x) &= 38.40\\ 40+2x-2x-0.1x^2 &= 38.40\\ 40-0.1x^2 &= 38.40\\ -0.1x^2 &= -1.6\\ -10\left(-0.1x^2\right) &= -10(-1.6)\\ x^2 = 16 &\Rightarrow x = \pm 4\end{aligned}$$

The negative solution is not meaningful. The farmer should wait 4 weeks to get an average revenue of \$38.40 per tree.

64. Let x = number of days the scouts should wait. Then $4-0.1x$ = the price the scouts will receive per hundred pounds, and $120+4x$ = the number of hundreds of pounds of cans the scouts can collect.
(Price per hundred pounds)(Number of pounds) = Revenue

$$\begin{aligned}(4-0.1x)(120+4x) &= 490\\ 480+16x-12x-0.4x^2 &= 490\\ 480+4x-0.4x^2 &= 490\\ 0 &= 0.4x^2-4x+10\\ 10\cdot 0 &= 10\left(0.4x^2-4x+10\right)\\ 0 &= 4x^2-40x+100\end{aligned}$$

$0 = x^2-10x+25$
$0 = (x-5)^2 \Rightarrow 5 = x$

The scouts should wait 5 days in order to receive \$490 for their cans.

Section 1.6 Other Types of Equations and Applications

1. A <u>rational equation</u> is an equation that has a rational expression for one or more terms.

2. Proposed solutions for which any denominator equals <u>0</u> are excluded from the solutions set of a rational equation.

3. If a job can be completed in 4 hours, then the rate of work is <u>1/4</u> of the job per hour.

4. When the power property is used to solve an equation, it is essential to check all proposed solutions in the <u>original equation</u>.

5. An equation such as $x^{3/2} = 8$ is an equation with a <u>rational exponent</u> because it contains a variable raised to an exponent that is a rational number.

6. B **7.** D

8. C **9.** E

10. A

11. $\dfrac{5}{2x+3}-\dfrac{1}{x-6}=0$
$2x+3\neq 0 \Rightarrow x\neq -\frac{3}{2}$ and $x-6\neq 0\Rightarrow x\neq 6$.

12. $\dfrac{2}{x+1}+\dfrac{3}{5x-2}=0$
$x+1\neq 0\Rightarrow x\neq -1$ and $5x-2\neq 0\Rightarrow x\neq \frac{2}{5}$

13. $\dfrac{3}{x-2}+\dfrac{1}{x+1}=\dfrac{3}{x^2-x-2}$
or $\dfrac{3}{x-2}+\dfrac{1}{x+1}=\dfrac{3}{(x-2)(x+1)}$
$x-2\neq 0\Rightarrow x\neq 2$ and $x+1\neq 0\Rightarrow x\neq -1$

14. $\dfrac{2}{x+3}-\dfrac{5}{x-1}=\dfrac{-5}{x^2+2x-3}$ or
$\dfrac{2}{x+3}-\dfrac{5}{x-1}=\dfrac{-5}{(x+3)(x-1)}$
$x+3\neq 0\Rightarrow x\neq -3$ and $x-1\neq 0\Rightarrow x\neq 1$

15. $\dfrac{1}{4x}-\dfrac{2}{x}=3$
$4x\neq 0\Rightarrow x\neq 0$

16. $\dfrac{5}{2x}+\dfrac{2}{x}=6$
$2x\neq 0\Rightarrow x\neq 0$

17. $\dfrac{2x+5}{2}-\dfrac{3x}{x-2}=x$
The least common denominator is $2(x-2)$, which is equal to 0 if $x = 2$. Therefore, 2 cannot possibly be a solution of this equation.

$$\begin{aligned}2(x-2)\left[\frac{2x+5}{2}-\frac{3x}{x-2}\right] &= 2(x-2)(x)\\ (x-2)(2x+5)-2(3x) &= 2x(x-2)\\ 2x^2+5x-4x-10-6x &= 2x^2-4x\\ -5x-10 = -4x &\Rightarrow -10 = x\end{aligned}$$

The restriction $x\neq 2$ does not affect the result. Therefore, the solution set is $\{-10\}$.

18. $\dfrac{4x+3}{4}-\dfrac{2x}{x+1}=x$
The least common denominator is $4(x+1)$, which is equal to 0 if $x = -1$. Therefore, –1 cannot possibly be a solution of this equation.

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(*continued*)

$$4(x+1)\left[\frac{4x+3}{4}-\frac{2x}{x+1}\right]=4(x+1)(x)$$
$$(x+1)(4x+3)-4(2x)=4x(x+1)$$
$$4x^2+3x+4x+3-8x=4x^2+4x$$
$$-x+3=4x$$
$$3=5x\Rightarrow\frac{3}{5}=x$$

The restriction $x\neq-1$ does not affect the result. Therefore, the solution set is $\left\{\frac{3}{5}\right\}$.

19. $\dfrac{x}{x-3}=\dfrac{3}{x-3}+3$

The least common denominator is $x-3$, which is equal to 0 if $x=3$. Therefore, 3 cannot possibly be a solution of this equation.

$$(x-3)\left(\frac{x}{x-3}\right)=(x-3)\left[\frac{3}{x-3}+3\right]$$
$$x=3+3(x-3)$$
$$x=3+3x-9$$
$$x=3x-6\Rightarrow-2x=-6\Rightarrow x=3$$

The only possible solution is 3. However, the variable is restricted to real numbers except 3. Therefore, the solution set is: $\varnothing$.

20. $\dfrac{x}{x-4}=\dfrac{4}{x-4}+4$

The least common denominator is $x-4$, which is equal to 0 if $x=4$. Therefore, 4 cannot possibly be a solution of this equation.

$$(x-4)\left(\frac{x}{x-4}\right)=(x-4)\left[\frac{4}{x-4}+4\right]$$
$$x=4+4(x-4)$$
$$x=4+4x-16$$
$$x=4x-12$$
$$-3x=-12\Rightarrow x=4$$

The only possible solution is 4. However, the variable is restricted to real numbers except 4. Therefore, the solution set is: $\varnothing$.

21. $\dfrac{-2}{x-3}+\dfrac{3}{x+3}=\dfrac{-12}{x^2-9}$ or
$\dfrac{-2}{x-3}+\dfrac{3}{x+3}=\dfrac{-12}{(x+3)(x-3)}$

The least common denominator is $(x+3)(x-3)$, which is equal to 0 if $x=-3$ or $x=3$. Therefore, –3 and 3 cannot possibly be solutions of this equation.

$$(x+3)(x-3)\left[\frac{-2}{x-3}+\frac{3}{x+3}\right]$$
$$=(x+3)(x-3)\left(\frac{-12}{(x+3)(x-3)}\right)$$
$$-2(x+3)+3(x-3)=-12$$
$$-2x-6+3x-9=-12$$
$$-15+x=-12\Rightarrow x=3$$

The only possible solution is 3. However, the variable is restricted to real numbers except –3 and 3. Therefore, the solution set is: $\varnothing$.

22. $\dfrac{3}{x-2}+\dfrac{1}{x+2}=\dfrac{12}{x^2-4}$ or
$\dfrac{3}{x-2}+\dfrac{1}{x+2}=\dfrac{12}{(x+2)(x-2)}$

The least common denominator is $(x+2)(x-2)$, which is equal to 0 if $x=-2$ or $x=2$. Therefore, -2 and 2 cannot possibly be solutions of this equation.

$$(x+2)(x-2)\left[\frac{3}{x-2}+\frac{1}{x+2}\right]$$
$$=(x+2)(x-2)\left(\frac{12}{(x+2)(x-2)}\right)$$
$$3(x+2)+(x-2)=12$$
$$3x+6+x-2=12$$
$$4x+4=12$$
$$4x=8\Rightarrow x=2$$

The only possible solution is 2. However, the variable is restricted to real numbers except -2 and 2. Therefore, the solution set is: $\varnothing$.

23. $\dfrac{4}{x^2+x-6}-\dfrac{1}{x^2-4}=\dfrac{2}{x^2+5x+6}$ or
$\dfrac{4}{(x+3)(x-2)}-\dfrac{1}{(x+2)(x-2)}=\dfrac{2}{(x+2)(x+3)}$

The least common denominator is $(x+3)(x-2)(x+2)$, which is equal to 0 if $x=-3$ or $x=2$ or $x=-2$. Therefore, -3 and 2 and -2 cannot possibly be solutions of this equation.

$$(x+3)(x-2)(x+2)$$
$$\cdot\left[\frac{4}{(x+3)(x-2)}-\frac{1}{(x+2)(x-2)}\right]$$
$$=(x+3)(x-2)(x+2)\left(\frac{2}{(x+2)(x+3)}\right)$$

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$4(x+2)-1(x+3)=2(x-2)$
$4x+8-x-3=2x-4$
$3x+5=2x-4 \Rightarrow x+5=-4 \Rightarrow x=-9$

The restrictions $x \neq -3$, $x \neq 2$, and $x \neq -2$ do not affect the result. Therefore, the solution set is $\{-9\}$.

24. $$\frac{3}{x^2+x-2}-\frac{1}{x^2-1}=\frac{7}{2x^2+6x+4}$$
$$\frac{3}{(x+2)(x-1)}-\frac{1}{(x+1)(x-1)}=\frac{7}{2(x^2+3x+2)}$$
$$\frac{3}{(x+2)(x-1)}-\frac{1}{(x+1)(x-1)}=\frac{7}{2(x+2)(x+1)}$$

The least common denominator is $2(x+1)(x-1)(x+2)$, which is equal to 0 if $x=-1$ or $x=1$ or $x=-2$. Therefore, –1 and 1 and –2 cannot possibly be solutions of this equation.

$$2(x+1)(x-1)(x+2) \cdot \left[\frac{3}{(x+2)(x-1)}-\frac{1}{(x+1)(x-1)}\right] = 2(x+1)(x-1)(x+2)\left(\frac{7}{2(x+2)(x+1)}\right)$$
$$2(3)(x+1)-2(x+2)=7(x-1)$$
$$6(x+1)-2(x+2)=7(x-1)$$
$$6x+6-2x-4=7x-7$$
$$4x+2=7x-7$$
$$2=3x-7$$
$$9=3x \Rightarrow 3=x$$

The restrictions $x \neq -1$, $x \neq 1$, and $x \neq -2$ do not affect the result. Therefore, the solution set is $\{3\}$.

25. $\frac{2x+1}{x-2}+\frac{3}{x}=\frac{-6}{x^2-2x}$ or
$$\frac{2x+1}{x-2}+\frac{3}{x}=\frac{-6}{x(x-2)}$$

Multiply each term in the equation by the least common denominator, $x(x-2)$, assuming $x \neq 0, 2$.

$$x(x-2)\left[\frac{2x+1}{x-2}+\frac{3}{x}\right]=x(x-2)\left(\frac{-6}{x(x-2)}\right)$$
$$x(2x+1)+3(x-2)=-6$$
$$2x^2+x+3x-6=-6$$
$$2x^2+4x-6=-6$$
$$2x^2+4x=0 \Rightarrow 2x(x+2)=0$$
$$2x=0 \Rightarrow x=0 \quad \text{or} \quad x+2=0 \Rightarrow x=-2$$

Because of the restriction $x \neq 0$, the only valid solution is −2. The solution set is $\{-2\}$.

26. $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^2+x}$ or $\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x(x+1)}$

Multiply each term in the equation by the least common denominator, $x(x+1)$, assuming $x \neq 0, -1$.

$$x(x+1)\left[\frac{4x+3}{x+1}+\frac{2}{x}\right]=x(x+1)\left(\frac{1}{x(x+1)}\right)$$
$$x(4x+3)+2(x+1)=1$$
$$4x^2+3x+2x+2=1$$
$$4x^2+5x+2=1 \Rightarrow 4x^2+5x+1=0$$
$$(4x+1)(x+1)=0$$
$$4x+1=0 \Rightarrow x=-\tfrac{1}{4} \quad \text{or} \quad x+1=0 \Rightarrow x=-1$$

Because of the restriction $x \neq -1$, the only valid solution is $-\frac{1}{4}$. The solution set is $\left\{-\frac{1}{4}\right\}$.

27. $\frac{x}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$ or
$$\frac{x}{x-1}-\frac{1}{x+1}=\frac{2}{(x+1)(x-1)}$$

Multiply each term in the equation by the least common denominator, $(x+1)(x-1)$, assuming $x \neq \pm 1$.

$$(x+1)(x-1)\left[\frac{x}{x-1}-\frac{1}{x+1}\right] = (x+1)(x-1)\left(\frac{2}{(x+1)(x-1)}\right)$$
$$x(x+1)-(x-1)=2 \Rightarrow x^2+x-x+1=2$$
$$x^2+1=2 \Rightarrow x^2-1=0$$
$$(x+1)(x-1)=0 \Rightarrow x+1=0 \Rightarrow x=-1 \text{ or}$$
$$x-1=0 \Rightarrow x=1$$

Because of the restriction $x \neq \pm 1$, the solution set is $\varnothing$.

28. $\frac{-x}{x+1}-\frac{1}{x-1}=\frac{-2}{x^2-1}$ or

$\frac{-x}{x+1}-\frac{1}{x-1}=\frac{-2}{(x+1)(x-1)}$

Multiply each term in the equation by the least common denominator, $(x+1)(x-1)$, assuming $x \neq \pm 1$.

$$(x+1)(x-1)\left[\frac{-x}{x+1}-\frac{1}{x-1}\right] = (x+1)(x-1)\left(\frac{-2}{(x+1)(x-1)}\right)$$

$$-x(x-1)-(x+1)=-2$$

$$-x^2+x-x-1=-2 \Rightarrow -x^2-1=-2 \Rightarrow$$

$$-x^2+1=0 \Rightarrow x^2-1=0 \Rightarrow$$

$$(x+1)(x-1)=0 \Rightarrow x+1=0 \Rightarrow x=-1 \text{ or}$$

$$x-1=0 \Rightarrow x=1$$

Because of the restriction $x \neq \pm 1$, the solution set is: $\varnothing$.

29. $\frac{5}{x^2}-\frac{43}{x}=18$

Multiply each term in the equation by the least common denominator, x^2, assuming $x \neq 0$.

$$x^2\left[\frac{5}{x^2}-\frac{43}{x}\right]=x^2(18)$$

$$5-43x=18x^2 \Rightarrow 0=18x^2+43x-5$$

$$0=(2x+5)(9x-1)$$

$$2x+5=0 \Rightarrow x=-\tfrac{5}{2} \text{ or}$$

$$9x-1=0 \Rightarrow x=\tfrac{1}{9}$$

The restriction $x \neq 0$ does not affect the result. Therefore, the solution set is $\left\{-\frac{5}{2}, \frac{1}{9}\right\}$.

30. $\frac{7}{x^2}+\frac{19}{x}=6$

Multiply each term in the equation by the least common denominator, x^2, assuming $x \neq 0$.

$$x^2\left[\frac{7}{x^2}+\frac{19}{x}\right]=x^2(6)$$

$$7+19x=6x^2$$

$$0=6x^2-19x-7$$

$$0=(3x+1)(2x-7)$$

$$3x+1=0 \Rightarrow x=-\tfrac{1}{3} \quad \text{or} \quad 2x-7=0 \Rightarrow x=\tfrac{7}{2}$$

The restriction $x \neq 0$ does not affect the result. Therefore, the solution set is $\left\{-\frac{1}{3}, \frac{7}{2}\right\}$.

31. $2=\frac{3}{2x-1}+\frac{-1}{(2x-1)^2}$

Multiply each term in the equation by the least common denominator, $(2x-1)^2$, assuming $x \neq \frac{1}{2}$.

$$(2x-1)^2(2)=(2x-1)^2\left[\frac{3}{2x-1}+\frac{-1}{(2x-1)^2}\right]$$

$$2\left(4x^2-4x+1\right)=3(2x-1)-1$$

$$8x^2-8x+2=6x-3-1$$

$$8x^2-8x+2=6x-4 \Rightarrow 8x^2-14x+6=0$$

$$2\left(4x^2-7x+3\right)=0 \Rightarrow 2(4x-3)(x-1)=0$$

$$4x-3=0 \Rightarrow x=\tfrac{3}{4} \text{ or } x-1=0 \Rightarrow x=1$$

The restriction $x \neq \frac{1}{2}$ does not affect the result. Therefore the solution set is $\left\{\frac{3}{4}, 1\right\}$.

32. $6=\frac{7}{2x-3}+\frac{3}{(2x-3)^2}$

Multiply each term in the equation by the least common denominator, $(2x-3)^2$, assuming $x \neq \frac{3}{2}$.

$$(2x-3)^2(6)=(2x-3)^2\left[\frac{7}{2x-3}+\frac{3}{(2x-3)^2}\right]$$

$$6\left(4x^2-12x+9\right)=7(2x-3)+3$$

$$24x^2-72x+54=14x-21+3$$

$$24x^2-72x+54=14x-18$$

$$24x^2-86x+72=0$$

$$2\left(12x^2-43x+36\right)=0 \Rightarrow 2(4x-9)(3x-4)=0$$

$$4x-9=0 \Rightarrow x=\tfrac{9}{4} \quad \text{or} \quad 3x-4=0 \Rightarrow x=\tfrac{4}{3}$$

The restriction $x \neq \frac{3}{2}$ does not affect the result. Therefore, the solution set is $\left\{\frac{9}{4}, \frac{4}{3}\right\}$.

33. $\frac{2x-5}{x}=\frac{x-2}{3}$

Multiply each term in the equation by the least common denominator, $3x$, assuming $x \neq 0$.

$$3x\left(\frac{2x-5}{x}\right)=3x\left(\frac{x-2}{3}\right)$$

$$3(2x-5)=x(x-2) \Rightarrow 6x-15=x^2-2x \Rightarrow$$

$$0=x^2-8x+15=(x-3)(x-5)$$

$$x-3=0 \Rightarrow x=3 \quad \text{or} \quad x-5=0 \Rightarrow x=5$$

The restriction $x \neq 0$ does not affect the result. Therefore, the solution set is $\{3, 5\}$.

34. $\frac{x+4}{2x}=\frac{x-1}{3}$

Multiply each term in the equation by the least common denominator, $6x$, assuming $x\neq 0$.

$$6x\left(\frac{x+4}{2x}\right)=6x\left(\frac{x-1}{3}\right)$$
$$3(x+4)=2x(x-1)$$
$$3x+12=2x^2-2x$$
$$0=2x^2-5x-12$$
$$0=(2x+3)(x-4)$$

$2x+3=0\Rightarrow x=-\frac{3}{2}$ or $x-4=0\Rightarrow x=4$

The restriction $x\neq 0$ does not affect the result.

Therefore the solution set is $\left\{-\frac{3}{2},4\right\}$.

35. $\frac{2x}{x-2}=5+\frac{4x^2}{x-2}$

Multiply each term in the equation by the least common denominator, $x-2$, assuming $x\neq 2$.

$$(x-2)\left(\frac{2x}{x-2}\right)=(x-2)\left[5+\frac{4x^2}{x-2}\right]$$
$$2x=5(x-2)+4x^2$$
$$2x=5x-10+4x^2$$
$$0=4x^2+3x-10$$
$$0=(x+2)(4x-5)$$

$x+2=0\Rightarrow x=-2$ or $4x-5=0\Rightarrow x=\frac{5}{4}$

The restriction $x\neq 2$ does not affect the result.

Therefore the solution set is $\left\{-2,\frac{5}{4}\right\}$.

36. $\frac{3x^2}{x-1}+2=\frac{x}{x-1}$

Multiply each term in the equation by the least common denominator, $x-1$, assuming $x\neq 1$.

$$(x-1)\left(\frac{3x^2}{x-1}+2\right)=(x-1)\left(\frac{x}{x-1}\right)$$
$$3x^2+2(x-1)=x$$
$$3x^2+2x-2=x$$
$$3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$

$3x-2=0\Rightarrow x=\frac{2}{3}$ or $x+1=0\Rightarrow x=-1$

The restriction $x\neq 1$ does not affect the result.

Therefore, the solution set is $\left\{-1,\frac{2}{3}\right\}$.

37. Let x = the amount of time (in hours) it takes Joe and Sam to paint the house.

	r	t	Part of the Job Accomplished
Joe	$\frac{1}{3}$	x	$\frac{1}{3}x$
Sam	$\frac{1}{5}$	x	$\frac{1}{5}x$

Because Joe and Sam must accomplish 1 job (painting a house), we must solve the following equation.

$$\tfrac{1}{3}x+\tfrac{1}{5}x=1$$
$$15\left[\tfrac{1}{3}x+\tfrac{1}{5}x\right]=15\cdot 1$$
$$5x+3x=15\Rightarrow 8x=15\Rightarrow x=\frac{15}{8}=1\tfrac{7}{8}$$

It takes Joe and Sam $1\frac{7}{8}$ hr working together to paint the house.

38. Let x = the amount of time (in hours) it takes Joe and Sam to paint the house.

	Rate	Time	Part of the Job Accomplished
Joe	$\frac{1}{6}$	x	$\frac{1}{6}x$
Sam	$\frac{1}{8}$	x	$\frac{1}{8}x$

Because Joe and Sam must accomplish 1 job (painting a house), we must solve the following equation.

$$\tfrac{1}{6}x+\tfrac{1}{8}x=1$$
$$24\left[\tfrac{1}{6}x+\tfrac{1}{8}x\right]=24\cdot 1$$
$$4x+3x=24$$
$$7x=24\Rightarrow x=\frac{24}{7}=3\tfrac{3}{7}$$

It takes Joe and Sam $3\frac{3}{7}$ hr working together to paint the house.

39. Let x = the amount of time (in hours) it takes plant A to produce the pollutant. Then $2x$ = the amount of time (in hours) it takes plant B to produce the pollutant.

	Rate	Time	Part of the Job Accomplished
Pollution from A	$\frac{1}{x}$	26	$26\left(\frac{1}{x}\right)$
Pollution from B	$\frac{1}{2x}$	26	$26\left(\frac{1}{2x}\right)$

Because plant A and B accomplish 1 job (producing the pollutant), we must solve the following equation.

$$26\left(\tfrac{1}{x}\right)+26\left(\tfrac{1}{2x}\right)=1$$
$$\tfrac{26}{x}+\tfrac{13}{x}=1$$
$$x\left[\tfrac{39}{x}\right]=x\cdot 1 \Rightarrow 39=x$$

Plant B will take $2\cdot 39=78$ hr to produce the pollutant.

40. Let x = the amount of time (in hours) the first (faster) pipe operates. Then $3x$ = the second (slower) pipe operates.

	Rate	Time	Part of the Job Accomplished
First pipe	$\frac{1}{x}$	12	$\frac{12}{x}$
Second pipe	$\frac{1}{3x}$	12	$\frac{12}{3x}$

Because the two pipes are working together, we must solve the following equation.

$$\frac{12}{x}+\frac{12}{3x}=1\Rightarrow\frac{36}{3x}+\frac{12}{3x}=1\Rightarrow\frac{48}{3x}=1\Rightarrow$$
$$48=3x\Rightarrow 16=x$$

The faster pipe can fill the pond in 16 hours working alone.

41. Let x = the amount of time (in hours) to fill the pool with both pipes open.

	Rate	Time	Part of the Job Accomplished
Inlet pipe	$\frac{1}{5}$	x	$\frac{1}{5}x$
Outlet pipe	$\frac{1}{8}$	x	$\frac{1}{8}x$

Filling the pool is 1 whole job, but because the outlet pipe empties the pool, its contribution should be subtracted from the contribution of the inlet pipe.

$$\tfrac{1}{5}x-\tfrac{1}{8}x=1$$
$$40\left[\tfrac{1}{5}x-\tfrac{1}{8}x\right]=40\cdot 1\Rightarrow 8x-5x=40\Rightarrow$$
$$3x=40\Rightarrow x=\frac{40}{3}=13\tfrac{1}{3}\text{hr}$$

It took $13\frac{1}{3}$ hr to fill the pool.

42. We need to determine how much of the pool was filled after 1 hour. To do this, we evaluate $\frac{1}{5}x-\frac{1}{8}x$ when $x=1$. After 1 hour, $\frac{1}{5}\cdot 1-\frac{1}{8}\cdot 1=\frac{1}{5}-\frac{1}{8}=\frac{8}{40}-\frac{5}{40}=\frac{3}{40}$ of the pool has been filled. What remains to be filled is $1-\frac{3}{40}=\frac{40}{40}-\frac{3}{40}=\frac{37}{40}$. If we now let x be the amount of time it takes to complete filling the pool, we must solve the following.

$$\tfrac{1}{5}x=\tfrac{37}{40}\Rightarrow 5\left(\tfrac{1}{5}x\right)=5\left(\tfrac{37}{40}\right)\Rightarrow$$
$$x=\tfrac{37}{8}=4\tfrac{5}{8}\text{ hr}$$

It will take $4\frac{5}{8}$ hr more to fill the pool.

43. Let x = the amount of time (in minutes) to fill the sink with both pipes open.

	Rate	Time	Part of the Job Accomplished
Tap	$\frac{1}{5}$	x	$\frac{1}{5}x$
Drain	$\frac{1}{10}$	x	$\frac{1}{10}x$

Filling the sink is 1 whole job, but because the sink is draining, its contribution should be subtracted from the contribution of the taps.

$$\tfrac{1}{5}x-\tfrac{1}{10}x=1$$
$$10\left[\tfrac{1}{5}x-\tfrac{1}{10}x\right]=10\cdot 1\Rightarrow 2x-x=10\Rightarrow x=10$$

It will take 10 minutes to fill the sink if Mark forgets to put in the stopper.

44. We need to determine how much of the sink was filled after 1 minute. To do this, we evaluate $\frac{1}{5}x-\frac{1}{10}x$ when $x=1$. After 1 minute, $\frac{1}{5}\cdot 1-\frac{1}{10}\cdot 1=\frac{1}{5}-\frac{1}{10}=\frac{2}{10}-\frac{1}{10}=\frac{1}{10}$ of the sink has been filled. What remains to be filled is $1-\frac{1}{10}=\frac{10}{10}-\frac{1}{10}=\frac{9}{10}$. If we now let x be the amount of time it takes to complete filling the sink, we must solve the following.

$$\tfrac{1}{5}x=\tfrac{9}{10}$$
$$5\left(\tfrac{1}{5}x\right)=5\left(\tfrac{9}{10}\right)\Rightarrow x=\frac{45}{10}=4\tfrac{1}{2}\text{ min}$$

It will take $4\frac{1}{2}$ min more to fill the sink.

45. $x - \sqrt{2x+3} = 0$

$x = \sqrt{2x+3} \Rightarrow x^2 = \left(\sqrt{2x+3}\right)^2$

$x^2 = 2x+3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow$

$(x+1)(x-3) = 0 \Rightarrow x = -1$ or $x = 3$

Check $x = -1$.

$x - \sqrt{2x+3} = 0$

$-1 - \sqrt{2(-1)+3} \stackrel{?}{=} 0$

$-1 - \sqrt{-2+3} = 0$

$-1 - \sqrt{1} = 0 \Rightarrow -1 - 1 = 0 \Rightarrow -2 = 0$

This is a false statement. -1 is not a solution.

Check $x = 3$.

$x - \sqrt{2x+3} = 0$

$3 - \sqrt{2(3)+3} \stackrel{?}{=} 0$

$3 - \sqrt{6+3} = 0$

$3 - \sqrt{9} = 0 \Rightarrow 3 - 3 = 0 \Rightarrow 0 = 0$

This is a true statement. 3 is a solution.

Solution set: $\{3\}$

46. $x - \sqrt{3x+18} = 0$

$x = \sqrt{3x+18} \Rightarrow x^2 = \left(\sqrt{3x+18}\right)^2$

$x^2 = 3x+18 \Rightarrow x^2 - 3x - 18 = 0$

$(x+3)(x-6) = 0 \Rightarrow x = -3$ or $x = 6$

Check $x = -3$.

$x - \sqrt{3x+18} = 0$

$-3 - \sqrt{3(-3)+18} \stackrel{?}{=} 0$

$-3 - \sqrt{-9+18} = 0$

$-3 - \sqrt{9} = 0 \Rightarrow -3 - 3 = 0 \Rightarrow -6 = 0$

This is a false statement. –3 is not a solution.

Check $x = 6$.

$x - \sqrt{3x+18} = 0$

$6 - \sqrt{3(6)+18} \stackrel{?}{=} 0$

$6 - \sqrt{18+18} = 0$

$6 - \sqrt{36} = 0 \Rightarrow 6 - 6 = 0 \Rightarrow 0 = 0$

This is a true statement. 6 is a solution

Solution set: $\{6\}$

47. $\sqrt{3x+7} = 3x+5$

$\left(\sqrt{3x+7}\right)^2 = (3x+5)^2$

$3x + 7 = 9x^2 + 30x + 25$

$0 = 9x^2 + 27x + 18$

$0 = 9\left(x^2 + 3x + 2\right) = 9(x+2)(x+1)$

$x = -2$ or $x = -1$

Check $x = -2$.

$\sqrt{3x+7} = 3x+5$

$\sqrt{3(-2)+7} \stackrel{?}{=} 3(-2)+5$

$\sqrt{-6+7} = -6+5$

$\sqrt{1} = -1 \Rightarrow 1 = -1$

This is a false statement. -2 is not a solution.

Check $x = -1$

$\sqrt{3x+7} = 3x+5$

$\sqrt{3(-1)+7} \stackrel{?}{=} 3(-1)+5$

$\sqrt{-3+7} = -3+5$

$\sqrt{4} = 2 \Rightarrow 2 = 2$

This is a true statement. −1 is a solution.

Solution set: $\{-1\}$

48. $\sqrt{4x+13} = 2x-1$

$\left(\sqrt{4x+13}\right)^2 = (2x-1)^2$

$4x + 13 = 4x^2 - 4x + 1$

$0 = 4x^2 - 8x - 12$

$0 = 4\left(x^2 - 2x - 3\right)$

$0 = 4(x+1)(x-3)$

$x = -1$ or $x = 3$

Check $x = -1$.

$\sqrt{4x+13} = 2x-1$

$\sqrt{4(-1)+13} \stackrel{?}{=} 2(-1)-1$

$\sqrt{-4+13} = -2-1$

$\sqrt{9} = -3 \Rightarrow 3 = -3$

This is a false statement. -1 is not a solution.

Check $x = 3$.

$\sqrt{4x+13} = 2x-1$

$\sqrt{4(3)+13} \stackrel{?}{=} 2(3)-1$

$\sqrt{12+13} = 6-1$

$\sqrt{25} = 5 \Rightarrow 5 = 5$

This is a true statement. 3 is a solution.

Solution set: $\{3\}$

49. $\sqrt{4x+5} - 6 = 2x - 11$

$\sqrt{4x+5} = 2x-5$

$\left(\sqrt{4x+5}\right)^2 = (2x-5)^2$

$4x + 5 = 4x^2 - 20x + 25$

$0 = 4x^2 - 24x + 20$

$0 = 4\left(x^2 - 6x + 5\right) = 4(x-1)(x-5)$

$x = 1$ or $x = 5$

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(*continued*)

Check $x = 1$.

$\sqrt{4x+5} - 6 = 2x - 11$

$\sqrt{4(1)+5} - 6 \stackrel{?}{=} 2(1) - 11$

$\sqrt{4+5} - 6 = 2 - 11$

$\sqrt{9} - 6 = -9$

$3 - 6 = -9 \Rightarrow -3 = -9$

This is a false statement. 1 is not a solution.

Check $x = 5$.

$\sqrt{4x+5} - 6 = 2x - 11$

$\sqrt{4(5)+5} - 6 \stackrel{?}{=} 2(5) - 11$

$\sqrt{20+5} - 6 = 10 - 11$

$\sqrt{25} - 6 = -1 \Rightarrow 5 - 6 = -1 \Rightarrow -1 = -1$

This is a true statement. 5 is a solution.

Solution set: $\{5\}$

50. $\sqrt{6x+7} - 9 = x - 7$

$\sqrt{6x+7} = x + 2$

$\left(\sqrt{6x+7}\right)^2 = (x+2)^2$

$6x + 7 = x^2 + 4x + 4$

$0 = x^2 - 2x - 3 = (x+1)(x-3)$

$x = -1$ or $x = 3$

Check $x = -1$.

$\sqrt{6x+7} - 9 = x - 7$

$\sqrt{6(-1)+7} - 9 \stackrel{?}{=} -1 - 7$

$\sqrt{-6+7} - 1 = 0$

$\sqrt{1} - 1 = 0 \Rightarrow 1 - 1 = 0 \Rightarrow 0 = 0$

This is a true statement. -1 is a solution.

Check $x = 3$.

$\sqrt{6x+7} - 9 = x - 7$

$\sqrt{6(3)+7} - 9 \stackrel{?}{=} 3 - 7$

$\sqrt{18+7} - 9 = -4$

$\sqrt{25} - 9 = -4 \Rightarrow 5 - 9 = -4 \Rightarrow -4 = -4$

This is a true statement. 3 is a solution.

Solution set: $\{-1, 3\}$

51. $\sqrt{4x} - x + 3 = 0$

$\sqrt{4x} = x - 3$

$\left(\sqrt{4x}\right)^2 = (x-3)^2$

$4x = x^2 - 6x + 9$

$0 = x^2 - 10x + 9 = (x-1)(x-9)$

$x = 1$ or $x = 9$

Check $x = 1$.

$\sqrt{4x} - x + 3 = 0$

$\sqrt{4(1)} - 1 + 3 \stackrel{?}{=} 0$

$\sqrt{4} - 1 + 3 = 0$

$2 - 1 + 3 = 0 \Rightarrow 4 = 0$

This is a false statement. 1 is not a solution.

Check $x = 9$.

$\sqrt{4x} - x + 3 = 0$

$\sqrt{4(9)} - 9 + 3 \stackrel{?}{=} 0$

$\sqrt{36} - 9 + 3 = 0$

$6 - 9 + 3 = 0 \Rightarrow 0 = 0$

This is a true statement. 9 is a solution.

Solution set: $\{9\}$

52. $\sqrt{2x} - x + 4 = 0$

$\sqrt{2x} = x - 4$

$\left(\sqrt{2x}\right)^2 = (x-4)^2$

$2x = x^2 - 8x + 16$

$0 = x^2 - 10x + 16 = (x-2)(x-8)$

$x = 2$ or $x = 8$

Check $x = 2$.

$\sqrt{2x} - x + 4 = 0$

$\sqrt{2(2)} - 2 + 4 \stackrel{?}{=} 0$

$\sqrt{4} - 2 + 4 = 0$

$2 - 2 + 4 = 0 \Rightarrow 4 = 0$

This is a false statement. 2 is not a solution.

Check $x = 8$.

$\sqrt{2x} - x + 4 = 0$

$\sqrt{2(8)} - 8 + 4 \stackrel{?}{=} 0$

$\sqrt{16} - 8 + 4 = 0$

$4 - 8 + 4 = 0 \Rightarrow 0 = 0$

This is a true statement. 8 is a solution.

Solution set: $\{8\}$

53. $\sqrt{x}-\sqrt{x-5}=1$

$\sqrt{x}=1+\sqrt{x-5}\Rightarrow\left(\sqrt{x}\right)^2=\left(1+\sqrt{x-5}\right)^2$

$x=1+2\sqrt{x-5}+(x-5)$

$x=x+2\sqrt{x-5}-4\Rightarrow 4=2\sqrt{x-5}$

$2=\sqrt{x-5}\Rightarrow 2^2=\left(\sqrt{x-5}\right)^2$

$4=x-5\Rightarrow 9=x$

Check $x=9$.

$\sqrt{x}-\sqrt{x-5}=1$

$\sqrt{9}-\sqrt{9-5}\overset{?}{=}1$

$3-\sqrt{4}=1\Rightarrow 3-2=1\Rightarrow 1=1$

This is a true statement.
Solution set is: $\{9\}$

54. $\sqrt{x}-\sqrt{x-12}=2$

$\sqrt{x}=2+\sqrt{x-12}\Rightarrow\left(\sqrt{x}\right)^2=\left(2+\sqrt{x-12}\right)^2$

$x=4+4\sqrt{x-12}+(x-12)$

$x=x+4\sqrt{x-12}-8\Rightarrow 8=4\sqrt{x-12}$

$2=\sqrt{x-12}\Rightarrow 2^2=\left(\sqrt{x-12}\right)^2$

$4=x-12\Rightarrow 16=x$

Check $x=16$.

$\sqrt{x}-\sqrt{x-12}=2$

$\sqrt{16}-\sqrt{16-12}\overset{?}{=}2$

$4-\sqrt{4}=2\Rightarrow 4-2=2\Rightarrow 2=2$

This is a true statement.
Solution set: $\{16\}$

55. $\sqrt{x+7}+3=\sqrt{x-4}$

$\left(\sqrt{x+7}+3\right)^2=\left(\sqrt{x-4}\right)^2$

$(x+7)+6\sqrt{x+7}+9=x-4$

$x+6\sqrt{x+7}+16=x-4\Rightarrow 6\sqrt{x+7}=-20$

$3\sqrt{x+7}=-10\Rightarrow\left(3\sqrt{x+7}\right)^2=(-10)^2$

$9(x+7)=100\Rightarrow 9x+63=100$

$9x=37\Rightarrow x=\dfrac{37}{9}$

Check $x=\frac{37}{9}$.

$\sqrt{x+7}+3=\sqrt{x-4}$

$\sqrt{\frac{37}{9}+7}+3\overset{?}{=}\sqrt{\frac{37}{9}-4}$

$\sqrt{\frac{37}{9}+\frac{63}{9}}+3=\sqrt{\frac{37}{9}-\frac{36}{9}}$

$\sqrt{\frac{100}{9}}+3=\sqrt{\frac{1}{9}}$

$\frac{10}{3}+3=\frac{1}{3}\Rightarrow\frac{10}{3}+\frac{9}{3}=\frac{1}{3}\Rightarrow\frac{19}{3}=\frac{1}{3}$

This is a false statement.
Solution set: $\varnothing$

56. $\sqrt{x+5}+2=\sqrt{x-1}$

$\left(\sqrt{x+5}+2\right)^2=\left(\sqrt{x-1}\right)^2$

$(x+5)+4\sqrt{x+5}+4=x-1$

$x+9+4\sqrt{x+5}=x-1\Rightarrow 4\sqrt{x+5}=-10$

$2\sqrt{x+5}=-5\Rightarrow\left(2\sqrt{x+5}\right)^2=(-5)^2$

$4(x+5)=25\Rightarrow 4x+20=25$

$4x=5\Rightarrow x=\dfrac{5}{4}$

Check $x=\frac{5}{4}$.

$\sqrt{x+5}+2=\sqrt{x-1}$

$\sqrt{\frac{5}{4}+5}+2\overset{?}{=}\sqrt{\frac{5}{4}-1}$

$\sqrt{\frac{5}{4}+\frac{20}{4}}+2=\sqrt{\frac{5}{4}-\frac{4}{4}}$

$\sqrt{\frac{25}{4}}+2=\sqrt{\frac{1}{4}}$

$\frac{5}{2}+2=\frac{1}{2}\Rightarrow\frac{9}{2}=\frac{1}{2}$

This is a false statement.
Solution set: $\varnothing$

57. $\sqrt{2x+5}-\sqrt{x+2}=1$

$\sqrt{2x+5}=\sqrt{x+2}+1$

$\left(\sqrt{2x+5}\right)^2=\left(\sqrt{x+2}+1\right)^2$

$2x+5=(x+2)+2\sqrt{x+2}+1$

$2x+5=x+3+2\sqrt{x+2}$

$x+2=2\sqrt{x+2}$

$(x+2)^2=\left(2\sqrt{x+2}\right)^2$

$x^2+4x+4=4(x+2)$

$x^2+4x+4=4x+8$

$0=x^2-4$

$0=(x+2)(x-2)\Rightarrow x=\pm 2$

Check $x=2$.

$\sqrt{2x+5}-\sqrt{x+2}=1$

$\sqrt{2(2)+5}-\sqrt{2+2}\overset{?}{=}1$

$\sqrt{4+5}-\sqrt{4}\overset{?}{=}1$

$\sqrt{9}-\sqrt{4}\overset{?}{=}1$

$3-2\overset{?}{=}1\Rightarrow 1=1$

This is a true statement. 2 is a solution.

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Check $x = -2$.

$$\sqrt{2x+5} - \sqrt{x+2} = 1$$
$$\sqrt{2(-2)+5} - \sqrt{-2+2} \stackrel{?}{=} 1$$
$$\sqrt{-4+5} - \sqrt{0} \stackrel{?}{=} 1$$
$$\sqrt{1} - \sqrt{0} \stackrel{?}{=} 1$$
$$1 - 0 \stackrel{?}{=} 1 \Rightarrow 1 = 1$$

This is a true statement. −2 is a solution.
Solution set: $\{\pm 2\}$

58. $$\sqrt{4x+1} - \sqrt{x-1} = 2$$
$$\sqrt{4x+1} = \sqrt{x-1} + 2$$
$$\left(\sqrt{4x+1}\right)^2 = \left(\sqrt{x-1} + 2\right)^2$$
$$4x+1 = (x-1) + 4\sqrt{x-1} + 4$$
$$4x+1 = x+3+4\sqrt{x-1}$$
$$3x-2 = 4\sqrt{x-1}$$
$$(3x-2)^2 = \left(4\sqrt{x-1}\right)^2$$
$$9x^2 - 12x + 4 = 16(x-1)$$
$$9x^2 - 12x + 4 = 16x - 16$$
$$9x^2 - 28x + 20 = 0 \Rightarrow (9x-10)(x-2) = 0$$
$$x = \tfrac{10}{9} \text{ or } x = 2$$

Check $x = \frac{10}{9}$.

$$\sqrt{4x+1} = \sqrt{x-1} + 2$$
$$\sqrt{4\left(\tfrac{10}{9}\right)+1} \stackrel{?}{=} \sqrt{\tfrac{10}{9} - 1} + 2$$
$$\sqrt{\tfrac{40}{9}+1} = \sqrt{\tfrac{10}{9} - \tfrac{9}{9}} + 2$$
$$\sqrt{\tfrac{40}{9}+\tfrac{9}{9}} = \sqrt{\tfrac{1}{9}} + 2 \Rightarrow \sqrt{\tfrac{49}{9}} = \tfrac{1}{3} + 2$$
$$\tfrac{7}{3} = \tfrac{1}{3} + \tfrac{6}{3} \Rightarrow \tfrac{7}{3} = \tfrac{7}{3}$$

This is a true statement. $\frac{10}{9}$ is a solution.

Check $x = 2$.

$$\sqrt{4x+1} = \sqrt{x-1} + 2$$
$$\sqrt{4(2)+1} \stackrel{?}{=} \sqrt{2-1} + 2$$
$$\sqrt{8+1} = \sqrt{1} + 2$$
$$\sqrt{9} = 1 + 2 \Rightarrow 3 = 3$$

This is a true statement. 2 is a solution.
Solution set: $\left\{\frac{10}{9}, 2\right\}$

59. $$\sqrt{3x} = \sqrt{5x+1} - 1$$
$$\left(\sqrt{3x}\right)^2 = \left(\sqrt{5x+1} - 1\right)^2$$
$$3x = (5x+1) - 2\sqrt{5x+1} + 1$$
$$3x = 5x + 2 - 2\sqrt{5x+1}$$
$$2\sqrt{5x+1} = 2 + 2x \Rightarrow \sqrt{5x+1} = 1 + x$$
$$\left(\sqrt{5x+1}\right)^2 = (1+x)^2 \Rightarrow 5x+1 = 1 + 2x + x^2$$
$$0 = x^2 - 3x \Rightarrow 0 = x(x-3) \Rightarrow$$
$$x = 0 \text{ or } x = 3$$

Check $x = 0$.

$$\sqrt{3x} = \sqrt{5x+1} - 1$$
$$\sqrt{3(0)} \stackrel{?}{=} \sqrt{5(0)+1} - 1$$
$$\sqrt{0} = \sqrt{0+1} - 1 \Rightarrow 0 = \sqrt{1} - 1$$
$$0 = 1 - 1 \Rightarrow 0 = 0$$

This is a true statement. 0 is a solution.

Check $x = 3$.

$$\sqrt{3x} = \sqrt{5x+1} - 1$$
$$\sqrt{3(3)} \stackrel{?}{=} \sqrt{5(3)+1} - 1$$
$$\sqrt{9} = \sqrt{15+1} - 1 \Rightarrow 3 = \sqrt{16} - 1$$
$$3 = 4 - 1 \Rightarrow 3 = 3$$

This is a true statement. 3 is a solution.
Solution set: $\{0, 3\}$

60. $$\sqrt{2x} = \sqrt{3x+12} - 2$$
$$\left(\sqrt{2x}\right)^2 = \left(\sqrt{3x+12} - 2\right)^2$$
$$2x = 3x + 12 - 4\sqrt{3x+12} + 4$$
$$2x = 3x + 16 - 4\sqrt{3x+12}$$
$$4\sqrt{3x+12} = x + 16$$
$$\left(4\sqrt{3x+12}\right)^2 = (x+16)^2$$
$$16(3x+12) = x^2 + 32x + 256$$
$$48x + 192 = x^2 + 32x + 256$$
$$0 = x^2 - 16x + 64$$
$$0 = (x-8)^2 \Rightarrow x = 8$$

Check $x = 8$.

$$\sqrt{2x} = \sqrt{3x+12} - 2$$
$$\sqrt{2(8)} \stackrel{?}{=} \sqrt{3(8)+12} - 2$$
$$\sqrt{16} = \sqrt{24+12} - 2$$
$$4 = \sqrt{36} - 2 \Rightarrow 4 = 6 - 2 \Rightarrow 4 = 4$$

This is a true statement.
Solution set: $\{8\}$

61. $\sqrt{x+2}=1-\sqrt{3x+7}$

$\left(\sqrt{x+2}\right)^2=(1-\sqrt{3x+7})^2$

$x+2=1-2\sqrt{3x+7}+(3x+7)$

$x+2=3x+8-2\sqrt{3x+7}$

$2\sqrt{3x+7}=2x+6$

$2\sqrt{3x+7}=2(x+3)$

$\sqrt{3x+7}=x+3\Rightarrow\left(\sqrt{3x+7}\right)^2=(x+3)^2$

$3x+7=x^2+6x+9\Rightarrow 0=x^2+3x+2$

$0=(x+2)(x+1)$

$x=-2$ or $x=-1$

Check $x=-2$.

$\sqrt{x+2}=1-\sqrt{3x+7}$

$\sqrt{-2+2}\overset{?}{=}1-\sqrt{3(-2)+7}$

$\sqrt{0}=1-\sqrt{-6+7}$

$0=1-\sqrt{1}$

$0=1-1\Rightarrow 0=0$

This is a true statement. -2 is a solution.

Check $x=-1$.

$\sqrt{x+2}=1-\sqrt{3x+7}$

$\sqrt{-1+2}\overset{?}{=}1-\sqrt{3(-1)+7}$

$\sqrt{1}=1-\sqrt{-3+7}$

$1=1-\sqrt{4}$

$1=1-2\Rightarrow 1=-1$

This is a false statement.

-1 is not a solution.

Solution set: $\{-2\}$

62. $\sqrt{2x-5}=2+\sqrt{x-2}$

$\left(\sqrt{2x-5}\right)^2=\left(2+\sqrt{x-2}\right)^2$

$2x-5=4+4\sqrt{x-2}+(x-2)$

$2x-5=x+2+4\sqrt{x-2}$

$x-7=4\sqrt{x-2}$

$(x-7)^2=\left(4\sqrt{x-2}\right)^2$

$x^2-14x+49=16(x-2)$

$x^2-14x+49=16x-32$

$x^2-30x+81=0\Rightarrow(x-3)(x-27)=0\Rightarrow$

$x=3$ or $x=27$

Check $x=3$.

$\sqrt{2x-5}=2+\sqrt{x-2}$

$\sqrt{2(3)-5}\overset{?}{=}2+\sqrt{3-2}$

$\sqrt{6-5}=2+\sqrt{1}$

$\sqrt{1}=2+1\Rightarrow 1=3$

This is a false statement. 3 is not a solution.

Check $x=27$.

$\sqrt{2x-5}=2+\sqrt{27-2}$

$\sqrt{2(27)-5}\overset{?}{=}2+\sqrt{27-2}$

$\sqrt{54-5}=2+\sqrt{25}$

$\sqrt{49}=2+5\Rightarrow 7=7$

This is a true statement. 27 is a solution.

Solution set: $\{27\}$

63. $\sqrt{2\sqrt{7x+2}}=\sqrt{3x+2}$

$\left(\sqrt{2\sqrt{7x+2}}\right)^2=\left(\sqrt{3x+2}\right)^2$

$2\sqrt{7x+2}=3x+2$

$\left(2\sqrt{7x+2}\right)^2=(3x+2)^2$

$4(7x+2)=9x^2+12x+4$

$28x+8=9x^2+12x+4$

$0=9x^2-16x-4$

$0=(9x+2)(x-2)$

$x=-\frac{2}{9}$ or $x=2$

Check $x=-\frac{2}{9}$.

$\sqrt{2\sqrt{7x+2}}=\sqrt{3x+2}$

$\sqrt{2\sqrt{7\left(-\frac{2}{9}\right)+2}}\overset{?}{=}\sqrt{3\left(-\frac{2}{9}\right)+2}$

$\sqrt{2\sqrt{-\frac{14}{9}+2}}=\sqrt{-\frac{2}{3}+2}$

$\sqrt{2\sqrt{-\frac{14}{9}+\frac{18}{9}}}=\sqrt{-\frac{2}{3}+\frac{6}{3}}$

$\sqrt{2\sqrt{\frac{4}{9}}}=\sqrt{\frac{4}{3}}\Rightarrow\sqrt{2\left(\frac{2}{3}\right)}=\frac{\sqrt{4}}{\sqrt{3}}$

$\sqrt{\frac{4}{3}}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\Rightarrow\frac{\sqrt{4}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}\Rightarrow\frac{2\sqrt{3}}{3}=\frac{2\sqrt{3}}{3}$

This is a true statement.

$-\frac{2}{9}$ is a solution.

Check $x=2$.

$\sqrt{2\sqrt{7x+2}}=\sqrt{3x+2}$

$\sqrt{2\sqrt{7(2)+2}}\overset{?}{=}\sqrt{3(2)+2}$

$\sqrt{2\sqrt{14+2}}=\sqrt{6+2}$

$\sqrt{2\sqrt{16}}=\sqrt{8}$

$\sqrt{2(4)}=2\sqrt{2}$

$\sqrt{8}=2\sqrt{2}\Rightarrow 2\sqrt{2}=2\sqrt{2}$

This is a true statement. 2 is a solution.

Solution set: $\left\{-\frac{2}{9},2\right\}$

64.
$$\sqrt{3\sqrt{2x+3}} = \sqrt{5x-6}$$
$$\left(\sqrt{3\sqrt{2x+3}}\right)^2 = \left(\sqrt{5x-6}\right)^2$$
$$3\sqrt{2x+3} = 5x-6$$
$$\left(3\sqrt{2x+3}\right)^2 = (5x-6)^2$$
$$9(2x+3) = 25x^2-60x+36$$
$$18x+27 = 25x^2-60x+36$$
$$0 = 25x^2-78x+9$$
$$0 = (25x-3)(x-3)$$
$$x = \tfrac{3}{25} \text{ or } x = 3$$

Check $x = \frac{3}{25}$.
$$\sqrt{3\sqrt{2x+3}} = \sqrt{5x-6}$$
$$\sqrt{3\sqrt{2\left(\tfrac{3}{25}\right)+3}} \stackrel{?}{=} \sqrt{5\left(\tfrac{3}{25}\right)-6}$$
$$\sqrt{3\sqrt{\tfrac{6}{25}+3}} = \sqrt{\tfrac{3}{5}-6}$$
$$\sqrt{3\sqrt{\tfrac{6}{25}+\tfrac{75}{25}}} = \sqrt{\tfrac{3}{5}-\tfrac{30}{5}}$$
$$\sqrt{3\sqrt{\tfrac{81}{25}}} = \sqrt{-\tfrac{27}{5}} \Rightarrow \sqrt{3\left(\tfrac{9}{5}\right)} = \frac{\sqrt{-27}}{\sqrt{5}}$$
$$\sqrt{\tfrac{27}{5}} = \frac{3i\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$$
$$\frac{\sqrt{27}}{\sqrt{5}} = \frac{3i\sqrt{15}}{5}$$
$$\frac{3\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{3i\sqrt{15}}{5} \Rightarrow \frac{3\sqrt{15}}{5} = \frac{3i\sqrt{15}}{5}$$

This is a false statement. $\frac{3}{25}$ is not a solution.

Check $x = 3$.
$$\sqrt{3\sqrt{2x+3}} = \sqrt{5x-6}$$
$$\sqrt{3\sqrt{2(3)+3}} \stackrel{?}{=} \sqrt{5(3)-6}$$
$$\sqrt{3\sqrt{6+3}} = \sqrt{15-6}$$
$$\sqrt{3\sqrt{9}} = \sqrt{9}$$
$$\sqrt{3(3)} = 3 \Rightarrow \sqrt{9} = 3 \Rightarrow 3 = 3$$

This is a true statement. 3 is a solution
Solution set: $\{3\}$

65.
$$3-\sqrt{x} = \sqrt{2\sqrt{x}-3}$$
$$\left(3-\sqrt{x}\right)^2 = \left(\sqrt{2\sqrt{x}-3}\right)^2$$
$$9-6\sqrt{x}+x = 2\sqrt{x}-3$$
$$12+x = 8\sqrt{x}$$
$$(12+x)^2 = \left(8\sqrt{x}\right)^2$$
$$144+24x+x^2 = 64x$$
$$x^2-40x+144 = 0$$
$$(x-36)(x-4) = 0 \Rightarrow x = 36 \text{ or } x = 4$$

Check $x = 36$.
$$3-\sqrt{x} = \sqrt{2\sqrt{x}-3}$$
$$3-\sqrt{36} \stackrel{?}{=} \sqrt{2\sqrt{36}-3}$$
$$3-6 = \sqrt{2(6)-3}$$
$$-3 = \sqrt{12-3} \Rightarrow -3 = \sqrt{9} \Rightarrow -3 = 3$$

This is a false statement. 36 is not a solution.
Check $x = 4$.
$$3-\sqrt{x} = \sqrt{2\sqrt{x}-3}$$
$$3-\sqrt{4} \stackrel{?}{=} \sqrt{2\sqrt{4}-3}$$
$$3-2 = \sqrt{2(2)-3}$$
$$1 = \sqrt{4-3} \Rightarrow 1 = \sqrt{1} \Rightarrow 1 = 1$$

This is a true statement. 4 is a solution.
Solution set: $\{4\}$

66.
$$\sqrt{x}+2 = \sqrt{4+7\sqrt{x}}$$
$$\left(\sqrt{x}+2\right)^2 = \left(\sqrt{4+7\sqrt{x}}\right)^2$$
$$x+4\sqrt{x}+4 = 4+7\sqrt{x}$$
$$x = 3\sqrt{x} \Rightarrow x^2 = \left(3\sqrt{x}\right)^2$$
$$x^2 = 9x \Rightarrow x^2-9x = 0$$
$$x(x-9) = 0 \Rightarrow x = 0 \text{ or } x = 9$$

Check $x = 0$.
$$\sqrt{x}+2 = \sqrt{4+7\sqrt{x}}$$
$$\sqrt{0}+2 \stackrel{?}{=} \sqrt{4+7\sqrt{0}}$$
$$0+2 = \sqrt{4+7(0)}$$
$$2 = \sqrt{4+0} \Rightarrow 2 = \sqrt{4} \Rightarrow 2 = 2$$

This is a true statement. 0 is a solution.
Check $x = 9$.
$$\sqrt{x}+2 = \sqrt{4+7\sqrt{x}}$$
$$\sqrt{9}+2 \stackrel{?}{=} \sqrt{4+7\sqrt{9}}$$
$$3+2 = \sqrt{4+7(3)}$$
$$5 = \sqrt{4+21} \Rightarrow 5 = \sqrt{25} \Rightarrow 5 = 5$$

This is a true statement. 9 is a solution.
Solution set: $\{0, 9\}$

67.
$$\sqrt[3]{4x+3} = \sqrt[3]{2x-1}$$
$$\left(\sqrt[3]{4x+3}\right)^3 = \left(\sqrt[3]{2x-1}\right)^3$$
$$4x+3 = 2x-1 \Rightarrow 2x = -4 \Rightarrow x = -2$$

Check $x = -2$.
$$\sqrt[3]{4(-2)+3} = \sqrt[3]{2(-2)-1}$$
$$\sqrt[3]{-5} \stackrel{?}{=} \sqrt[3]{-5} \Rightarrow -\sqrt[3]{5} = -\sqrt[3]{5}$$

This is a true statement. -2 is a solution.
Solution set: $\{-2\}$

68. $\sqrt[3]{2x}=\sqrt[3]{5x+2}\Rightarrow\left(\sqrt[3]{2x}\right)^3=\left(\sqrt[3]{5x+2}\right)^3$

$2x=5x+2\Rightarrow -3x=2\Rightarrow x=-\dfrac{2}{3}$

Check $x=-\frac{2}{3}$.

$\sqrt[3]{2\left(-\frac{2}{3}\right)}=\sqrt[3]{5\left(-\frac{2}{3}\right)+2}$

$\sqrt[3]{-\frac{4}{3}}\stackrel{?}{=}\sqrt[3]{-\frac{10}{3}+2}\Rightarrow -\sqrt[3]{\frac{4}{3}}=\sqrt[3]{-\frac{10}{3}+\frac{6}{3}}\Rightarrow$

$-\dfrac{\sqrt[3]{4}}{\sqrt[3]{3}}\cdot\dfrac{\sqrt[3]{9}}{\sqrt[3]{9}}=\sqrt[3]{-\frac{4}{3}}\Rightarrow -\dfrac{\sqrt[3]{36}}{3}=-\sqrt[3]{\frac{4}{3}}\Rightarrow$

$-\dfrac{\sqrt[3]{36}}{3}=-\dfrac{\sqrt[3]{4}}{\sqrt[3]{3}}\cdot\dfrac{\sqrt[3]{9}}{\sqrt[3]{9}}\Rightarrow -\dfrac{\sqrt[3]{36}}{3}=-\dfrac{\sqrt[3]{36}}{3}$

This is a true statement.

Solution set: $\left\{-\frac{2}{3}\right\}$

69. $\sqrt[3]{5x^2-6x+2}-\sqrt[3]{x}=0$

$\sqrt[3]{5x^2-6x+2}=\sqrt[3]{x}$

$\left(\sqrt[3]{5x^2-6x+2}\right)^3=\left(\sqrt[3]{x}\right)^3$

$5x^2-6x+2=x$

$5x^2-7x+2=0$

$(5x-2)(x-1)=0\Rightarrow x=\frac{2}{5}$ or $x=1$

Check $x=\frac{2}{5}$.

$\sqrt[3]{5x^2-6x+2}-\sqrt[3]{x}=0$

$\sqrt[3]{5\left(\frac{2}{5}\right)^2-6\left(\frac{2}{5}\right)+2}-\sqrt[3]{\frac{2}{5}}\stackrel{?}{=}0$

$\sqrt[3]{5\left(\frac{4}{25}\right)-\frac{12}{5}+2}-\sqrt[3]{\frac{2}{5}}=0$

$\sqrt[3]{\frac{4}{5}-\frac{12}{5}+\frac{10}{5}}-\sqrt[3]{\frac{2}{5}}=0$

$\sqrt[3]{\frac{2}{5}}-\sqrt[3]{\frac{2}{5}}=0\Rightarrow 0=0$

This is a true statement. $\frac{2}{5}$ is a solution.

Check $x=1$.

$\sqrt[3]{5x^2-6x+2}-\sqrt[3]{x}=0$

$\sqrt[3]{5(1)^2-6(1)+2}-\sqrt[3]{1}\stackrel{?}{=}0$

$\sqrt[3]{5(1)-6+2}-1=0$

$\sqrt[3]{5-6+2}-1=0$

$\sqrt[3]{1}-1=0\Rightarrow 1-1=0\Rightarrow 0=0$

This is a true statement. 1 is a solution.

Solution set: $\left\{\frac{2}{5},1\right\}$

70. $\sqrt[3]{3x^2-9x+8}=\sqrt[3]{x}$

$\left(\sqrt[3]{3x^2-9x+8}\right)^3=\left(\sqrt[3]{x}\right)^3$

$3x^2-9x+8=x\Rightarrow 3x^2-10x+8=0\Rightarrow$

$(3x-4)(x-2)=0\Rightarrow x=\frac{4}{3}$ or $x=2$

Check $x=\frac{4}{3}$.

$\sqrt[3]{3x^2-9x+8}=\sqrt[3]{x}$

$\sqrt[3]{3\left(\frac{4}{3}\right)^2-9\left(\frac{4}{3}\right)+8}\stackrel{?}{=}\sqrt[3]{\frac{4}{3}}$

$\sqrt[3]{3\left(\frac{16}{9}\right)-12+8}=\frac{\sqrt[3]{4}}{\sqrt[3]{3}}\cdot\frac{\sqrt[3]{9}}{\sqrt[3]{9}}\Rightarrow\sqrt[3]{\frac{16}{3}-4}=\frac{\sqrt[3]{36}}{3}$

$\sqrt[3]{\frac{16}{3}-\frac{12}{3}}=\frac{\sqrt[3]{36}}{3}\Rightarrow\sqrt[3]{\frac{4}{3}}=\frac{\sqrt[3]{36}}{3}$

$\frac{\sqrt[3]{4}}{\sqrt[3]{3}}\cdot\frac{\sqrt[3]{9}}{\sqrt[3]{9}}=\frac{\sqrt[3]{36}}{3}\Rightarrow\frac{\sqrt[3]{36}}{3}=\frac{\sqrt[3]{36}}{3}$

This is a true statement. $\frac{4}{3}$ is a solution.

Check $x=2$.

$\sqrt[3]{3x^2-9x+8}=\sqrt[3]{x}$

$\sqrt[3]{3(2)^2-9(2)+8}\stackrel{?}{=}\sqrt[3]{2}$

$\sqrt[3]{3(4)-18+8}=\sqrt[3]{2}$

$\sqrt[3]{12-10}=\sqrt[3]{2}\Rightarrow\sqrt[3]{2}=\sqrt[3]{2}$

This is a true statement. 2 is a solution.

Solution set: $\left\{\frac{4}{3},2\right\}$

71. $\sqrt[4]{x-15}=2\Rightarrow\left(\sqrt[4]{x-15}\right)^4=2^4\Rightarrow$

$x-15=16\Rightarrow x=31$

Check $x=31$.

$\sqrt[4]{x-15}=2\Rightarrow\sqrt[4]{31-15}\stackrel{?}{=}2$

$\sqrt[4]{16}=2\Rightarrow 2=2$

This is a true statement.

Solution set: $\{31\}$

72. $\sqrt[4]{3x+1}=1$

$\left(\sqrt[4]{3x+1}\right)^4=1^4\Rightarrow 3x+1=1$

$3x=0\Rightarrow x=0$

Check $x=0$.

$\sqrt[4]{3x+1}=1\Rightarrow\sqrt[4]{3(0)+1}\stackrel{?}{=}1$

$\sqrt[4]{0+1}=1\Rightarrow\sqrt[4]{1}=1\Rightarrow 1=1$

This is a true statement.

Solution set: $\{0\}$

73. $\sqrt[4]{x^2+2x}=\sqrt[4]{3}\Rightarrow\left(\sqrt[4]{x^2+2x}\right)^4=\left(\sqrt[4]{3}\right)^4$

$x^2+2x=3\Rightarrow x^2+2x-3=0$

$(x+3)(x-1)=0\Rightarrow x=-3$ or $x=1$

Check $x=-3$.

$\sqrt[4]{x^2+2x}=\sqrt[4]{3}$

$\sqrt[4]{(-3)^2+2(-3)}\stackrel{?}{=}\sqrt[4]{3}$

$\sqrt[4]{9-6}=\sqrt[4]{3}\Rightarrow\sqrt[4]{3}=\sqrt[4]{3}$

This is a true statement. -3 is a solution.

Check $x=1$.

$\sqrt[4]{x^2+2x}=\sqrt[4]{3}$

$\sqrt[4]{1^2+2(1)}\stackrel{?}{=}\sqrt[4]{3}$

$\sqrt[4]{1+2}=\sqrt[4]{3}\Rightarrow\sqrt[4]{3}=\sqrt[4]{3}$

This is a true statement. 1 is a solution.

Solution set: $\{-3, 1\}$

74. $\sqrt[4]{x^2+6x}=2\Rightarrow\left(\sqrt[4]{x^2+6x}\right)^4=2^4$

$x^2+6x=16\Rightarrow x^2+6x-16=0$

$(x+8)(x-2)=0\Rightarrow x=-8$ or $x=2$

Check $x=-8$.

$\sqrt[4]{x^2+6x}=2$

$\sqrt[4]{(-8)^2+6(-8)}\stackrel{?}{=}2$

$\sqrt[4]{64-48}=2\Rightarrow\sqrt[4]{16}=2\Rightarrow 2=2$

This is a true statement. -8 is a solution.

Check $x=2$.

$\sqrt[4]{x^2+6x}=2$

$\sqrt[4]{2^2+6(2)}\stackrel{?}{=}2$

$\sqrt[4]{4+12}=2\Rightarrow\sqrt[4]{16}=2\Rightarrow 2=2$

This is a true statement. 2 is a solution.

Solution set: $\{-8,2\}$

75. $x^{3/2}=125$

$\left(x^{3/2}\right)^{2/3}=125^{2/3}$

$x=5^2=25$

Check $x=25$.

$x^{3/2}=125$

$25^{3/2}\stackrel{?}{=}125$

$125=125$

This is a true statement. 25 is a solution.

Solution set: $\{25\}$

76. $x^{5/4}=32$

$\left(x^{5/4}\right)^{4/5}=32^{4/5}$

$x=2^4=16$

Check $x=16$.

$x^{5/4}=32$

$16^{5/4}\stackrel{?}{=}32$

$32=32$

This is a true statement. 16 is a solution.

Solution set: $\{16\}$

77. $(x^2+24x)^{1/4}=3\Rightarrow\left[\left(x^2+24x\right)^{1/4}\right]^4=3^4\Rightarrow$

$x^2+24x=81\Rightarrow x^2+24x-81=0\Rightarrow$

$(x+27)(x-3)=0\Rightarrow x+27=0\Rightarrow x=-27$ or

$x-3=0\Rightarrow x=3$

Check $x=-27$.

$(x^2+24x)^{1/4}=3$

$\left[(-27)^2+24(-27)\right]^{1/4}\stackrel{?}{=}3$

$(729-648)^{1/4}=3$

$81^{1/4}=3\Rightarrow 3=3$

This is a true statement. –27 is a solution.

Check $x=3$.

$(x^2+24x)^{1/4}=3$

$\left[3^2+24(3)\right]^{1/4}\stackrel{?}{=}3$

$(9+72)^{1/4}=3\Rightarrow 81^{1/4}=3\Rightarrow 3=3$

This is a true statement. 3 is a solution.

Solution set: $\{-27, 3\}$

78. $\left(3x^2+52x\right)^{1/4}=4\Rightarrow\left[\left(3x^2+52x\right)^{1/4}\right]^4=4^4$

$3x^2+52x=256\Rightarrow 3x^2+52x-256=0$

$(3x+64)(x-4)=0\Rightarrow x=-\frac{64}{3}$ or $x=4$

Check $x=-\frac{64}{3}$.

$\left(3x^2+52x\right)^{1/4}=4$

$\left[3\left(-\frac{64}{3}\right)^2+52\left(-\frac{64}{3}\right)\right]^{1/4}\stackrel{?}{=}4$

$\left[3\left(\frac{4096}{9}\right)-\frac{3328}{3}\right]^{1/4}=4$

$\left(\frac{4096}{3}-\frac{3328}{3}\right)^{1/4}=4$

$\left(\frac{768}{3}\right)^{1/4}=4$

$256^{1/4}=4\Rightarrow 4=4$

This is a true statement.

$-\frac{64}{3}$ is a solution.

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Check $x = 4$.

$$\left(3x^2 + 52x\right)^{1/4} = 4$$
$$\left[3(4)^2 + 52(4)\right]^{1/4} \stackrel{?}{=} 4$$
$$\left[3(16) + 208\right]^{1/4} = 4$$
$$(48 + 208)^{1/4} = 4$$
$$256^{1/4} = 4 \Rightarrow 4 = 4$$

This is a true statement. 4 is a solution.

Solution set: $\left\{-\frac{64}{3}, 4\right\}$

79. $$(x-3)^{2/5} = 4$$
$$\left[(x-3)^{2/5}\right]^{5/2} = \pm 4^{5/2}$$
$$x - 3 = \pm 32$$
$$x - 3 = 32 \Rightarrow x = 35$$
$$x - 3 = -32 \Rightarrow x = -29$$

Check $x = 35$.

$$(x-3)^{2/5} = 4$$
$$(35-3)^{2/5} \stackrel{?}{=} 4$$
$$32^{2/5} = 4 \Rightarrow 4 = 4$$

This is a true statement. 35 is a solution.

Check $x = -29$.

$$(x-3)^{2/5} = 4$$
$$(-29-3)^{2/5} \stackrel{?}{=} 4$$
$$(-32)^{2/5} = 4$$
$$(-2)^2 = 4 \Rightarrow 4 = 4$$

This is a true statement. −29 is a solution.

Solution set: {−29, 35}

80. $$(x+200)^{2/3} = 36$$
$$\left[(x+200)^{2/3}\right]^{3/2} = \pm 36^{3/2}$$
$$x + 200 = \pm 216$$
$$x + 200 = 216 \Rightarrow x = 16$$
$$x + 200 = -216 \Rightarrow x = -416$$

Check $x = 16$.

$$(x+200)^{2/3} = 36$$
$$(16+200)^{2/3} \stackrel{?}{=} 36$$
$$216^{2/3} = 36 \Rightarrow 36 = 36$$

This is a true statement. 16 is a solution.

Check $x = -416$.

$$(x+200)^{2/3} = 36$$
$$(-416+200)^{2/3} \stackrel{?}{=} 36$$
$$(-216)^{2/3} = 36$$
$$(-6)^2 = 36 \Rightarrow 36 = 36$$

This is a true statement. −416 is a solution.

Solution set: {−416, 16}

81. $$(2x+5)^{1/3} - (6x-1)^{1/3} = 0$$
$$(2x+5)^{1/3} = (6x-1)^{1/3}$$
$$\left[(2x+5)^{1/3}\right]^3 = \left[(6x-1)^{1/3}\right]^3$$
$$2x + 5 = 6x - 1$$
$$-4x = -6$$
$$x = \frac{3}{2}$$

Check $x = \frac{3}{2}$.

$$(2x+5)^{1/3} - (6x-1)^{1/3} = 0$$
$$\left(2\cdot\frac{3}{2}+5\right)^{1/3} - \left(6\cdot\frac{3}{2}-1\right)^{1/3} \stackrel{?}{=} 0$$
$$8^{1/3} - 8^{1/3} = 0 \Rightarrow 0 = 0$$

This is a true statement. $\frac{3}{2}$ is a solution.

Solution set: $\left\{\frac{3}{2}\right\}$

82. $$(3x+7)^{1/3} - (4x+2)^{1/3} = 0$$
$$(3x+7)^{1/3} = (4x+2)^{1/3}$$
$$\left[(3x+7)^{1/3}\right]^3 = \left[(4x+2)^{1/3}\right]^3$$
$$3x + 7 = 4x + 2$$
$$5 = x$$

Check $x = 5$.

$$(3x+7)^{1/3} - (4x+2)^{1/3} = 0$$
$$(3\cdot 5+7)^{1/3} - (4\cdot 5+2)^{1/3} \stackrel{?}{=} 0$$
$$22^{1/3} - 22^{1/3} = 0 \Rightarrow 0 = 0$$

This is a true statement. 5 is a solution.

Solution set: {5}

83. $(2x-1)^{2/3} = x^{1/3}$
$[(2x-1)^{2/3}]^3 = (x^{1/3})^3$
$(2x-1)^2 = x \Rightarrow 4x^2 - 4x + 1 = x$
$4x^2 - 5x + 1 = 0 \Rightarrow (4x-1)(x-1) = 0 \Rightarrow$
$x = \dfrac{1}{4}$ or $x = 1$

Check $x = \frac{1}{4}$.

$(2x-1)^{2/3} = x^{1/3} \Rightarrow \left[2\left(\frac{1}{4}\right)-1\right]^{2/3} \overset{?}{=} \left(\frac{1}{4}\right)^{1/3}$
$\left[\frac{1}{2}-1\right]^{2/3} = \frac{1}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \Rightarrow \left[-\frac{1}{2}\right]^{2/3} = \frac{\sqrt[3]{2}}{2} \Rightarrow$
$\left[\left(-\frac{1}{2}\right)^2\right]^{1/3} = \frac{\sqrt[3]{2}}{2} \Rightarrow \left(\frac{1}{4}\right)^{1/3} = \frac{\sqrt[3]{2}}{2}$
$\frac{1}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}}{2} \Rightarrow \frac{\sqrt[3]{2}}{2} = \frac{\sqrt[3]{2}}{2}$

This is a true statement. $\frac{1}{4}$ is a solution.

Check $x = 1$.

$(2x-1)^{2/3} = x^{1/3} \Rightarrow \left[2(1)-1\right]^{2/3} \overset{?}{=} (1)^{1/3}$
$\left[2-1\right]^{2/3} = 1 \Rightarrow 1^{2/3} = 1 \Rightarrow 1 = 1$

This is a true statement. 1 is a solution.
Solution set: $\left\{\frac{1}{4}, 1\right\}$

84. $(x-3)^{2/5} = (4x)^{1/5}$
$\left[(x-3)^{2/5}\right]^5 = \left[(4x)^{1/5}\right]^5$
$(x-3)^2 = 4x$
$x^2 - 6x + 9 = 4x$
$x^2 - 10x + 9 = 0$
$(x-1)(x-9) = 0 \Rightarrow x = 1$ or $x = 9$

Check $x = 1$.

$(x-3)^{2/5} = (4x)^{1/5}$
$(1-3)^{2/5} \overset{?}{=} (4 \cdot 1)^{1/5}$
$(-2)^{2/5} = 4^{1/5}$
$\left[(-2)^2\right]^{1/5} = \sqrt[5]{4} \Rightarrow 4^{1/5} = \sqrt[5]{4} \Rightarrow \sqrt[5]{4} = \sqrt[5]{4}$

This is a true statement. 1 is a solution.
Check $x = 9$.

$(x-3)^{2/5} = (4x)^{1/5}$
$(9-3)^{2/5} \overset{?}{=} (4 \cdot 9)^{1/5}$
$6^{2/5} = 36^{1/5} \Rightarrow \left[6^2\right]^{1/5} = \sqrt[5]{36}$
$36^{1/5} = \sqrt[5]{36} \Rightarrow \sqrt[5]{36} = \sqrt[5]{36}$

This is a true statement. 9 is a solution.
Solution set: $\{1, 9\}$

85. $x^{2/3} = 2x^{1/3} \Rightarrow \left(x^{2/3}\right)^3 = \left(2x^{1/3}\right)^3 \Rightarrow$
$x^2 = 8x \Rightarrow x^2 - 8x = 0 \Rightarrow x(x-8) = 0 \Rightarrow$
$x = 0$ or $x = 8$

Check $x = 0$.
$x^{2/3} = 2x^{1/3}$
$0^{2/3} \overset{?}{=} 2\left(0^{1/3}\right) \Rightarrow 0 = 2 \cdot 0 \Rightarrow 0 = 0$

This is a true statement. 0 is a solution.
Check $x = 8$.

$x^{2/3} = 2x^{1/3} \Rightarrow 8^{2/3} \overset{?}{=} 2\left(8^{1/3}\right)$
$\left(8^2\right)^{1/3} = 2 \cdot 2 \Rightarrow 64^{1/3} = 4 \Rightarrow 4 = 4$

This is a true statement. 8 is a solution.
Solution set: $\{0, 8\}$

86. $3x^{3/4} = x^{1/2} \Rightarrow \left(3x^{3/4}\right)^4 = \left(x^{1/2}\right)^4 \Rightarrow$
$81x^3 = x^2 \Rightarrow 81x^3 - x^2 = 0 \Rightarrow$
$x^2(81x-1) = 0 \Rightarrow x = 0$ or $x = \frac{1}{81}$

Check $x = 0$.
$3x^{3/4} = x^{1/2}$
$3(0)^{3/4} \overset{?}{=} 0^{1/2} \Rightarrow 3 \cdot 0 = 0 \Rightarrow 0 = 0$

This is a true statement. 0 is a solution.
Check $x = \frac{1}{81}$.

$3x^{3/4} = x^{1/2}$
$3\left(\frac{1}{81}\right)^{3/4} \overset{?}{=} \left(\frac{1}{81}\right)^{1/2} \Rightarrow 3 \cdot \left[\left(\frac{1}{81}\right)^{1/4}\right]^3 = \frac{1}{9}$
$3 \cdot \left(\frac{1}{3}\right)^3 = \frac{1}{9} \Rightarrow 3 \cdot \frac{1}{27} = \frac{1}{9} \Rightarrow \frac{1}{9} = \frac{1}{9}$

This is a true statement. $\frac{1}{81}$ is a solution.

Solution set: $\left\{0, \frac{1}{81}\right\}$

87. $2x^4 - 7x^2 + 5 = 0$
Let $u = x^2$; then $u^2 = x^4$. With this substitution, the equation becomes
$2u^2 - 7u + 5 = 0$.
$2u^2 - 7u + 5 = 0 \Rightarrow (u-1)(2u-5) = 0 \Rightarrow$
$u = 1$ or $u = \frac{5}{2}$

To find *x*, replace *u* with x^2.
$x^2 = 1 \Rightarrow x = \pm 1$ or
$x^2 = \frac{5}{2} \Rightarrow x = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{10}}{2}$

Solution set: $\left\{\pm 1, \pm\frac{\sqrt{10}}{2}\right\}$

88. $4x^4 - 8x^2 + 3 = 0$

Let $u = x^2$; then $u^2 = x^4$.

$4u^2 - 8u + 3 = 0 \Rightarrow (2u-1)(2u-3) = 0 \Rightarrow$.

$u = \frac{1}{2}$ or $u = \frac{3}{2}$

To find *x*, replace *u* with x^2.

$x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x = \pm\frac{\sqrt{6}}{2}$ or

$x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x = \pm\frac{\sqrt{2}}{2}$

Solution set: $\left\{\pm\frac{\sqrt{6}}{2}, \pm\frac{\sqrt{2}}{2}\right\}$

89. $x^4 + 2x^2 - 15 = 0$

Let $u = x^2$; then $u^2 = x^4$.

$u^2 + 2u - 15 = 0 \Rightarrow (u-3)(u+5) = 0$.

$u = 3$ or $u = -5$

To find *x*, replace *u* with x^2.

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ or

$x^2 = -5 \Rightarrow x = \pm\sqrt{-5} = \pm i\sqrt{5}$

Solution set: $\left\{\pm\sqrt{3}, \pm i\sqrt{5}\right\}$

90. $3x^4 + 10x^2 - 25 = 0$

Let $u = x^2$; then $u^2 = x^4$.

$3u^2 + 10u - 25 = 0 \Rightarrow (u+5)(3u-5) = 0 \Rightarrow$.

$u = -5$ or $u = \frac{5}{3}$

To find *x*, replace *u* with x^2.

$x^2 = -5 \Rightarrow x = \pm\sqrt{-5} = \pm i\sqrt{5}$ or

$x^2 = \frac{5}{3} \Rightarrow x = \pm\sqrt{\frac{5}{3}} \Rightarrow x = \pm\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{15}}{3}$

Solution set: $\left\{\pm i\sqrt{5}, \pm\frac{\sqrt{15}}{3}\right\}$

91. $(x-1)^{2/3} + (x-1)^{1/3} - 12 = 0$

Let $u = (x-1)^{1/3}$. Then

$u^2 = \left[(x-1)^{1/3}\right]^2 = (x-1)^{2/3}$.

$u^2 + u - 12 = 0 \Rightarrow (u+4)(u-3) = 0 \Rightarrow$

$u = -4$ or $u = 3$

To find *x*, replace *u* with $(x-1)^{1/3}$.

$(x-1)^{1/3} = -4 \Rightarrow \left[(x-1)^{1/3}\right]^3 = (-4)^3 \Rightarrow$

$x - 1 = -64 \Rightarrow x = -63$ or

$(x-1)^{1/3} = 3 \Rightarrow \left[(x-1)^{1/3}\right]^3 = 3^3 \Rightarrow$

$x - 1 = 27 \Rightarrow x = 28$

Check $x = -63$.

$(x-1)^{2/3} + (x-1)^{1/3} - 12 = 0$

$(-63-1)^{2/3} + (-63-1)^{1/3} - 12 \stackrel{?}{=} 0$

$(-64)^{2/3} + (-64)^{1/3} - 12 = 0$

$\left[(-64)^{1/3}\right]^2 - 4 - 12 = 0$

$(-4)^2 - 4 - 12 = 0$

$16 - 4 - 12 = 0 \Rightarrow 0 = 0$

This is a true statement. -63 is a solution.

Check $x = 28$.

$(x-1)^{2/3} + (x-1)^{1/3} - 12 = 0$

$(28-1)^{2/3} + (28-1)^{1/3} - 12 \stackrel{?}{=} 0$

$27^{2/3} + 27^{1/3} - 12 = 0$

$\left[27^{1/3}\right]^2 + 3 - 12 = 0$

$3^2 + 3 - 12 = 0$

$9 + 3 - 12 = 0 \Rightarrow 0 = 0$

This is a true statement. 28 is a solution.

Solution set: $\{-63, 28\}$

92. $(2x-1)^{2/3} + 2(2x-1)^{1/3} - 3 = 0$

Let $u = (2x-1)^{1/3}$. Then

$u^2 = \left[(2x-1)^{1/3}\right]^2 = (2x-1)^{2/3}$.

$u^2 + 2u - 3 = 0 \Rightarrow (u+3)(u-1) = 0 \Rightarrow$

$u = -3$ or $u = 1$

To find *x*, replace *u* with $(2x-1)^{1/3}$.

$(2x-1)^{1/3} = -3 \Rightarrow \left[(2x-1)^{1/3}\right]^3 = (-3)^3 \Rightarrow$

$2x - 1 = -27 \Rightarrow 2x = -26 \Rightarrow x = -13$ or

$(2x-1)^{1/3} = 1 \Rightarrow \left[(2x-1)^{1/3}\right]^3 = 1^3 \Rightarrow$

$2x - 1 = 1 \Rightarrow 2x = 2 \Rightarrow x = 1$

Check $x = -13$.

$(2x-1)^{2/3} + 2(2x-1)^{1/3} - 3 = 0$

$[2(-13)-1]^{2/3} + 2[2(-13)-1]^{1/3} - 3 \stackrel{?}{=} 0$

$(-26-1)^{2/3} + 2(-26-1)^{1/3} - 3 = 0$

$(-27)^{2/3} + 2(-27)^{1/3} - 3 = 0$

$\left[(-27)^{1/3}\right]^2 + 2(-3) - 3 = 0$

$(-3)^2 - 6 - 3 = 0$

$9 - 6 - 3 = 0$

$0 = 0$

This is a true statement. -13 is a solution.

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Check $x = 1$.

$$(2x-1)^{2/3} + 2(2x-1)^{1/3} - 3 = 0$$
$$\left[2(1)-1\right]^{2/3} + 2\left[2(1)-1\right]^{1/3} - 3 \overset{?}{=} 0$$
$$(2-1)^{2/3} + 2(2-1)^{1/3} - 3 = 0$$
$$1^{2/3} + 2(1)^{1/3} - 3 = 0$$
$$1 + 2(1) - 3 = 0$$
$$1 + 2 - 3 = 0$$
$$0 = 0$$

This is a true statement. 1 is a solution

Solution set: $\{-13, 1\}$.

93. $(x+1)^{2/5} - 3(x+1)^{1/5} + 2 = 0$

Let $u = (x+1)^{1/5}$. Then

$u^2 = \left[(x+1)^{1/5}\right]^2 = (x+1)^{2/5}$.

$u^2 - 3u + 2 = 0 \Rightarrow (u-1)(u-2) = 0 \Rightarrow$

$u = 1$ or $u = 2$

To find x, replace u with $(x+1)^{1/5}$.

$(x+1)^{1/5} = 1 \Rightarrow \left[(x+1)^{1/5}\right]^5 = 1^5 \Rightarrow$ or

$x + 1 = 1 \Rightarrow x = 0$

$(x+1)^{1/5} = 2 \Rightarrow \left[(x+1)^{1/5}\right]^5 = 2^5 \Rightarrow$

$x + 1 = 32 \Rightarrow x = 31$

Check $x = 0$.

$$(x+1)^{2/5} - 3(x+1)^{1/5} + 2 = 0$$
$$(0+1)^{2/5} - 3(0+1)^{1/5} + 2 \overset{?}{=} 0$$
$$1^{2/5} - 3(1)^{1/5} + 2 = 0$$
$$1 - 3(1) + 2 = 0 \Rightarrow 0 = 0$$

This is a true statement. 0 is a solution.

Check $x = 31$.

$$(x+1)^{2/5} - 3(x+1)^{1/5} + 2 = 0$$
$$(31+1)^{2/5} - 3(31+1)^{1/5} + 2 \overset{?}{=} 0$$
$$32^{2/5} - 3(32)^{1/5} + 2 = 0$$
$$\left[(32)^{1/5}\right]^2 - 3(2) + 2 = 0$$
$$2^2 - 6 + 2 = 0$$
$$4 - 6 + 2 = 0 \Rightarrow 0 = 0$$

This is a true statement. 31 is a solution.

Solution set: $\{0, 31\}$

94. $(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$

Let $u = (x+5)^{1/3}$. Then

$u^2 = \left[(x+5)^{1/3}\right]^2 = (x+5)^{2/3}$.

$u^2 + u - 20 = 0 \Rightarrow (u+5)(u-4) = 0 \Rightarrow$

$u = -5$ or $u = 4$

To find x, replace u with $(x+5)^{1/3}$.

$(x+5)^{1/3} = -5 \Rightarrow \left[(x+5)^{1/3}\right]^3 = (-5)^3 \Rightarrow$

$x + 5 = -125 \Rightarrow x = -130$ or

$(x+5)^{1/3} = 4 \Rightarrow \left[(x+5)^{1/3}\right]^3 = 4^3 \Rightarrow$

$x + 5 = 64 \Rightarrow x = 59$

Check $x = -130$.

$$(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$$
$$(-130+5)^{2/3} + (-130+5)^{1/3} - 20 \overset{?}{=} 0$$
$$(-125)^{2/3} + (-125)^{1/3} - 20 = 0$$
$$25 + (-5) - 20 = 0$$
$$0 = 0$$

This is a true statement. −130 is a solution.

Check $x = 59$.

$$(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$$
$$(59+5)^{2/3} + (59+5)^{1/3} - 20 \overset{?}{=} 0$$
$$(64)^{2/3} + (64)^{1/3} - 20 = 0$$
$$16 + 4 - 20 = 0$$
$$0 = 0$$

This is a true statement. 59 is a solution.

Solution set: $\{-130, 59\}$

95. $4(x+1)^4 - 13(x+1)^2 = -9$

$4(x+1)^4 - 13(x+1)^2 + 9 = 0$

Let $u = (x+1)^2$. Then $u^2 = (x+1)^4$.

$4u^2 - 13u + 9 = 0$

$(4u-9)(u-1) = 0 \Rightarrow u = \frac{9}{4}$ or $u = 1$

To find x, replace u with $(x+1)^2$.

$(x+1)^2 = \frac{9}{4} \Rightarrow x + 1 = \pm\frac{3}{2} \Rightarrow$

$x = -1 \pm \frac{3}{2} \Rightarrow x = -\frac{5}{2}$ or $x = \frac{1}{2}$

$(x+1)^2 = 1 \Rightarrow x + 1 = \pm 1 \Rightarrow$

$x = -1 \pm 1 \Rightarrow x = -2$ or $x = 0$

Be sure to check all possible solutions in the original equation.

Solution set: $\left\{-\frac{5}{2}, -2, 0, \frac{1}{2}\right\}$

96. $25(x-5)^4 - 116(x-5)^2 = -64$

$25(x-5)^4 - 116(x-5)^2 + 64 = 0$

Let $u = (x-5)^2$. Then $u^2 = (x-5)^4$.

$25u^2 - 116u + 64 = 0$

$(25u-16)(u-4) = 0 \Rightarrow u = \frac{16}{25}$ or $u = 4$

To find *x*, replace *u* with $(x-5)^2$.

$(x-5)^2 = \frac{16}{25} \Rightarrow x-5 = \pm\frac{4}{5} \Rightarrow$
$x = 5 \pm \frac{4}{5} \Rightarrow x = \frac{21}{5}$ or $x = \frac{29}{5}$

$(x-5)^2 = 4 \Rightarrow x-5 = \pm 2 \Rightarrow$
$x = 5 \pm 2 \Rightarrow x = 7$ or $x = 3$

Be sure to check all possible solutions in the original equation.

Solution set: $\left\{3, \frac{21}{5}, \frac{29}{5}, 7\right\}$

97. $6(x+2)^4 - 11(x+2)^2 = -4$

$6(x+2)^4 - 11(x+2)^2 + 4 = 0$

Let $u = (x+2)^2$; then $u^2 = (x+2)^4$.

$6u^2 - 11u + 4 = 0 \Rightarrow (3u-4)(2u-1) = 0 \Rightarrow$

$u = \frac{4}{3}$ or $u = \frac{1}{2}$

To find *x*, replace *u* with $(x+2)^2$.

$(x+2)^2 = \frac{4}{3} \Rightarrow x+2 = \pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3}$ or

$x = -2 \pm \frac{2\sqrt{3}}{3} = -\frac{6}{3} \pm \frac{2\sqrt{3}}{3} = \frac{-6\pm 2\sqrt{3}}{3}$

$(x+2)^2 = \frac{1}{2} \Rightarrow x+2 = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$x = -2 \pm \frac{\sqrt{2}}{2} = -\frac{4}{2} \pm \frac{\sqrt{2}}{2} = \frac{-4\pm\sqrt{2}}{2}$

Solution set: $\left\{\frac{-6\pm 2\sqrt{3}}{3}, \frac{-4\pm\sqrt{2}}{2}\right\}$

98. $8(x-4)^4 - 10(x-4)^2 = -3$

$8(x-4)^4 - 10(x-4)^2 + 3 = 0$

Let $u = (x-4)^2$; then $u^2 = (x-4)^4$.

$8u^2 - 10u + 3 = 0 \Rightarrow (2u-1)(4u-3) = 0 \Rightarrow$

$u = \frac{1}{2}$ or $u = \frac{3}{4}$

$(x-4)^2 = \frac{1}{2} \Rightarrow x-4 = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ or

$x = 4 \pm \frac{\sqrt{2}}{2} = \frac{8}{2} \pm \frac{\sqrt{2}}{2} = \frac{8\pm\sqrt{2}}{2}$

$(x-4)^2 = \frac{3}{4} \Rightarrow x-4 = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$x = 4 \pm \frac{\sqrt{3}}{2} = \frac{8}{2} \pm \frac{\sqrt{3}}{2} = \frac{8\pm\sqrt{3}}{2}$

Solution set: $\left\{\frac{8\pm\sqrt{2}}{2}, \frac{8\pm\sqrt{3}}{2}\right\}$

99. $10x^{-2} + 33x^{-1} - 7 = 0$

Let $u = x^{-1}$; then $u^2 = x^{-2}$.

$10u^2 + 33u - 7 = 0 \Rightarrow (2u+7)(5u-1) = 0$

$u = -\frac{7}{2}$ or $u = \frac{1}{5}$

To find *x*, replace *u* with x^{-1}.

$x^{-1} = -\frac{7}{2} \Rightarrow x = -\frac{2}{7}$ or $x^{-1} = \frac{1}{5} \Rightarrow x = 5$

Solution set: $\left\{-\frac{2}{7}, 5\right\}$

100. $7x^{-2} - 10x^{-1} - 8 = 0$

Let $u = x^{-1}$; then $u^2 = x^{-2}$.

$7u^2 - 10u - 8 = 0 \Rightarrow (7u+4)(u-2) = 0$

$u = -\frac{4}{7}$ or $u = 2$

To find *x*, replace *u* with x^{-1}.

$x^{-1} = -\frac{4}{7} \Rightarrow x = -\frac{7}{4}$ or $x^{-1} = 2 \Rightarrow x = \frac{1}{2}$

Solution set: $\left\{-\frac{7}{4}, \frac{1}{2}\right\}$

101. $x^{-2/3} + x^{-1/3} - 6 = 0$

Let $u = x^{-1/3}$; then $u^2 = \left(x^{-1/3}\right)^2 = x^{-2/3}$.

$u^2 + u - 6 = 0 \Rightarrow (u+3)(u-2) = 0$

$u = -3$ or $u = 2$

To find *x*, replace *u* with $x^{-1/3}$.

$x^{-1/3} = -3 \Rightarrow \left(x^{-1/3}\right)^{-3} = (-3)^{-3} \Rightarrow$ or

$x = \frac{1}{(-3)^3} \Rightarrow x = -\frac{1}{27}$

$x^{-1/3} = 2 \Rightarrow \left(x^{-1/3}\right)^{-3} = 2^{-3} \Rightarrow$

$x = \frac{1}{2^3} \Rightarrow x = \frac{1}{8}$

Check $x = -\frac{1}{27}$.

$$x^{-2/3} + x^{-1/3} - 6 = 0$$

$$\left(-\frac{1}{27}\right)^{-2/3} + \left(-\frac{1}{27}\right)^{-1/3} - 6 \stackrel{?}{=} 0$$

$$(-27)^{2/3} + (-27)^{1/3} - 6 = 0$$

$$\left[(-27)^{1/3}\right]^2 - 3 - 6 = 0$$

$$(-3)^2 - 3 - 6 = 0$$

$$9 - 3 - 6 = 0 \Rightarrow 0 = 0$$

This is a true statement.

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Check $x = \frac{1}{8}$.

$$x^{-2/3} + x^{-1/3} - 6 = 0$$
$$\left(\tfrac{1}{8}\right)^{-2/3} + \left(\tfrac{1}{8}\right)^{-1/3} - 6 \stackrel{?}{=} 0$$
$$8^{2/3} + 8^{1/3} - 6 = 0$$
$$\left(8^{1/3}\right)^2 + 2 - 6 = 0$$
$$2^2 + 2 - 6 = 0$$
$$4 + 2 - 6 = 0 \Rightarrow 0 = 0$$

This is a true statement.

Solution set: $\left\{-\frac{1}{27}, \frac{1}{8}\right\}$

102. $2x^{-2/5} - x^{-1/5} - 1 = 0$

Let $u = x^{-1/5}$; then $u^2 = \left(x^{-1/5}\right)^2 = x^{-2/5}$.

$2u^2 - u - 1 = 0 \Rightarrow (2u+1)(u-1) = 0$

$u = -\frac{1}{2}$ or $u = 1$

To find x, replace u with $x^{-1/5}$.

$$x^{-1/5} = -\tfrac{1}{2}$$
$$\left(x^{-1/5}\right)^{-5} = \left(-\tfrac{1}{2}\right)^{-5}$$
$$x = -32$$

or $x^{-1/5} = 1 \Rightarrow \left(x^{-1/5}\right)^{-5} = 1^{-5} \Rightarrow x = 1$

Solution set: $\{-32, 1\}$

103. $16x^{-4} - 65x^{-2} + 4 = 0$

Let $u = x^{-2}$; then $u^2 = x^{-4}$. Solve the resulting equation by factoring:

$16u^2 - 65u + 4 = 0 \Rightarrow (u-4)(16u-1) = 0 \Rightarrow$
$u = 4$ or $u = \frac{1}{16}$

Find x by replacing u with x^{-2}:

$x^{-2} = 4 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$

$x^{-2} = \frac{1}{16} \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

Check $x = \frac{1}{2}$

$$16\left(\tfrac{1}{2}\right)^{-4} - 65\left(\tfrac{1}{2}\right)^{-2} + 4 = 0$$
$$16(2)^4 - 65(2)^2 + 4 \stackrel{?}{=} 0$$
$$16(16) - 65(4) + 4 = 0$$
$$256 - 260 + 4 = 0$$
$$0 = 0$$

This is a true statement, so $\frac{1}{2}$ is a solution.

Check $x = -\frac{1}{2}$

$$16\left(-\tfrac{1}{2}\right)^{-4} - 65\left(-\tfrac{1}{2}\right)^{-2} + 4 = 0$$
$$16(-2)^4 - 65(-2)^2 + 4 \stackrel{?}{=} 0$$
$$16(16) - 65(4) + 4 = 0$$
$$256 - 260 + 4 = 0$$
$$0 = 0$$

This is a true statement, so $-\frac{1}{2}$ is a solution.

Check $x = 4$

$$16(4)^{-4} - 65(4)^{-2} + 4 = 0$$
$$16\left(\tfrac{1}{4}\right)^4 - 65\left(\tfrac{1}{4}\right)^2 + 4 \stackrel{?}{=} 0$$
$$16\left(\tfrac{1}{256}\right) - 65\left(\tfrac{1}{16}\right) + 4 = 0$$
$$\tfrac{1}{16} - \tfrac{65}{16} + 4 = 0$$
$$0 = 0$$

This is a true statement, so 4 is a solution.

Check $x = -4$

$$16(-4)^{-4} - 65(-4)^{-2} + 4 = 0$$
$$16\left(-\tfrac{1}{4}\right)^4 - 65\left(-\tfrac{1}{4}\right)^2 + 4 \stackrel{?}{=} 0$$
$$16\left(\tfrac{1}{256}\right) - 65\left(\tfrac{1}{16}\right) + 4 = 0$$
$$\tfrac{1}{16} - \tfrac{65}{16} + 4 = 0$$
$$0 = 0$$

This is a true statement, so −4 is a solution.

Solution set: $\left\{\pm\frac{1}{2}, \pm 4\right\}$

104. $625x^{-4} - 125x^{-2} + 4 = 0$

Let $u = x^{-2}$; then $u^2 = x^{-4}$. Solve the resulting equation by factoring:

$625u^2 - 125u + 4 = 0 \Rightarrow$
$(25u-4)(25u-1) = 0 \Rightarrow u = \frac{4}{25}$ or $u = \frac{1}{25}$

Find x by replacing u with x^{-2}.

$x^{-2} = \frac{4}{25} \Rightarrow x^2 = \frac{25}{4} \Rightarrow x = \pm\frac{5}{2}$

$x^{-2} = \frac{1}{25} \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

Check $x = \frac{5}{2}$

$$625\left(\tfrac{5}{2}\right)^{-4} - 125\left(\tfrac{5}{2}\right)^{-2} + 4 \stackrel{?}{=} 0$$
$$625\left(\tfrac{2}{5}\right)^4 - 125\left(\tfrac{2}{5}\right)^2 + 4 = 0$$
$$625\left(\tfrac{16}{625}\right) - 125\left(\tfrac{4}{25}\right) + 4 = 0$$
$$16 - 20 + 4 = 0 \Rightarrow 0 = 0$$

This is a true statement, so $\frac{5}{2}$ is a solution.

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Check $x = -\frac{5}{2}$

$$625\left(-\tfrac{5}{2}\right)^{-4} - 125\left(-\tfrac{5}{2}\right)^{-2} + 4 \stackrel{?}{=} 0$$
$$625\left(-\tfrac{2}{5}\right)^{4} - 125\left(-\tfrac{2}{5}\right)^{2} + 4 = 0$$
$$625\left(\tfrac{16}{625}\right) - 125\left(\tfrac{4}{25}\right) + 4 = 0$$
$$16 - 20 + 4 = 0 \Rightarrow 0 = 0$$

This is a true statement, so $-\frac{5}{2}$ is a solution.

Check $x = 5$

$$625(5)^{-4} - 125(5)^{-2} + 4 \stackrel{?}{=} 0$$
$$625\left(\tfrac{1}{5}\right)^{4} - 125\left(\tfrac{1}{5}\right)^{2} + 4 = 0$$
$$625\left(\tfrac{1}{625}\right) - 125\left(\tfrac{1}{25}\right) + 4 = 0$$
$$1 - 5 + 4 = 0 \Rightarrow 0 = 0$$

This is a true statement, so 5 is a solution.

Check $x = -5$

$$625(-5)^{-4} - 125(-5)^{-2} + 4 \stackrel{?}{=} 0$$
$$625\left(-\tfrac{1}{5}\right)^{4} - 125\left(-\tfrac{1}{5}\right)^{2} + 4 = 0$$
$$625\left(\tfrac{1}{625}\right) - 125\left(\tfrac{1}{25}\right) + 4 = 0$$
$$1 - 5 + 4 = 0 \Rightarrow 0 = 0$$

This is a true statement, so −5 is a solution.

Solution set: $\left\{\pm\frac{5}{2}, \pm 5\right\}$

105. $d = k\sqrt{h}$ for h

$$\frac{d}{k} = \sqrt{h} \Rightarrow \frac{d^2}{k^2} = h$$

So, $h = \dfrac{d^2}{k^2}$.

106. $x^{2/3} + y^{2/3} = a^{2/3}$ for y

$$y^{2/3} = a^{2/3} - x^{2/3}$$
$$\left(y^{2/3}\right)^3 = \left(a^{2/3} - x^{2/3}\right)^3$$
$$y^2 = \left(a^{2/3} - x^{2/3}\right)^3$$
$$y = \pm\sqrt{(a^{2/3} - x^{2/3})^3}$$
$$y = \pm\left(a^{2/3} - x^{2/3}\right)^{3/2}$$

107. $m^{3/4} + n^{3/4} = 1$ for m

$$m^{3/4} = 1 - n^{3/4}$$

Raise both sides to the $\frac{4}{3}$ power.

$$(m^{3/4})^{4/3} = (1 - n^{3/4})^{4/3}$$
$$m = (1 - n^{3/4})^{4/3}$$

108. $\dfrac{1}{R} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$ for R

$$Rr_1r_2\left(\frac{1}{R}\right) = Rr_1r_2\left(\frac{1}{r_1}\right) + Rr_1r_2\left(\frac{1}{r_2}\right)$$

Multiply both sides by Rr_1r_2.

$$r_1r_2 = Rr_2 + Rr_1$$
$$r_1r_2 = R\left(r_2 + r_1\right)$$
$$\frac{r_1r_2}{r_2 + r_1} = R$$

So, $R = \dfrac{r_1r_2}{r_1 + r_2}$.

109. $\dfrac{E}{e} = \dfrac{R + r}{r}$ for e

$$er\left(\frac{E}{e}\right) = er\left(\frac{R + r}{r}\right)$$

Multiply both sides by er.

$$Er = eR + er$$
$$Er = e(R + r)$$
$$\frac{Er}{R + r} = e$$

So, $e = \dfrac{Er}{R + r}$.

110. $a^2 + b^2 = c^2$ for b

$$b^2 = c^2 - a^2$$
$$b = \pm\sqrt{c^2 - a^2}$$

111. $x - \sqrt{x} - 12 = 0$

Let $u = \sqrt{x}$; then $u^2 = x$. Solve the resulting equation by factoring.

$u^2 - u - 12 = 0 \Rightarrow (u - 4)(u + 3) = 0$

$u = 4$ or $u = -3$

To find x, replace u with $\sqrt{x}$.

$\sqrt{x} = 4 \Rightarrow \left(\sqrt{x}\right)^2 = 4^2 \Rightarrow x = 16$ or

$\sqrt{x} = -3 \Rightarrow \left(\sqrt{x}\right)^2 = (-3)^2 \Rightarrow x = 9$

But $\sqrt{9} \neq -3$

So when $u = -3$, there is no solution for x.

Solution set: {16}

112. $x-\sqrt{x}-12=0$

Solve by isolating $\sqrt{x}$, then squaring both sides.

$x-12=\sqrt{x}$

$(x-12)^2=(\sqrt{x})^2 \Rightarrow x^2-24x+144=x$

$x^2-25x+144=0 \Rightarrow (x-16)(x-9)=0$

$x=16$ or $x=9$

Check $x=16$.

$x-\sqrt{x}-12=0$

$16-\sqrt{16}-12\stackrel{?}{=}0$

$16-4-12=0 \Rightarrow 0=0$

This is a true statement.

Check $x=9$.

$x-\sqrt{x}-12=0$

$9-\sqrt{9}-12\stackrel{?}{=}0 \Rightarrow 9-3-12=0 \Rightarrow -6=0$

This is a false statement. 9 does not satisfy the equation.

Solution set: $\{16\}$

113. Answers will vary.

114. $3x-2\sqrt{x}-8=0$

Solve by substitution.

Let $u=\sqrt{x}$; then $u^2=x$. Solve the resulting equation by factoring.

$3u^2-2u-8=0 \Rightarrow (3u+4)(u-2)=0$

$u=-\frac{4}{3}$ or $u=2$

To find x, replace u with $\sqrt{x}$.

$\sqrt{x}=-\dfrac{4}{3}$ has no solution, because the result of a square root is never a negative real number.

$\sqrt{x}=2 \Rightarrow x=4$

Check $x=4$.

$3x-2\sqrt{x}-8=0$

$3(4)-2\sqrt{4}-8\stackrel{?}{=}0$

$3(4)-2(2)-8=0$

$12-4-8=0 \Rightarrow 0=0$

This is a true statement.

Solution set: $\{4\}$

Summary Exercises on Solving Equations

1. $4x-3=2x+3 \Rightarrow 2x-3=3 \Rightarrow$

$2x=6 \Rightarrow x=3$

Solution set: $\{3\}$

2. $5-(6x+3)=2(2-2x)$

$5-6x-3=4-4x$

$2-6x=4-4x \Rightarrow 2=4+2x$

$-2=2x \Rightarrow -1=x$

Solution set: $\{-1\}$.

3. $x(x+6)=9 \Rightarrow x^2+6x=9 \Rightarrow x^2+6x-9=0$

Solve by completing the square.

$x^2+6x+9=9+9$

Note: $\left[\frac{1}{2}\cdot 6\right]^2=3^2=9$

$(x+3)^2=18 \Rightarrow x+3=\pm\sqrt{18} \Rightarrow$

$x+3=\pm 3\sqrt{2} \Rightarrow x=-3\pm 3\sqrt{2}$

Solve by the quadratic formula.

Let $a=1, b=6,$ and $c=-9.$

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-6\pm\sqrt{6^2-4(1)(-9)}}{2(1)} \\ &= \frac{-6\pm\sqrt{36+36}}{2}=\frac{-6\pm\sqrt{72}}{2} \\ &= \frac{-6\pm 6\sqrt{2}}{2}=-3\pm 3\sqrt{2} \end{aligned}$$

Solution set: $\{-3\pm 3\sqrt{2}\}$

4. $x^2=8x-12 \Rightarrow x^2-8x+12=0$

Solve by factoring.

$x^2-8x+12=0 \Rightarrow (x-2)(x-6)=0 \Rightarrow$

$x=2$ or $x=6$

Solve by completing the square.

$x^2-8x+16=-12+16$

Note: $\left[\frac{1}{2}\cdot(-8)\right]^2=(-4)^2=16$

$(x-4)^2=4 \Rightarrow x-4=\pm\sqrt{4} \Rightarrow$

$x-4=\pm 2 \Rightarrow x=4\pm 2 \Rightarrow$

$x=4-2=2$ or $x=4+2=6$

Solve by the quadratic formula.

Let $a=1,\ b=-8,$ and $c=12.$

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(12)}}{2(1)} \end{aligned}$$

$$x=\frac{8\pm\sqrt{64-48}}{2}=\frac{8\pm\sqrt{16}}{2}=\frac{8\pm 4}{2}=4\pm 2$$

$x=4-2=2$ or $x=4+2=6$

Solution set: $\{2,6\}$

5. $$\sqrt{x+2}+5=\sqrt{x+15}$$
$$\left(\sqrt{x+2}+5\right)^2=\left(\sqrt{x+15}\right)^2$$
$$(x+2)+10\sqrt{x+2}+25=x+15\Rightarrow$$
$$x+27+10\sqrt{x+2}=x+15\Rightarrow$$
$$27+10\sqrt{x+2}=15\Rightarrow$$
$$10\sqrt{x+2}=-12\Rightarrow 5\sqrt{x+2}=-6$$
$$\left(5\sqrt{x+2}\right)^2=(-6)^2\Rightarrow$$
$$25(x+2)=36\Rightarrow 25x+50=36$$
$$25x=-14\Rightarrow x=-\frac{14}{25}$$

Check $x=-\frac{14}{25}$.

$$\sqrt{x+2}+5=\sqrt{x+15}$$
$$\sqrt{-\frac{14}{25}+2}+5\overset{?}{=}\sqrt{-\frac{14}{25}+15}$$
$$\sqrt{-\frac{14}{25}+\frac{50}{25}}+5=\sqrt{-\frac{14}{25}+\frac{375}{25}}$$
$$\sqrt{\frac{36}{25}}+5=\sqrt{\frac{361}{25}}\Rightarrow\frac{6}{5}+\frac{25}{5}=\frac{19}{5}\Rightarrow\frac{31}{5}=\frac{19}{5}$$

This is a false statement. Solution set: $\varnothing$

6. $\dfrac{5}{x+3}-\dfrac{6}{x-2}=\dfrac{3}{x^2+x-6}$ or

$$\frac{5}{x+3}-\frac{6}{x-2}=\frac{3}{(x+3)(x-2)}$$

The least common denominator is $(x+3)(x-2)$, which is equal to 0 if $x=-3$ or $x=2$. Therefore, -3 and 2 cannot possibly be solutions of this equation.

$$(x+3)(x-2)\left[\frac{5}{x+3}-\frac{6}{x-2}\right]$$
$$=(x+3)(x-2)\left(\frac{3}{(x+3)(x-2)}\right)$$
$$5(x-2)-6(x+3)=3$$
$$5x-10-6x-18=3$$
$$-x-28=3\Rightarrow -x=31\Rightarrow x=-31$$

The restrictions $x\neq-3$ and $x\neq2$ do not affect the result. Therefore, the solution set is $\{-31\}$.

7. $\dfrac{3x+4}{3}-\dfrac{2x}{x-3}=x$

The least common denominator is $3(x-3)$, which is equal to 0 if $x=3$. Therefore, 3 cannot possibly be a solution of this equation.

$$3(x-3)\left[\frac{3x+4}{3}-\frac{2x}{x-3}\right]=3(x-3)(x)$$
$$(x-3)(3x+4)-3(2x)=3x(x-3)$$
$$3x^2+4x-9x-12-6x=3x^2-9x$$
$$3x^2-11x-12=3x^2-9x$$
$$-11x-12=-9x$$
$$-12=2x\Rightarrow -6=x$$

The restriction $x\neq3$ does not affect the result. Therefore, the solution set is $\{-6\}$.

8. $\dfrac{x}{2}+\dfrac{4}{3}x=x+5\Rightarrow 6\left(\dfrac{x}{2}+\dfrac{4}{3}x\right)=6(x+5)$

$$3x+8x=6x+30\Rightarrow 11x=6x+30$$
$$5x=30\Rightarrow x=6$$

Solution set: $\{6\}$

9. $5-\dfrac{2}{x}+\dfrac{1}{x^2}=0$

The least common denominator is x^2, which is equal to 0 if $x=0$. Therefore, 0 cannot possibly be a solution of this equation.

$$x^2\left[5-\frac{2}{x}+\frac{1}{x^2}\right]=x^2(0)\Rightarrow 5x^2-2x+1=0$$

Solve by completing the square.

$$x^2-\frac{2}{5}x+\frac{1}{5}=0 \qquad \text{Multiply by } \tfrac{1}{5}.$$
$$x^2-\frac{2}{5}x+\frac{1}{25}=-\frac{1}{5}+\frac{1}{25}$$
$$\text{Note:}\left[\frac{1}{2}\cdot\left(-\frac{2}{5}\right)\right]^2=\left(-\frac{1}{5}\right)^2=\frac{1}{25}$$
$$\left(x-\frac{1}{5}\right)^2=-\frac{5}{25}+\frac{1}{25}=\frac{-4}{25}$$
$$x-\frac{1}{5}=\pm\sqrt{\frac{-4}{25}}$$
$$x-\frac{1}{5}=\pm\frac{2}{5}i\Rightarrow x=\frac{1}{5}\pm\frac{2}{5}i$$

Solve by the quadratic formula.
Let $a=5$, $b=-2$, and $c=1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(5)(1)}}{2(5)}$$
$$=\frac{2\pm\sqrt{4-20}}{10}=\frac{2\pm\sqrt{-16}}{10}$$
$$=\frac{2\pm4i}{10}=\frac{2}{10}\pm\frac{4}{10}i=\frac{1}{5}\pm\frac{2}{5}i$$

The restriction $x\neq0$ does not affect the result. Therefore, the solution set is $\left\{\frac{1}{5}\pm\frac{2}{5}i\right\}$.

10. $(2x+1)^2=9\Rightarrow 2x+1=\pm\sqrt{9}\Rightarrow 2x+1=\pm3$

$$2x=-1\pm3\Rightarrow x=\frac{-1\pm3}{2}$$
$$x=\frac{-1-3}{2}=\frac{-4}{2}=-2 \text{ or } x=\frac{-1+3}{2}=\frac{2}{2}=1$$

Solution set: $\{-2,1\}$

11. $x^{-2/5} - 2x^{-1/5} - 15 = 0$

Let $u = x^{-1/5}$; then $u^2 = \left(x^{-1/5}\right)^2 = x^{-2/5}$.

$u^2 - 2u - 15 = 0 \Rightarrow (u+3)(u-5) = 0 \Rightarrow$
$u = -3$ or $u = 5$

To find x, replace u with $x^{-1/5}$.

$x^{-1/5} = -3 \Rightarrow \left(x^{-1/5}\right)^{-5} = (-3)^{-5}$
$x = \frac{1}{(-3)^5} \Rightarrow x = -\frac{1}{243}$ or

$x^{-1/5} = 5 \Rightarrow \left(x^{-1/5}\right)^{-5} = 5^{-5} \Rightarrow x = \frac{1}{5^5} \Rightarrow$
$x = \frac{1}{3125}$

Check $x = -\frac{1}{243}$.

$x^{-2/5} - 2x^{-1/5} - 15 = 0$
$\left(-\frac{1}{243}\right)^{-2/5} - 2\left(-\frac{1}{243}\right)^{-1/5} - 15 = 0\ ?$
$(-243)^{2/5} - 2(-243)^{1/5} - 15 = 0$
$\left[(-243)^{1/5}\right]^2 - 2(-3) - 15 = 0$
$(-3)^2 + 6 - 15 = 0 \Rightarrow 9 + 6 - 15 = 0 \Rightarrow 0 = 0$

This is a true statement. $-\frac{1}{243}$ is a solution.

Check $x = \frac{1}{3125}$.

$x^{-2/5} - 2x^{-1/5} - 15 = 0$
$\left(\frac{1}{3125}\right)^{-2/5} - 2\left(\frac{1}{3125}\right)^{-1/5} - 15 \stackrel{?}{=} 0$
$(3125)^{2/5} - 2(3125)^{1/5} - 15 = 0$
$\left[(3125)^{1/5}\right]^2 - 2(5) - 15 = 0$
$5^2 - 10 - 15 = 0$
$25 - 10 - 15 = 0 \Rightarrow 0 = 0$

This is a true statement. $\frac{1}{3125}$ is a solution.

Solution set: $\left\{-\frac{1}{243}, \frac{1}{3125}\right\}$

12. $\sqrt{x+2} + 1 = \sqrt{2x+6}$
$\left(\sqrt{x+2} + 1\right)^2 = \left(\sqrt{2x+6}\right)^2$
$x + 2 + 2\sqrt{x+2} + 1 = 2x + 6$
$x + 3 + 2\sqrt{x+2} = 2x + 6$
$2\sqrt{x+2} = x + 3$
$\left(2\sqrt{x+2}\right)^2 = (x+3)^2$
$4(x+2) = x^2 + 6x + 9$
$4x + 8 = x^2 + 6x + 9$
$0 = x^2 + 2x + 1 = (x+1)^2$
$0 = x + 1 \Rightarrow -1 = x$

Check $x = -1$.

$\sqrt{x+2} + 1 = \sqrt{2x+6}$
$\sqrt{-1+2} + 1 \stackrel{?}{=} \sqrt{2(-1)+6}$
$\sqrt{1} + 1 = \sqrt{-2+6} \Rightarrow 2 = \sqrt{4} \Rightarrow 2 = 2$

This is a true statement.

Solution set: $\{-1\}$

13. $x^4 - 3x^2 - 4 = 0$

Let $u = x^2$; then $u^2 = x^4$.

$u^2 - 3u - 4 = 0 \Rightarrow (u+1)(u-4) = 0 \Rightarrow$.
$u = -1$ or $u = 4$

To find x, replace x with x^2.

$x^2 = -1 \Rightarrow x = \pm\sqrt{-1} = \pm i$ or
$x^2 = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2$

Solution set: $\{\pm i, \pm 2\}$

14. $1.2x + 0.3 = 0.7x - 0.9$
$10[1.2x + 0.3] = 10[0.7x - 0.9]$
$12x + 3 = 7x - 9 \Rightarrow 5x + 3 = -9 \Rightarrow$
$5x = -12 \Rightarrow x = -2.4$

Solution set: $\{-2.4\}$

15. $\sqrt[3]{2x+1} = \sqrt[3]{9} \Rightarrow \left(\sqrt[3]{2x+1}\right)^3 = \left(\sqrt[3]{9}\right)^3$
$2x + 1 = 9 \Rightarrow 2x = 8 \Rightarrow x = 4$

Check $x = 4$.

$\sqrt[3]{2x+1} = \sqrt[3]{9} \Rightarrow \sqrt[3]{2(4)+1} \stackrel{?}{=} \sqrt[3]{9}$
$\sqrt[3]{8+1} = \sqrt[3]{9} \Rightarrow \sqrt[3]{9} = \sqrt[3]{9}$

This is a true statement.

Solution set: $\{4\}$

16. $3x^2 - 2x = -1 \Rightarrow 3x^2 - 2x + 1 = 0$

Solve by completing the square.

$3x^2 - 2x = -1$
$x^2 - \frac{2}{3}x = -\frac{1}{3}$ Multiply by $\frac{1}{3}$.
$x^2 - \frac{2}{3}x + \frac{1}{9} = -\frac{1}{3} + \frac{1}{9}$
Note: $\left[\frac{1}{2}\cdot\left(-\frac{2}{3}\right)\right]^2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$
$\left(x - \frac{1}{3}\right)^2 = -\frac{3}{9} + \frac{1}{9} = \frac{-2}{9}$
$x - \frac{1}{3} = \pm\sqrt{\frac{-2}{9}}$
$x - \frac{1}{3} = \pm\frac{\sqrt{2}}{3}i \Rightarrow x = \frac{1}{3} \pm \frac{\sqrt{2}}{3}i$

Solve by the quadratic formula.

Let $a = 3$, $b = -2$, and $c = 1$.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(1)}}{2(3)} = \frac{2 \pm \sqrt{4-12}}{6} = \frac{2 \pm \sqrt{-8}}{6} = \frac{2 \pm 2i\sqrt{2}}{6} = \frac{2}{6} \pm \frac{2\sqrt{2}}{6}i = \frac{1}{3} \pm \frac{\sqrt{2}}{3}i$$

Solution set: $\left\{\frac{1}{3} \pm \frac{\sqrt{2}}{3}i\right\}$.

17. $3[2x - (6 - 2x) + 1] = 5x$

$3(2x - 6 + 2x + 1) = 5x$

$3(4x - 5) = 5x$

$12x - 15 = 5x \Rightarrow -15 = -7x \Rightarrow$

$\frac{-15}{-7} = x \Rightarrow x = \frac{15}{7}$

Solution set: $\left\{\frac{15}{7}\right\}$

18. $\sqrt{x} + 1 = \sqrt{11 - \sqrt{x}}$

$\left(\sqrt{x} + 1\right)^2 = \left(\sqrt{11 - \sqrt{x}}\right)^2$

$x + 2\sqrt{x} + 1 = 11 - \sqrt{x}$

$x + 3\sqrt{x} + 1 = 11 \Rightarrow 3\sqrt{x} = 10 - x \Rightarrow$

$\left(3\sqrt{x}\right)^2 = (10 - x)^2$

$9x = 100 - 20x + x^2$

$0 = 100 - 29x + x^2$

$0 = x^2 - 29x + 100$

$0 = (x - 4)(x - 25) \Rightarrow$

$x = 4$ or $x = 25$

Check $x = 4$.

$\sqrt{x} + 1 = \sqrt{11 - \sqrt{x}}$

$\sqrt{4} + 1 \stackrel{?}{=} \sqrt{11 - \sqrt{4}}$

$2 + 1 = \sqrt{11 - 2} \Rightarrow 3 = \sqrt{9} \Rightarrow 3 = 3$

This is a true statement.

Check $x = 25$.

$\sqrt{x} + 1 = \sqrt{11 - \sqrt{x}}$

$\sqrt{25} + 1 \stackrel{?}{=} \sqrt{11 - \sqrt{25}}$

$5 + 1 = \sqrt{11 - 5} \Rightarrow 6 = \sqrt{6}$

This is a false statement.

Solution set: $\{4\}$

19. $(14 - 2x)^{2/3} = 4$

$[(14 - 2x)^{2/3}]^3 = 4^3$

$(14 - 2x)^2 = 64$

$196 - 56x + 4x^2 = 64$

$4x^2 - 56x + 132 = 0$

$4(x^2 - 14x + 33) = 0$

$4(x - 3)(x - 11) = 0 \Rightarrow x = 3$ or $x = 11$

Check $x = 3$.

$(14 - 2x)^{2/3} = 4$

$[14 - 2(3)]^{2/3} \stackrel{?}{=} 4$

$(14 - 6)^{2/3} = 4 \Rightarrow 8^{2/3} = 4$

$\left(8^{1/3}\right)^2 = 4 \Rightarrow 2^2 = 4 \Rightarrow 4 = 4$

This is a true statement.

Check $x = 11$.

$(14 - 2x)^{2/3} = 4$

$[14 - 2(11)]^{2/3} \stackrel{?}{=} 4$

$(14 - 22)^{2/3} = 4 \Rightarrow (-8)^{2/3} = 4$

$\left[(-8)^{1/3}\right]^2 = 4 \Rightarrow (-2)^2 = 4 \Rightarrow 4 = 4$

This is a true statement.

Solution set: $\{3, 11\}$

20. $-x^{-2} + 2x^{-1} = 1$

$-x^{-2} + 2x^{-1} - 1 = 0$

$x^{-2} - 2x^{-1} + 1 = 0$

Let $u = x^{-1}$; then $u^2 = x^{-2}$.

$u^2 - 2u + 1 = 0 \Rightarrow (u - 1)^2 = 0 \Rightarrow u = 1$

To find x, replace u with x^{-1}.

$x^{-1} = 1 \Rightarrow x = 1$

Solution set: $\{1\}$

21. $\frac{3}{x - 3} = \frac{3}{x - 3}$

The least common denominator is $(x - 3)$ which is equal to 0 if $x = 3$. Therefore, 3 cannot possibly be a solution of this equation.

Solution set: $\{x \mid x \neq 3\}$.

22. $a^2 + b^2 = c^2$ for a

$a^2 = c^2 - b^2 \Rightarrow a = \pm\sqrt{c^2 - b^2}$

Section 1.7 Inequalities

1. F. The inequality $x < -6$ includes all real numbers less than −6 not including −6. The correct interval notation is $(-\infty, -6)$.

2. J. The inequality $x \le 6$ includes all real numbers less than or equal to 6, so it includes 6. The correct interval notation is $(-\infty, 6]$.

3. A. The inequality $-2 < x \le 6$ includes all real numbers from –2 to 6, not including –2, but including 6. The correct interval notation is (–2, 6].

4. H. The inequality $x^2 \ge 0$ includes all real numbers because the square of any real number is greater than or equal to 0. The correct interval notation is $(-\infty, \infty)$.

5. I. The inequality $x \ge -6$ includes all real numbers greater than or equal to –6, so it includes –6. The correct interval notation is $[-6, \infty)$.

6. D. The inequality $6 \le x$ includes all real numbers greater than or equal to 6, so it includes 6. The correct interval notation is $[6, \infty)$.

7. B. The interval shown on the number line includes all real numbers between –2 and 6, including –2, but not including 6. The correct interval notation is [–2, 6).

8. G. The interval shown on the number line includes all real numbers between 0 and 8, not including 0 or 8. The correct interval notation is (0, 8).

9. E. The interval shown on the number line includes all real numbers less than −3, not including −3, and greater than 3, not including 3. The correct interval notation is $(-\infty, -3) \cup (3, \infty)$,

10. C. The interval includes all real numbers less than or equal to –6, so it includes –6, The correct interval notation is $(-\infty, -6]$.

11. Answers wil1 vary. Sample answer: A square bracket is used to show that a number is part of the solution set, while a parenthesis is used to indicate that a number is not part of the solution set.

12. D. $-8 < x < -10$ means $-8 < x$ and $x < -10$, which is equivalent to $x > -8$ and $x < -10$. There is no real number that is simultaneously to the right of –8 and to the left of –10 on a number line.

13. $-2x + 8 \le 16 \Rightarrow -2x + 8 - 8 \le 16 - 8 \Rightarrow$

$$-2x \le 8 \Rightarrow \frac{-2x}{-2} \ge \frac{8}{-2} \Rightarrow x \ge -4$$

Solution set: $[-4, \infty)$

14. $-3x - 8 \le 7 \Rightarrow -3x - 8 + 8 \le 7 + 8 \Rightarrow$

$$-3x \le 15 \Rightarrow \frac{-3x}{-3} \ge \frac{15}{-3} \Rightarrow x \ge -5$$

Solution set: $[-5, \infty)$

15.

$$-2x - 2 \le 1 + x$$
$$-2x - 2 + 2 \le 1 + x + 2$$
$$-2x \le x + 3 \Rightarrow -2x - x \le 3 \Rightarrow$$
$$-3x \le 3 \Rightarrow \frac{-3x}{-3} \ge \frac{3}{-3} \Rightarrow x \ge -1$$

Solution set: $[-1, \infty)$

16.

$$-4x + 3 \ge -2 + x$$
$$-4x + 3 - 3 \ge -2 - 3 + x$$
$$-4x \ge -5 + x \Rightarrow -4x - x \ge -5 \Rightarrow$$
$$-5x \ge -5 \Rightarrow \frac{-5x}{-5} \le \frac{-5}{-5} \Rightarrow x \le 1$$

Solution set: $(-\infty, 1]$

17. $3(x+5) + 1 \ge 5 + 3x$

$3x + 15 + 1 \ge 5 + 3x \Rightarrow 16 \ge 5$

The inequality is true when x is any real number.

Solution set: $(-\infty, \infty)$

18. $6x - (2x + 3) \ge 4x - 5$

$6x - 2x - 3 \ge 4x - 5 \Rightarrow 4x - 3 \ge 4x - 5$

$4x - 4x - 3 \ge 4x - 5 - 4x \Rightarrow -3 \ge -5$

The inequality is true when x is any real number.

Solution set: $(-\infty, \infty)$

19.

$$8x - 3x + 2 < 2(x + 7)$$
$$5x + 2 < 2x + 14$$
$$5x + 2 - 2x < 2x + 14 - 2x$$
$$3x + 2 < 14 \Rightarrow 3x + 2 - 2 < 14 - 2 \Rightarrow$$
$$3x < 12 \Rightarrow \frac{3x}{3} < \frac{12}{3} \Rightarrow x < 4$$

Solution set: $(-\infty, 4)$

20. $2-4x+5(x-1)<-6(x-2)$

$$2-4x+5x-5<-6x+12$$
$$x-3<-6x+12$$
$$x-3+6x<-6x+12+6x$$
$$7x-3<12 \Rightarrow 7x-3+3<12+3 \Rightarrow$$
$$7x<15 \Rightarrow \frac{7x}{7}<\frac{15}{7} \Rightarrow x<\frac{15}{7}$$

Solution set: $\left(-\infty, \frac{15}{7}\right)$

21. $\frac{4x+7}{-3} \le 2x+5$

$$(-3)\left(\frac{4x+7}{-3}\right) \ge (-3)(2x+5)$$
$$4x+7 \ge -6x-15$$
$$4x+7+6x \ge -6x-15+6x$$
$$10x+7 \ge -15$$
$$10x+7-7 \ge -15-7 \Rightarrow 10x \ge -22 \Rightarrow$$
$$\frac{10x}{10} \ge \frac{-22}{10} \Rightarrow x \ge -\frac{11}{5}$$

Solution set: $\left[-\frac{11}{5}, \infty\right)$

22. $\frac{2x-5}{-8} \le 1-x$

$$(-8)\left(\frac{2x-5}{-8}\right) \ge (-8)(1-x)$$
$$2x-5 \ge -8+8x$$
$$2x-5-8x \ge -8+8x-8x$$
$$-6x-5 \ge -8$$
$$-6x-5+5 \ge -8+5 \Rightarrow -6x \ge -3$$
$$\frac{-6x}{-6} \le \frac{-3}{-6} \Rightarrow x \le \frac{1}{2}$$

Solution set: $\left(-\infty, \frac{1}{2}\right]$

23. $\frac{1}{3}x+\frac{2}{5}x-\frac{1}{2}(x+3) \le \frac{1}{10}$

$$30\left[\frac{1}{3}x+\frac{2}{5}x-\frac{1}{2}(x+3)\right] \le 30\left[\frac{1}{10}\right]$$
$$10x+12x-15(x+3) \le 3$$
$$10x+12x-15x-45 \le 3$$
$$7x-45 \le 3$$
$$7x-45+45 \le 3+45$$
$$7x \le 48$$
$$\frac{7x}{7} \le \frac{48}{7} \Rightarrow x \le \frac{48}{7}$$

Solution set: $\left(-\infty, \frac{48}{7}\right]$

24. $-\frac{2}{3}x-\frac{1}{6}x+\frac{2}{3}(x+1) \le \frac{4}{3}$

$$(-6)\left[-\frac{2}{3}x-\frac{1}{6}x+\frac{2}{3}(x+1)\right] \ge (-6)\left[\frac{4}{3}\right]$$
$$4x+x-4(x+1) \ge -8$$
$$4x+x-4x-4 \ge -8$$
$$x-4 \ge -8$$
$$x-4+4 \ge -8+4$$
$$x \ge -4$$

Solution set: $[-4, \infty)$

25. $C = 50x + 5000$; $R = 60x$
The product will at least break even when $R \ge C$. Set $R \ge C$ and solve for x.
$60x \ge 50x+5000 \Rightarrow 10x \ge 5000 \Rightarrow x \ge 500$
The break-even point is at $x = 500$.
This product will at least break even if the number of units of picture frames produced is in the interval $[500, \infty)$.

26. $C = 100x + 6000$; $R = 500x$
The product will at least break even when $R \ge C$. Set $R \ge C$ and solve for x.
$500x \ge 100x+6000 \Rightarrow 400x \ge 6000 \Rightarrow x \ge 15$
The break-even point is $x = 15$.
The product will at least break even when the number of units of baseball caps produced is in the interval $[15, \infty)$.

27. $C = 105x + 900$; $R = 85x$
The product will at least break even when $R \ge C$. Set $R \ge C$ and solve for x.
$85x \ge 105x+900 \Rightarrow -20x \ge 900 \Rightarrow x \le -45$
The product will never break even.

28. $C = 70x + 500$; $R = 60x$
The product will at least break even when $R \ge C$. Set $R \ge C$ and solve for x.
$60x \ge 70x+500 \Rightarrow -10x \ge 500 \Rightarrow x \le -50$
The product will never break even.

29. $-5<5+2x<11$

$$-5-5<5+2x-5<11-5$$
$$-10<2x<6$$
$$\frac{-10}{2}<\frac{2x}{2}<\frac{6}{2}$$
$$-5<x<3$$

Solution set: $(-5, 3)$

30. $-7 < 2+3x < 5$
$-7-2 < 2+3x-2 < 5-2$
$-9 < 3x < 3$
$\frac{-9}{3} < \frac{3x}{3} < \frac{3}{3}$
$-3 < x < 1$
Solution set: $(-3, 1)$

31. $10 \le 2x+4 \le 16$
$10-4 \le 2x+4-4 \le 16-4$
$6 \le 2x \le 12$
$\frac{6}{2} \le \frac{2x}{2} \le \frac{12}{2}$
$3 \le x \le 6$
Solution set: $[3, 6]$

32. $-6 \le 6x+3 \le 21$
$-6-3 \le 6x+3-3 \le 21-3$
$-9 \le 6x \le 18$
$\frac{-9}{6} \le \frac{6x}{6} \le \frac{18}{6}$
$-\frac{3}{2} \le x \le 3$
Solution set: $\left[-\frac{3}{2}, 3\right]$

33. $-11 > -3x+1 > -17$
$-11-1 > -3x+1-1 > -17-1$
$-12 > -3x > -18$
$\frac{-12}{-3} < \frac{-3x}{-3} < \frac{-18}{-3}$
$4 < x < 6$
Solution set: $(4, 6)$

34. $2 > -6x+3 > -3$
$2-3 > -6x+3-3 > -3-3$
$-1 > -6x > -6$
$\frac{-1}{-6} < \frac{-6x}{-6} < \frac{-6}{-6}$
$\frac{1}{6} < x < 1$
Solution set: $\left(\frac{1}{6}, 1\right)$

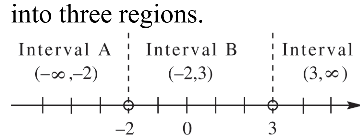
35. $-4 \le \frac{x+1}{2} \le 5$
$2(-4) \le 2\left(\frac{x+1}{2}\right) \le 2(5)$
$-8 \le x+1 \le 10$
$-8-1 \le x+1-1 \le 10-1$
$-9 \le x \le 9$
Solution set: $[-9, 9]$

36. $-5 \le \frac{x-3}{3} \le 1$
$3(-5) \le 3\left(\frac{x-3}{3}\right) \le 3(1)$
$-15 \le x-3 \le 3$
$-15+3 \le x-3+3 \le 3+3$
$-12 \le x \le 6$
Solution set: $[-12, 6]$

37. $-3 \le \frac{3x-4}{-5} < 4$
$(-5)(-3) \ge (-5)\left(\frac{3x-4}{-5}\right) > (-5)(4)$
$15 \ge 3x-4 > -20$
$15+4 \ge 3x-4+4 > -20+4$
$19 \ge 3x > -16$
$-\frac{16}{3} < x \le \frac{19}{3}$
Solution set: $\left(-\frac{16}{3}, \frac{19}{3}\right]$

38. $1 \le \frac{4x-5}{-2} < 9$
$(-2)(1) \ge (-2)\left(\frac{4x-5}{-2}\right) > (-2)(9)$
$-2 \ge 4x-5 > -18$
$-2+5 \ge 4x-5+5 > -18+5$
$3 \ge 4x > -13$
$\frac{3}{4} \ge \frac{4x}{4} > \frac{-13}{4} \Rightarrow -\frac{13}{4} < x \le \frac{3}{4}$
Solution set: $\left(-\frac{13}{4}, \frac{3}{4}\right]$

39. $x^2 - x - 6 > 0$
Step 1: Find the values of x that satisfy $x^2 - x - 6 = 0$.
$x^2 - x - 6 = 0 \Rightarrow (x+2)(x-3) = 0$
$x+2=0 \Rightarrow x=-2$ or $x-3=0 \Rightarrow x=3$
Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2 - x - 6 > 0$.

Interval	Test Value	Is $x^2 - x - 6 > 0$ True or False?
A: $(-\infty, -2)$	-3	$(-3)^2 - (-3) - 6 \overset{?}{>} 0$ $6 > 0$ True

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Interval	Test Value	Is $x^2 - x - 6 > 0$ True or False?
B: $(-2, 3)$	0	$0^2 - 0 - 6 \overset{?}{>} 0$ $-6 > 0$ False
C: $(3, \infty)$	4	$4^2 - 4 - 6 \overset{?}{>} 0$ $6 > 0$ True

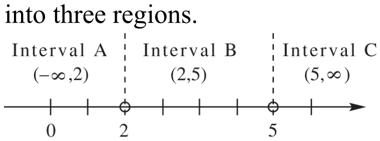
Solution set: $(-\infty, -2) \cup (3, \infty)$

40. $x^2 - 7x + 10 > 0$

Step 1: Find the values of *x* that satisfy the corresponding equation.

$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$
$$x - 2 = 0 \Rightarrow x = 2 \quad \text{or} \quad x - 5 = 0 \Rightarrow x = 5$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2 - 7x + 10 > 0$.

Interval	Test Value	Is $x^2 - 7x + 10 > 0$ True or False?
A: $(-\infty, 2)$	0	$0^2 - 7(0) + 10 \overset{?}{>} 0$ $10 > 0$ True
B: $(2, 5)$	3	$3^2 - 7(3) + 10 \overset{?}{>} 0$ $-2 > 0$ False
C: $(5, \infty)$	6	$6^2 - 7(6) + 10 \overset{?}{>} 0$ $4 > 0$ True

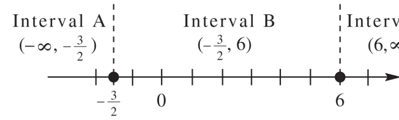
Solution set: $(-\infty, 2) \cup (5, \infty)$

41. $2x^2 - 9x \le 18$

Step 1: Find the values of *x* that satisfy the corresponding equation.

$$2x^2 - 9x = 18$$
$$2x^2 - 9x - 18 = 0$$
$$(2x+3)(x-6) = 0$$
$$2x + 3 = 0 \Rightarrow x = -\tfrac{3}{2} \quad \text{or} \quad x - 6 = 0 \Rightarrow x = 6$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $2x^2 - 9x \le 18$

Interval	Test Value	Is $2x^2 - 9x \le 18$ True or False?
A: $\left(-\infty, -\frac{3}{2}\right)$	−2	$2(-2)^2 - 9(-2) \overset{?}{\le} 18$ $26 \le 18$ False
B: $\left(-\frac{3}{2}, 6\right)$	0	$2(0)^2 - 9(0) \overset{?}{\le} 18$ $0 \le 18$ True
C: $(6, \infty)$	7	$2(7)^2 - 9(7) \overset{?}{\le} 18$ $35 \le 18$ False

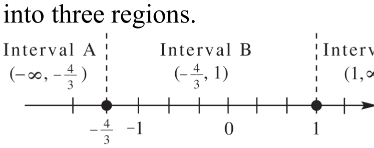
Solution set: $\left[-\frac{3}{2}, 6\right]$

42. $3x^2 + x \le 4$

Step 1: Find the values of *x* that satisfy the corresponding equation.

$$3x^2 + x = 4 \Rightarrow 3x^2 + x - 4 = 0 \Rightarrow$$
$$(3x+4)(x-1) = 0$$
$$3x + 4 = 0 \Rightarrow x = -\tfrac{4}{3} \quad \text{or} \quad x - 1 = 0 \Rightarrow x = 1$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $3x^2 + x \le 4$

Interval	Test Value	Is $3x^2 + x \le 4$ True or False?
A: $\left(-\infty, -\frac{4}{3}\right)$	−2	$3(-2)^2 + (-2) \overset{?}{\le} 4$ $10 \le 4$ False
B: $\left(-\frac{4}{3}, 1\right)$	0	$3(0)^2 + (0) \overset{?}{\le} 4$ $0 \le 4$ True
C: $(1, \infty)$	2	$3(2)^2 + 2 \overset{?}{\le} 4$ $14 \le 4$ False

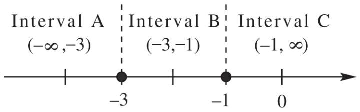
Solution set: $\left[-\frac{4}{3}, 1\right]$

43. $-x^2 - 4x - 6 \le -3$

Step 1: Find the values of x that satisfy the corresponding equation.

$-x^2 - 4x - 6 = -3$
$x^2 + 4x + 3 = 0$
$(x+3)(x+1) = 0$
$x + 3 = 0 \Rightarrow x = -3$ or $x + 1 = 0 \Rightarrow x = -1$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $-x^2 - 4x - 6 \le -3$

Interval	Test Value	Is $-x^2 - 4x - 6 \le -3$ True or False?
A: $(-\infty, -3)$	-4	$-(-4)^2 - 4(-4) - 6 \overset{?}{\le} -3$ $-6 \le -3$ True
B: $(-3, -1)$	-2	$-(-2)^2 - 4(-2) - 6 \overset{?}{\le} -3$ $-2 \le -3$ False
C: $(-1, \infty)$	0	$-(0)^2 - 4(0) - 6 \overset{?}{\le} -3$ $-6 \le -3$ True

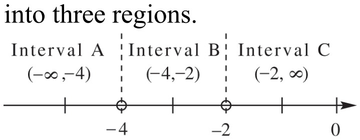
Solution set: $(-\infty, -3] \cup [-1, \infty)$

44. $-x^2 - 6x - 16 > -8$

Step 1: Find the values of x that satisfy the corresponding equation.

$-x^2 - 6x - 16 = -8$
$x^2 + 6x + 8 = 0$
$(x+4)(x+2) = 0$
$x + 4 = 0 \Rightarrow x = -4$ or $x + 2 = 0 \Rightarrow x = -2$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $-x^2 - 6x - 16 > -8$.

Interval	Test Value	Is $-x^2 - 6x - 16 > -8$ True or False?
A: $(-\infty, -4)$	-5	$-(-5)^2 - 6(-5) - 16 > -8$ $-11 > -8$ False
B: $(-4, -2)$	-3	$-(-3)^2 - 6(-3) - 16 > -8$ $-7 > -8$ True
C: $(-2, \infty)$	0	$-(0)^2 - 6(0) - 16 > -8$ $-16 > -8$ False

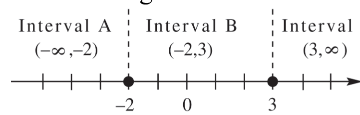
Solution set: $(-4, -2)$

45. $x(x-1) \le 6 \Rightarrow x^2 - x \le 6 \Rightarrow x^2 - x - 6 \le 0$

Step 1: Find the values of x that satisfy $x^2 - x - 6 = 0$.

$x^2 - x - 6 = 0$
$(x+2)(x-3) = 0$
$x + 2 = 0 \Rightarrow x = -2$ or $x - 3 = 0 \Rightarrow x = 3$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x(x-1) \le 6$.

Interval	Test Value	Is $x(x-1) \le 6$ True or False?
A: $(-\infty, -2)$	-3	$-3(-3-1) \overset{?}{\le} 6$ $12 \le 6$ False
B: $(-2, 3)$	0	$0(0-1) \overset{?}{\le} 6$ $0 \le 6$ True
C: $(3, \infty)$	4	$4(4-1) \overset{?}{\le} 6$ $12 \le 6$ False

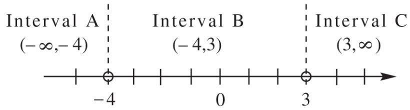
Solution set: $[-2, 3]$

46. $x(x+1)<12 \Rightarrow x(x+1)<12 \Rightarrow$
$x^2+x<12 \Rightarrow x^2+x-12<0$

Step 1: Find the values of x that satisfy $x^2+x-12=0$.

$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x+4=0 \Rightarrow x=-4 \quad \text{or} \quad x-3=0 \Rightarrow x=3$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x(x+1)<12$.

Interval	Test Value	Is $x(x+1)<12$ True or False?
A: $(-\infty,-4)$	−5	$-5(-5+1)\overset{?}{<}12$ $20<12$ False
B: $(-4,3)$	0	$0(0+1)\overset{?}{<}12$ $0<12$ True
C: $(3,\infty)$	4	$4(4+1)\overset{?}{<}12$ $20<12$ False

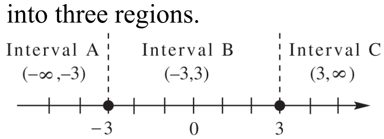
Solution set: $(-4,3)$

47. $x^2 \le 9$

Step 1: Find the values of x that satisfy $x^2 \le 9$

$$x^2=9$$
$$x^2-9=0$$
$$(x+3)(x-3)=0$$
$$x+3=0 \Rightarrow x=-3 \quad \text{or} \quad x-3=0 \Rightarrow x=3$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2 \le 9$.

Interval	Test Value	Is $x^2 \le 9$ True or False?
A: $(-\infty,-3)$	−4	$(-4)^2 \overset{?}{\le} 9$ $16 \le 9$ False
B: $(-3,3)$	0	$(0)^2 \overset{?}{\le} 9$ $0 \le 9$ True
C: $(3,\infty)$	4	$(4)^2 \overset{?}{\le} 9$ $16 \le 9$ False

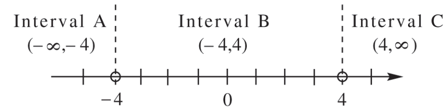
Solution set: [–3, 3]

48. $x^2>16 \Rightarrow x^2-16>0$

Step 1: Find the values of x that satisfy

$$x^2-16=0$$
$$(x+4)(x-4)=0$$
$$x+4=0 \Rightarrow x=-4 \quad \text{or} \quad x-4=0 \Rightarrow x=4$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2>16$.

Interval	Test Value	Is $x^2>16$ True or False?
A: $(-\infty,-4)$	−5	$(-5)^2 \overset{?}{>} 16$ $25>16$ True
B: $(-4,4)$	0	$(0)^2 \overset{?}{>} 16$ $0>16$ False
C: $(4,\infty)$	5	$(5)^2 \overset{?}{>} 16$ $25>16$ True

Solution set: $(-\infty,-4)\cup(4,\infty)$

49. $x^2+5x+7<0$

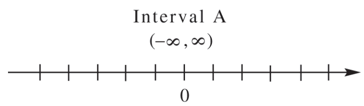
Step 1: Find the values of x that satisfy $x^2+5x+7=0$.

Use the quadratic formula to solve the equation.

Let $a = 1$, $b = 5$, and $c = 7$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-5\pm\sqrt{5^2-4(1)(7)}}{2(1)}$$
$$=\frac{-5\pm\sqrt{25-28}}{2}=\frac{-5\pm\sqrt{-3}}{2}$$
$$=\frac{-5\pm i\sqrt{3}}{2}=\frac{-5}{2}\pm\frac{\sqrt{3}}{2}i$$

Step 2: The number line is one region, $(-\infty,\infty)$.



Step 3: Because there are no real values of x that satisfy $x^2+5x+7=0$, x^2+5x+7 is either always positive or always negative. By substituting an arbitrary value such as $x=0$, we see that x^2+5x+7 will be positive and thus the solution set is $\varnothing$.

Interval	Test Value	Is $x^2+5x-2<0$ True or False?
A: $(-\infty,\infty)$	0	$0^2+5(0)+7\overset{?}{<}0$ $7<0$ False

Solution set: $\varnothing$

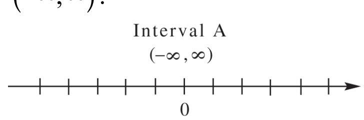
50. $4x^2+3x+1\le 0$

Step 1: Find the values of x that satisfy $4x^2+3x+1=0$.

Use the quadratic formula to solve the equation. Let $a=4$, $b=3$, and $c=1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{3^2-4(4)(1)}}{2(4)}$$
$$=\frac{-3\pm\sqrt{9-16}}{8}=\frac{-3\pm\sqrt{-7}}{8}=\frac{-3\pm i\sqrt{7}}{8}$$
$$=\frac{-3}{8}\pm\frac{\sqrt{7}}{8}i$$

Step 2: The number line is one region, $(-\infty,\infty)$.



Step 3: Because there are no real values of x that satisfy $4x^2+3x+1=0$, $4x^2+3x+1$ is either always positive or always negative. By substituting an arbitrary value such as $x=0$, we see that $4x^2+3x+1$ will be positive and thus the solution set is $\varnothing$.

Interval	Test Value	Is $4x^2+3x+1\le 0$ True or False?
A: $(-\infty,\infty)$	0	$4(0)^2+3(0)+1\overset{?}{\le}0$ $1\le 0$

Solution set: $\varnothing$

51. $x^2-2x\le 1\Rightarrow x^2-2x-1\le 0$

Step 1: Find the values of x that satisfy $x^2-2x-1=0$.

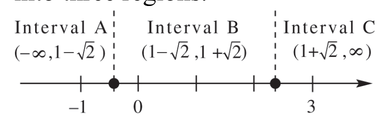
Use the quadratic formula to solve the equation.

Let $a=1$, $b=-2$, and $c=-1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm 2\sqrt{2}}{2}=1\pm\sqrt{2}$$

$1-\sqrt{2}\approx -0.4$ or $1+\sqrt{2}\approx 2.4$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2-2x\le 1$.

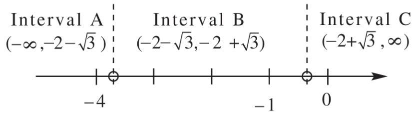
Interval	Test Value	Is $x^2-2x\le 1$ True or False?
A: $(-\infty,1-\sqrt{2})$	−1	$(-1)^2-2(-1)\overset{?}{\le}1$ $3\le 1$ False
B: $(1-\sqrt{2},1+\sqrt{2})$	0	$0^2-2(0)\overset{?}{\le}1\Rightarrow 0\le 1$ True
C: $(1+\sqrt{2},\infty)$	3	$3^2-2(3)\overset{?}{\le}1\Rightarrow 3\le 1$ False

Solution set: $\left[1-\sqrt{2},1+\sqrt{2}\right]$

52. $x^2+4x>-1 \Rightarrow x^2+4x+1>0$
Step 1: Find the values of *x* that satisfy $x^2+4x+1=0.$
Use the quadratic formula to solve the equation. Let $a=1$, $b=4$, and $c=1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{4^2-4(1)(1)}}{2(1)}$$
$$=\frac{-4\pm\sqrt{16-4}}{2}=\frac{-4\pm\sqrt{12}}{2}=\frac{-4\pm2\sqrt{3}}{2}$$
$$=-2\pm\sqrt{3}\Rightarrow x\approx-3.7 \text{ or } x\approx-0.3$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2+4x>-1$.

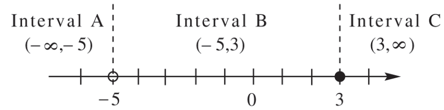
Interval	Test Value	Is $x^2+4x>-1$ True or False?
A: $\left(-\infty,-2-\sqrt{3}\right)$	−4	$(-4)^2+4(-4)\overset{?}{>}-1$ $0>-1$ True
B: $\left(-2-\sqrt{3},-2+\sqrt{3}\right)$	−1	$(-1)^2+4(-1)\overset{?}{>}-1$ $-3>-1$ False
C: $\left(-2+\sqrt{3},\infty\right)$	0	$0^2+4(0)\overset{?}{>}-1$ $0>-1$ True

Solution set: $\left(-\infty,-2-\sqrt{3}\right)\cup\left(-2+\sqrt{3},\infty\right)$

53. A. $(x+3)^2$ is equal to zero when $x=-3$. For any other real number, $(x+3)^2$ is positive. $(x+3)^2\ge0$ has solution set $(-\infty,\infty)$.

54. D. $(8x-7)^2$ is never negative, so $(8x-7)^2<0$ has solution set $\varnothing$.

55. $\dfrac{x-3}{x+5}\le0$
Because one side of the inequality is already 0, we start with Step 2.
Step 2: Determine the values that will cause either the numerator or denominator to equal 0.
$x-3=0\Rightarrow x=3$ or $x+5=0\Rightarrow x=-5$
The values –5 and 3 divide the number line into three regions. Use an open circle on –5 because it makes the denominator equal 0.

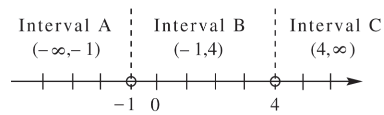


Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{x-3}{x+5}\le0.$

Interval	Test Value	Is $\frac{x-3}{x+5}\le0$ True or False?
A: $(-\infty,-5)$	–6	$\frac{-6-3}{-6+5}\overset{?}{\le}0$ $9\le0$ False
B: $(-5,3)$	0	$\frac{0-3}{0+5}\overset{?}{\le}0$ $-\frac{3}{5}\le0$ True
C: $(3,\infty)$	4	$\frac{4-3}{4+5}\overset{?}{\le}0$ $\frac{1}{9}\le0$ False

Interval B satisfies the inequality. The endpoint −5 is not included because it makes the denominator 0.
Solution set: (–5, 3]

56. $\dfrac{x+1}{x-4}>0$
Because one side of the inequality is already 0, we start with Step 2.
Step 2: Determine the values that will cause either the numerator or denominator to equal 0.
$x+1=0\Rightarrow x=-1$ or $x-4=0\Rightarrow x=4$
The values –1 and 4 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{x+1}{x-4}>0.$

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Interval	Test Value	Is $\frac{x+1}{x-4} > 0$ True or False?
A: $(-\infty,-1)$	-2	$\frac{-2+1}{-2-4} \overset{?}{>} 0$ $\frac{1}{6} > 0$ True
B: $(-1,4)$	0	$\frac{0+1}{0-4} \overset{?}{>} 0$ $-\frac{1}{4} > 0$ False
C: $(4,\infty)$	5	$\frac{5+1}{5-4} \overset{?}{>} 0$ $6 > 0$ True

Solution set: $(-\infty,-1)\cup(4,\infty)$

57. $\dfrac{1-x}{x+2} < -1$

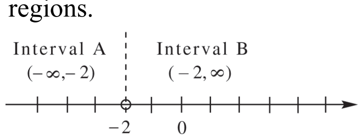
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{1-x}{x+2} < -1 \Rightarrow \frac{x-1}{x+2} > 1$$
$$\frac{x-1}{x+2} - 1 > 0 \Rightarrow \frac{x-1}{x+2} - \frac{x+2}{x+2} > 0$$
$$\frac{x-1-(x+2)}{x+2} > 0 \Rightarrow \frac{x-1-x-2}{x+2} > 0$$
$$\frac{-3}{x+2} > 0$$

Step 2: Because the numerator is a constant, determine the values that will cause denominator to equal 0.

$x+2=0 \Rightarrow x=-2$

The value –2 divides the number line into two regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{1-x}{x+2} < -1$

Interval	Test Value	Is $\frac{1-x}{x+2} < -1$ True or False?
A: $(-\infty,-2)$	-3	$\frac{-1-(-3)}{-3+2} \overset{?}{<} -1$ $-2 < -1$ True
B: $(-2,\infty)$	-1	$\frac{1-(-1)}{-1+2} \overset{?}{<} 1$ $2 < -1$ False

Solution set: $(-\infty,-2)$

58. $\dfrac{6-x}{x+2} > 1$

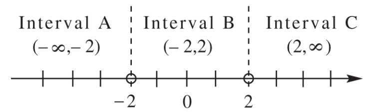
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{6-x}{x+2} > 1 \Rightarrow \frac{x-6}{x+2} < -1$$
$$\frac{x-6}{x+2} + 1 < 0 \Rightarrow \frac{x-6}{x+2} + \frac{x+2}{x+2} < 0$$
$$\frac{x-6+x+2}{x+2} < 0 \Rightarrow \frac{2x-4}{x+2} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$2x-4=0 \Rightarrow x=2$ or $x+2=0 \Rightarrow x=-2$

The values –2 and 2 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{6-x}{x+2} > 1$

Interval	Test Value	Is $\frac{6-x}{x+2} > 1$ True or False?
A: $(-\infty,-2)$	-3	$\frac{6-(-3)}{-3+2} \overset{?}{>} 1$ $-9 > -1$ False
B: $(-2,2)$	0	$\frac{6-0}{0+2} \overset{?}{>} 1$ $3 > 1$ True
C: $(2,\infty)$	3	$\frac{6-3}{3+2} \overset{?}{>} 1$ $\frac{3}{5} > 1$ False

Solution set: (–2, 2)

59. $\dfrac{3}{x-6} \le 2$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3}{x-6} - 2 \le 0 \Rightarrow \frac{3}{x-6} - \frac{2(x-6)}{x-6} \le 0$$
$$\frac{3-2(x-6)}{x-6} \le 0 \Rightarrow \frac{3-2x+12}{x-6} \le 0$$
$$\frac{15-2x}{x-6} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$15-2x=0 \Rightarrow x=\frac{15}{2}$ or $x-6=0 \Rightarrow x=6$

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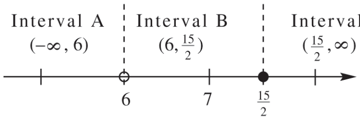
The values 6 and $\frac{15}{2}$ divide the number line into three regions. Use an open circle on 6 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{3}{x-6} \le 2$.

Interval	Test Value	Is $\frac{3}{x-6} \le 2$ True or False?
A: $(-\infty, 6)$	0	$\frac{3}{0-6} \overset{?}{\le} 2$ $-\frac{1}{2} \le 2$ True
B: $\left(6, \frac{15}{2}\right)$	7	$\frac{3}{7-6} \overset{?}{\le} 2$ $3 \le 2$ False
C: $\left(\frac{15}{2}, \infty\right)$	8	$\frac{3}{8-6} \overset{?}{\le} 2$ $\frac{3}{2} \le 2$ True

Intervals A and C satisfiy the inequality. The endpoint 6 is not included because it makes the denominator 0.

Solution set: $(-\infty, 6) \cup \left[\frac{15}{2}, \infty\right)$

60. $\frac{3}{x-2} < 1$

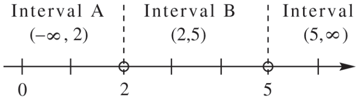
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3}{x-2} - 1 < 0$$
$$\frac{3}{x-2} - \frac{x-2}{x-2} < 0 \Rightarrow \frac{3-(x-2)}{x-2} < 0$$
$$\frac{3-x+2}{x-2} < 0 \Rightarrow \frac{5-x}{x-2} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$5 - x = 0 \Rightarrow 5 = x$ or $x - 2 = 0 \Rightarrow x = 2$

The values 2 and 5 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{3}{x-2} < 1$.

Interval	Test Value	Is $\frac{3}{x-2} < 1$ True or False?
A: $(-\infty, 2)$	0	$\frac{3}{0-2} \overset{?}{<} 1$ $-\frac{3}{2} < 1$ True
B: $(2, 5)$	3	$\frac{3}{3-2} \overset{?}{<} 1$ $3 < 1$ False
C: $(5, \infty)$	6	$\frac{3}{6-2} \overset{?}{<} 1$ $\frac{3}{4} < 1$ True

Solution set: $(-\infty, 2) \cup (5, \infty)$

61. $\frac{-4}{1-x} < 5$

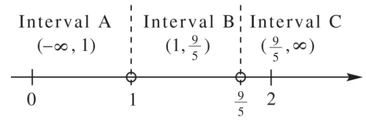
Step 1: Rewrite the inequality to compare a single fraction to 0:

$$\frac{-4}{1-x} < 5 \Rightarrow \frac{-4}{1-x} - 5 < 0$$
$$\frac{-4}{1-x} - \frac{5(1-x)}{1-x} < 0 \Rightarrow \frac{-4-5(1-x)}{1-x} < 0$$
$$\frac{-4-5+5x}{1-x} < 0 \Rightarrow \frac{-9+5x}{1-x} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-9 + 5x = 0 \Rightarrow x = \frac{9}{5}$ or $1 - x = 0 \Rightarrow x = 1$

The values 1 and $\frac{9}{5}$ divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{-4}{1-x} < 5$

Interval	Test Value	Is $\frac{-4}{1-x} < 5$ True or False?
A: $(-\infty, 1)$	0	$\frac{-4}{1-0} \overset{?}{<} 5$ $-4 < 5$ True
B: $\left(1, \frac{9}{5}\right)$	$\frac{6}{5}$	$\frac{-4}{1-\frac{6}{5}} \overset{?}{<} 5$ $20 < 5$ False
C: $\left(\frac{9}{5}, \infty\right)$	2	$\frac{-4}{1-2} \overset{?}{<} 5$ $4 < 5$ True

Solution set: $(-\infty, 1) \cup \left(\frac{9}{5}, \infty\right)$

62. $\dfrac{-6}{3x-5} \le 2$

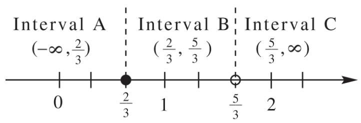
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{-6}{3x-5} \le 2 \Rightarrow \frac{-6}{3x-5} - \frac{2(3x-5)}{3x-5} \le 0$$
$$\frac{-6-2(3x-5)}{3x-5} \le 0 \Rightarrow \frac{-6-6x+10}{3x-5} \le 0$$
$$\frac{-6x+4}{3x-5} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-6x+4=0 \Rightarrow x=\frac{2}{3}$ or $3x-5=0 \Rightarrow x=\frac{5}{3}$

The values $\frac{2}{3}$ and $\frac{5}{3}$ divide the number line into three regions. Use an open circle on $\frac{5}{3}$ because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{6}{5-3x} \le 2$.

Interval	Test Value	Is $\frac{6}{5-3x} \le 2$ True or False?
A: $\left(-\infty, \frac{2}{3}\right)$	0	$\frac{6}{5-3\cdot 0} \overset{?}{\le} 2$ $\frac{6}{5} \le 2$ True
B: $\left(\frac{2}{3}, \frac{5}{3}\right)$	1	$\frac{6}{5-3\cdot 1} \overset{?}{\le} 2$ $3 \le 2$ False
C: $\left(\frac{5}{3}, \infty\right)$	2	$\frac{6}{5-3\cdot 2} \overset{?}{\le} 2$ $-6 \le 2$ True

Intervals A and C satisfy the inequality. The endpoint $\frac{5}{3}$ is not included because it makes the denominator 0.

Solution set: $\left(-\infty, \frac{2}{3}\right] \cup \left(\frac{5}{3}, \infty\right)$

63. $\dfrac{10}{3+2x} \le 5$

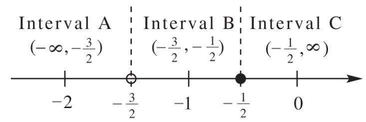
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{10}{3+2x} - 5 \le 0 \Rightarrow \frac{10}{3+2x} - \frac{5(3+2x)}{3+2x} \le 0$$
$$\frac{10-5(3+2x)}{3+2x} \le 0 \Rightarrow \frac{10-15-10x}{3+2x} \le 0$$
$$\frac{-10x-5}{3+2x} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-10x-5=0 \Rightarrow x=-\frac{1}{2}$ or $3+2x=0 \Rightarrow x=-\frac{3}{2}$

The values $-\frac{3}{2}$ and $-\frac{1}{2}$ divide the number line into three regions. Use an open circle on $-\frac{3}{2}$ because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{10}{3+2x} \le 5$.

Interval	Test Value	Is $\frac{10}{3+2x} \le 5$ True or False?
A: $\left(-\infty, -\frac{3}{2}\right)$	−2	$\frac{10}{3+2(-2)} \overset{?}{\le} 5$ $-10 \le 5$ True
B: $\left(-\frac{3}{2}, -\frac{1}{2}\right)$	−1	$\frac{10}{3+2(-1)} \overset{?}{\le} 5$ $10 \le 5$ False
C: $\left(-\frac{1}{2}, \infty\right)$	0	$\frac{10}{3+2(0)} \overset{?}{\le} 5$ $\frac{10}{3} \le 5$ True

Intervals A and C satisfy the inequality. The endpoint $-\frac{3}{2}$ is not included because it makes the denominator 0.

Solution set: $\left(-\infty, -\frac{3}{2}\right) \cup \left[-\frac{1}{2}, \infty\right)$

64. $\dfrac{1}{x+2} \ge 3$

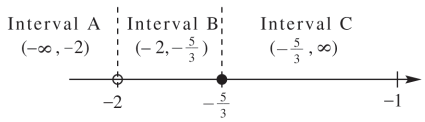
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{1}{x+2} - 3 \ge 0 \Rightarrow \frac{1}{x+2} - \frac{3(x+2)}{x+2} \ge 0$$
$$\frac{1-3(x+2)}{x+2} \ge 0 \Rightarrow \frac{1-3x-6}{x+2} \ge 0$$
$$\frac{-3x-5}{x+2} \ge 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-3x-5=0 \Rightarrow x=-\frac{5}{3}$ or $x+2=0 \Rightarrow x=-2$

The values –2 and $-\frac{5}{3}$ divide the number line into three regions. Use an open circle on –2 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{1}{x+2} \ge 3$.

Interval	Test Value	Is $\frac{1}{x+2} \ge 3$ True or False?
A: $(-\infty, -2)$	–3	$\frac{1}{-3+2} \overset{?}{\ge} 3$ $-1 \ge 3$ False
B: $\left(-2, -\frac{5}{3}\right)$	$-\frac{11}{6}$	$\frac{1}{-\frac{11}{6}+2} \overset{?}{\ge} 3$ $6 \ge 3$ True
C: $\left(-\frac{5}{3}, \infty\right)$	0	$\frac{1}{0+2} \overset{?}{\ge} 3$ $\frac{1}{2} \ge 3$ False

Interval B satisfies the inequality. The endpoint –2 is not included because it makes the denominator 0.

Solution set: $\left(-2, -\frac{5}{3}\right]$

65. $\dfrac{7}{x+2} \ge \dfrac{1}{x+2}$

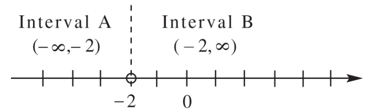
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{7}{x+2} - \frac{1}{x+2} \ge 0 \Rightarrow \frac{6}{x+2} \ge 0$$

Step 2: Because the numerator is a constant, determine the value that will cause the denominator to equal 0.

$x+2=0 \Rightarrow x=-2$

The value -2 divides the number line into two regions. Use an open circle on –2 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{7}{x+2} \ge \dfrac{1}{x+2}$.

Interval	Test Value	Is $\frac{7}{x+2} \ge \frac{1}{x+2}$ True or False?
A: $(-\infty, -2)$	–3	$\frac{7}{-3+2} \overset{?}{\ge} \frac{1}{-3+2}$ $-7 \ge -1$ False
B: $(-2, \infty)$	0	$\frac{7}{0+2} \overset{?}{\ge} \frac{1}{0+2}$ $\frac{7}{2} \ge \frac{1}{2}$ True

Interval B satisfies the inequality. The endpoint −2 is not included because it makes the denominator 0.

Solution set: $(-2, \infty)$

66. $\dfrac{5}{x+1} > \dfrac{12}{x+1}$

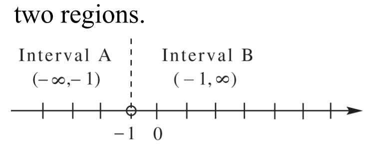
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{5}{x+1} - \frac{12}{x+1} > 0 \Rightarrow \frac{-7}{x+1} > 0$$

Step 2: Because the numerator is a constant, determine the value that will cause the denominator to equal 0.

$x+1=0 \Rightarrow x=-1$

The value -1 divides the number line into two regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{5}{x+1} > \dfrac{12}{x+1}$.

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Interval	Test Value	Is $\frac{5}{x+1} > \frac{12}{x+1}$ True or False?
A: $(-\infty, -1)$	-2	$\frac{5}{-2+1} \overset{?}{>} \frac{12}{-2+1}$ $-5 > -12$ True
B: $(-1, \infty)$	0	$\frac{5}{0+1} \overset{?}{>} \frac{12}{0+1}$ $5 > 12$ False

Solution set: $(-\infty, -1)$

67. $\dfrac{3}{2x-1} > \dfrac{-4}{x}$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3}{2x-1} + \frac{4}{x} > 0$$

$$\frac{3x}{x(2x-1)} + \frac{4(2x-1)}{x(2x-1)} > 0$$

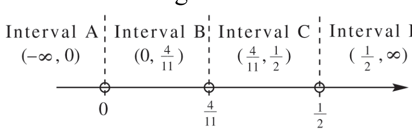
$$\frac{3x + 4(2x-1)}{x(2x-1)} > 0$$

$$\frac{3x + 8x - 4}{x(2x-1)} > 0 \Rightarrow \frac{11x-4}{x(2x-1)} > 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$11x - 4 = 0 \Rightarrow x = \frac{4}{11}$ or $x = 0$ or
$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

The values 0, $\frac{4}{11}$, and $\frac{1}{2}$ divide the number line into four regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{3}{2x-1} > \dfrac{-4}{x}$.

Interval	Test Value	Is $\frac{3}{2x-1} > \frac{-4}{x}$ True or False?
A: $(-\infty, 0)$	-1	$\frac{3}{2(-1)-1} \overset{?}{>} \frac{-4}{-1}$ $-1 > 4$ False
B: $\left(0, \frac{4}{11}\right)$	$\frac{1}{11}$	$\frac{3}{2\left(\frac{1}{11}\right)-1} \overset{?}{>} \frac{-4}{\frac{1}{11}}$ or $-\frac{11}{3} \overset{?}{>} -44$ $-3\frac{2}{3} > -44$ True
C: $\left(\frac{4}{11}, \frac{1}{2}\right)$	$\frac{9}{22}$	$\frac{3}{2\left(\frac{9}{22}\right)-1} \overset{?}{>} \frac{-4}{\frac{9}{22}}$ or $-\frac{33}{2} \overset{?}{>} -\frac{88}{9}$ $-16\frac{1}{2} > -9\frac{7}{9}$ False
D: $\left(\frac{1}{2}, \infty\right)$	1	$\frac{3}{2(1)-1} \overset{?}{>} \frac{-4}{1}$ $3 > -4$ True

Solution set: $\left(0, \frac{4}{11}\right) \cup \left(\frac{1}{2}, \infty\right)$

68. $\dfrac{-5}{3x+2} \ge \dfrac{5}{x}$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{-5}{3x+2} - \frac{5}{x} \ge 0$$

$$\frac{-5x}{x(3x+2)} - \frac{5(3x+2)}{x(3x+2)} \ge 0$$

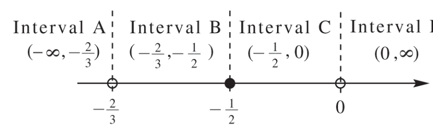
$$\frac{-5x - 5(3x+2)}{x(3x+2)} \ge 0$$

$$\frac{-5x - 15x - 10}{x(3x+2)} \ge 0 \Rightarrow \frac{-20x-10}{x(3x+2)} \ge 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-20x - 10 = 0$ or $x = 0$ or $3x + 2 = 0$
$x = -\frac{1}{2}$ or $x = 0$ or $x = -\frac{2}{3}$

The values $-\frac{2}{3}$, $-\frac{1}{2}$, and 0 divide the number line into four regions. Use an open circle on 0 and $-\frac{2}{3}$ because they makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{-5}{3x+2} \ge \dfrac{5}{x}$.

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Interval	Test Value	Is $\frac{-5}{3x+2} \ge \frac{5}{x}$ True or False?
A: $\left(-\infty, -\frac{2}{3}\right)$	-1	$\frac{-5}{3(-1)+2} \overset{?}{\ge} \frac{5}{-1}$ $5 \ge -5$ True
B: $\left(-\frac{2}{3}, -\frac{1}{2}\right)$	$-\frac{7}{12}$	$\frac{-5}{3\left(-\frac{7}{12}\right)+2} \overset{?}{\ge} \frac{5}{-\frac{7}{12}}$ $-20 \ge -\frac{60}{7}$ $-20 \ge -8\frac{4}{7}$ False
C: $\left(-\frac{1}{2}, 0\right)$	$-\frac{1}{4}$	$\frac{-5}{3\left(-\frac{1}{4}\right)+2} \overset{?}{\ge} \frac{5}{-\frac{1}{4}}$ $-4 \ge -20$ True
D: $(0, \infty)$	1	$\frac{-5}{3(1)+2} \overset{?}{\ge} \frac{5}{1}$ $-1 \ge 5$ False

Intervals A and C satisfy the inequality. The endpoints $-\frac{2}{3}$ and 0 are not included because they make the denominator 0.

Solution set: $\left(-\infty, -\frac{2}{3}\right) \cup \left[-\frac{1}{2}, 0\right)$

69. $\frac{4}{2-x} \ge \frac{3}{1-x}$

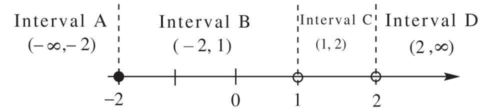
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{4}{2-x} \ge \frac{3}{1-x}$$
$$\frac{4}{2-x} - \frac{3}{1-x} \ge 0$$
$$\frac{4(1-x)}{(2-x)(1-x)} - \frac{3(2-x)}{(1-x)(2-x)} \ge 0$$
$$\frac{4(1-x)-3(2-x)}{(x-2)(1-x)} \ge 0$$
$$\frac{4-4x-6+3x}{(2-x)(1-x)} \ge 0$$
$$\frac{-2-x}{(2-x)(1-x)} \ge 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-2-x=0 \Rightarrow x=-2$ or $2-x=0 \Rightarrow x=2$ or $1-x=0 \Rightarrow x=1$

The values –2, 1, and 2 divide the number line into four regions. Use an open circle on 1 and 2 because they make the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{4}{2-x} \ge \frac{3}{1-x}$

Interval	Test Value	Is $\frac{4}{2-x} \ge \frac{3}{1-x}$ True or False?
A: $(-\infty, -2)$	-3	$\frac{4}{-2-(-3)} \overset{?}{\ge} \frac{3}{1-(-3)}$ $4 \ge \frac{3}{4}$ True
B: $(-2, 1)$	0	$\frac{4}{2-0} \overset{?}{\ge} \frac{3}{1-0}$ $2 \ge 3$ False
C: $(1, 2)$	1.5	$\frac{4}{2-1.5} \overset{?}{\ge} \frac{3}{1-1.5}$ $8 \ge -6$ True
D: $(2, \infty)$	3	$\frac{4}{2-3} \overset{?}{\ge} \frac{3}{1-3}$ $-4 \ge -\frac{3}{2}$ False

Intervals A and C satisfy the inequality. The endpoints 1 and 2 are not included because they make the denominator 0.

Solution set: $(-\infty, -2] \cup (1, 2)$

70. $\frac{4}{x+1} < \frac{2}{x+3}$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{4}{x+1} - \frac{2}{x+3} < 0$$
$$\frac{4(x+3)}{(x+1)(x+3)} - \frac{2(x+1)}{(x+3)(x+1)} < 0$$
$$\frac{4(x+3)-2(x+1)}{(x+1)(x+3)} < 0$$
$$\frac{4x+12-2x-2}{(x+1)(x+3)} < 0$$
$$\frac{2x+10}{(x+1)(x+3)} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$2x+10=0 \Rightarrow x=-5$ or $x+1=0 \Rightarrow x=-1$ or $x+3=0 \Rightarrow x=-3$

The values –5, -3, and -1 divide the number line into four regions.

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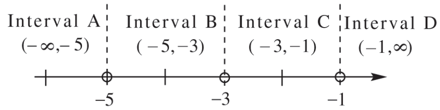


Step 3: Choose a test value to see if it satisfies the inequality, $\frac{4}{x+1} < \frac{2}{x+3}$.

Interval	Test Value	Is $\frac{4}{x+1} < \frac{2}{x+3}$ True or False?
A: $(-\infty,-5)$	−6	$\frac{4}{-6+1} \overset{?}{<} \frac{2}{-6+3}$ or $-\frac{4}{5} \overset{?}{<} -\frac{2}{3}$ $-\frac{12}{15} < -\frac{10}{15}$ True
B: $(-5,-3)$	−4	$\frac{4}{-4+1} \overset{?}{<} \frac{2}{-4+3}$ $-\frac{4}{3} < -2$ False
C: $(-3,-1)$	−2	$\frac{4}{-2+1} \overset{?}{<} \frac{2}{-2+3}$ $-4 < 2$ True
D: $(-1,\infty)$	0	$\frac{4}{0+1} \overset{?}{<} \frac{2}{0+3}$ $4 < \frac{2}{3}$ False

Solution set: $(-\infty,-5)\cup(-3,-1)$

71. $\frac{x+3}{x-5} \le 1$

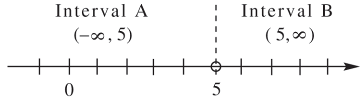
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{x+3}{x-5} - 1 \le 0 \Rightarrow \frac{x+3}{x-5} - \frac{x-5}{x-5} \le 0$$
$$\frac{x+3-(x-5)}{x-5} \le 0 \Rightarrow \frac{x+3-x+5}{x-5} \le 0$$
$$\frac{8}{x-5} \le 0$$

Step 2: Because the numerator is a constant, determine the value that will cause the denominator to equal 0.

$x-5=0 \Rightarrow x=5$

The value 5 divides the number line into two regions. Use an open circle on 5 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{x+3}{x-5} \le 1$.

Interval	Test Value	Is $\frac{x+3}{x-5} \le 1$ True or False?
A: $(-\infty,5)$	0	$\frac{0+3}{0-5} \overset{?}{\le} 1$ $-\frac{3}{5} \le 1$ True
B: $(5,\infty)$	6	$\frac{6+3}{6-5} \overset{?}{\le} 1$ $9 \le 1$ False

Interval A satisfies the inequality. The endpoint 5 is not included because it makes the denominator 0.

Solution set: $(-\infty,5)$

72. $\frac{x+2}{3+2x} \le 5$

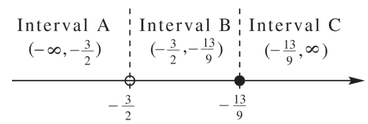
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{x+2}{3+2x} - 5 \le 0$$
$$\frac{x+2}{3+2x} - \frac{5(3+2x)}{3+2x} \le 0$$
$$\frac{x+2-5(3+2x)}{3+2x} \le 0$$
$$\frac{x+2-15-10x}{3+2x} \le 0$$
$$\frac{-9x-13}{3+2x} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-9x-13=0 \Rightarrow x=-\frac{13}{9}$ or $3+2x=0 \Rightarrow x=-\frac{3}{2}$

The values $-\frac{3}{2}$ and $-\frac{13}{9}$ divide the number line into three regions. Use an open circle on $-\frac{3}{2}$ because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{x+2}{3+2x} \le 5$.

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Interval	Test Value	Is $\frac{x+2}{3+2x} \le 5$ True or False?
A: $\left(-\infty, -\frac{3}{2}\right)$	-2	$\frac{-2+2}{3+2(-2)} \overset{?}{\le} 5$ $0 \le 5$ True
B: $\left(-\frac{3}{2}, -\frac{13}{9}\right)$	-1.45	$\frac{-1.45+2}{3+2(-1.45)} \overset{?}{\le} 5$ $5.5 \le 5$ False
C: $\left(-\frac{13}{9}, \infty\right)$	0	$\frac{0+2}{3+2(0)} \overset{?}{\le} 5$ $\frac{2}{3} \le 5$ True

Intervals A and C satisfy the inequality. The endpoint $-\frac{3}{2}$ is not included because it makes the denominator 0.

Solution set: $\left(-\infty, -\frac{3}{2}\right) \cup \left[-\frac{13}{9}, \infty\right)$

73. $\dfrac{2x-3}{x^2+1} \ge 0$

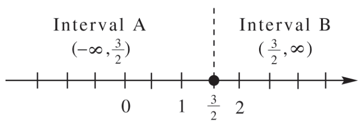
Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$2x - 3 = 0$ or $x^2 + 1 = 0$

$x = \frac{3}{2}$ has no real solutions

$\frac{3}{2}$ divides the number line into two intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{2x-3}{x^2+1} \ge 0$.

Interval	Test Value	Is $\frac{2x-3}{x^2+1} \ge 0$ True or False?
A: $\left(-\infty, \frac{3}{2}\right)$	0	$\frac{2(0)-3}{0^2+1} \overset{?}{\ge} 0$ $-3 \ge 0$ False
B: $\left(\frac{3}{2}, \infty\right)$	2	$\frac{2(2)-3}{2^2+1} \overset{?}{\ge} 0$ $\frac{1}{5} \ge 0$ True

Solution set: $\left[\frac{3}{2}, \infty\right)$

74. $\dfrac{9x-8}{4x^2+25} < 0$

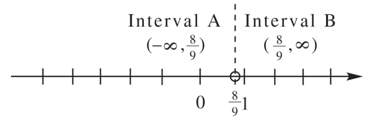
Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$9x - 8 = 0 \Rightarrow x = \frac{8}{9}$ or

$4x^2 + 25 = 0$, which has no real solutions

The value $\frac{8}{9}$ divides the number line into two intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{9x-8}{4x^2+25} < 0$.

Interval	Test Value	Is $\frac{9x-8}{4x^2+25} < 0$ True or False?
A: $\left(-\infty, \frac{8}{9}\right)$	0	$\frac{9(0)-8}{4(0)^2+25} \overset{?}{<} 0$ $-\frac{8}{25} < 0$ True
B: $\left(\frac{8}{9}, \infty\right)$	1	$\frac{9(1)-8}{4(1)^2+25} \overset{?}{<} 0$ $\frac{1}{29} < 0$ False

Solution set: $\left(-\infty, \frac{8}{9}\right)$

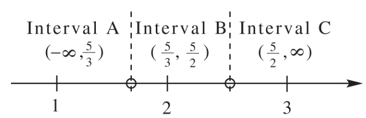
75. $\dfrac{(5-3x)^2}{(2x-5)^3} > 0$

Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$5 - 3x = 0 \Rightarrow x = \frac{5}{3}$ or $2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

The values $\frac{5}{3}$ and $\frac{5}{2}$ divide the number line into three intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{(5-3x)^2}{(2x-5)^3} > 0$.

(*continued on next page*)

(*continued*)

Interval	Test Value	Is $\frac{(5-3x)^2}{(2x-5)^3} > 0$ True or False?
A: $\left(-\infty, \frac{5}{3}\right)$	0	$\frac{(5-3\cdot 0)^2}{(2\cdot 0-5)^3} \overset{?}{>} 0$ $-\frac{1}{5} > 0$ False
B: $\left(\frac{5}{3}, \frac{5}{2}\right)$	2	$\frac{(5-3\cdot 2)^2}{(2\cdot 2-5)^3} \overset{?}{>} 0$ $-1 > 0$ False
C: $\left(\frac{5}{2}, \infty\right)$	3	$\frac{(5-3\cdot 3)^2}{(2\cdot 3-5)^3} \overset{?}{>} 0$ $16 > 0$ True

Solution set: $\left(\frac{5}{2}, \infty\right)$

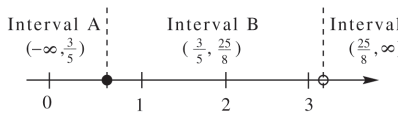
76. $\frac{(5x-3)^3}{(25-8x)^2} \le 0$

Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$5x-3=0 \Rightarrow x=\frac{3}{5}$ or $25-8x=0 \Rightarrow x=\frac{25}{8}$

The values $\frac{3}{5}$ and $\frac{25}{8}$ divide the number line into three intervals. Use an open circle on $\frac{25}{8}$ because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{(5x-3)^3}{(25-8x)^2} \le 0.$

Interval	Test Value	Is $\frac{(5x-3)^3}{(25-8x)^2} \le 0$ True or False?
A: $\left(-\infty, \frac{3}{5}\right)$	0	$\frac{(5\cdot 0-3)^3}{(25-8\cdot 0)^2} \overset{?}{\le} 0$ $-\frac{27}{625} \le 0$ True
B: $\left(\frac{3}{5}, \frac{25}{8}\right)$	2	$\frac{(5\cdot 2-3)^3}{(25-8\cdot 2)^2} \overset{?}{\le} 0$ $\frac{343}{81} \le 0$ False
C: $\left(\frac{25}{8}, \infty\right)$	4	$\frac{(5\cdot 4-3)^3}{(25-8\cdot 4)^2} \overset{?}{\le} 0$ $\frac{4913}{49} \le 0$ False

Solution set: $\left(-\infty, \frac{3}{5}\right]$

77. $\frac{(2x-3)(3x+8)}{(x-6)^3} \ge 0$

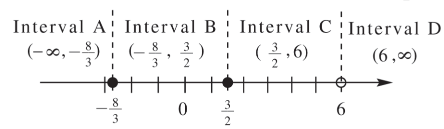
Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$2x-3=0$ or $3x+8=0$ or $x-6=0$

$x=\frac{3}{2}$ or $x=-\frac{8}{3}$ or $x=6$

The values $-\frac{8}{3}$, $\frac{3}{2}$, and 6 divide the number line into four intervals. Use an open circle on 6 because it makes the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{(2x-3)(3x+8)}{(x-6)^3} \ge 0.$

Interval	Test Value	Is $\frac{(2x-3)(3x+8)}{(x-6)^3} \ge 0$ True or False?
A: $\left(-\infty, -\frac{8}{3}\right)$	−3	$\frac{[2(-3)-3][3(-3)+8]}{(-3-6)^3} \overset{?}{\ge} 0$ $-\frac{1}{81} \ge 0$ False
B: $\left(-\frac{8}{3}, \frac{3}{2}\right)$	0	$\frac{(2\cdot 0-3)(3\cdot 0+8)}{(0-6)^3} \overset{?}{\ge} 0$ $\frac{1}{9} \ge 0$ True
C: $\left(\frac{3}{2}, 6\right)$	2	$\frac{(2\cdot 2-3)(3\cdot 2+8)}{(2-6)^3} \overset{?}{\ge} 0$ $-\frac{7}{32} \ge 0$ False
D: $(6, \infty)$	7	$\frac{(2\cdot 7-3)(3\cdot 7+8)}{(7-6)^3} \overset{?}{\ge} 0$ $319 \ge 0$ True

Solution set: $\left[-\frac{8}{3}, \frac{3}{2}\right] \cup (6, \infty)$

78. $\dfrac{(9x-11)(2x+7)}{(3x-8)^3} > 0$

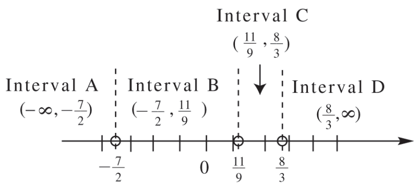
Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$9x-11=0$ or $2x+7=0$ or $3x-8=0$

$x=\frac{11}{9}$ or $x=-\frac{7}{2}$ or $x=\frac{8}{3}$

The values $-\frac{7}{2}$, $\frac{11}{9}$, and $\frac{8}{3}$ divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{(9x-11)(2x+7)}{(3x-8)^3} > 0$.

Interval	Test Value	Is $\frac{(9x-11)(2x+7)}{(3x-8)^3} > 0$ True or False?
A: $\left(-\infty, -\frac{7}{2}\right)$	-4	$\frac{[9(-4)-11][2(-4)+7]}{[3(-4)-8]^3} \overset{?}{>} 0$ $-\frac{47}{8000} > 0$ False
B: $\left(-\frac{7}{2}, \frac{11}{9}\right)$	0	$\frac{(9\cdot0-11)(2\cdot0+7)}{(3\cdot0-8)^3} \overset{?}{>} 0$ $\frac{77}{512} > 0$ True
C: $\left(\frac{11}{9}, \frac{8}{3}\right)$	2	$\frac{(9\cdot2-11)(2\cdot2+7)}{(3\cdot2-8)^3} \overset{?}{>} 0$ $-\frac{77}{8} > 0$ False
D: $\left(\frac{8}{3}, \infty\right)$	3	$\frac{(9\cdot3-11)(2\cdot3+7)}{(3\cdot3-8)^3} \overset{?}{>} 0$ $208 > 0$ True

Solution set: $\left(-\frac{7}{2}, \frac{11}{9}\right) \cup \left(\frac{8}{3}, \infty\right)$

79. **(a)** Let $R = 7.6$ and then solve for x.

$0.2844x + 5.535 > 7.6$

$0.2844x > 2.065 \Rightarrow x > 7.3$

The model predicts that the receipts exceeded \$7.6 billion about 7.3 years after 1993, which was in 2000.

(b) Let $R = 10$ and then solve for x.

$0.2844x + 5.535 > 10$

$0.2844x > 4.465 \Rightarrow x > 15.7$

The model predicts that the receipts exceeded \$10 billion about 15.7 years after 1993, which was in 2008.

80. **(a)** Let $W = 34$ and solve for x.

$W = 0.33x + 33.1$

$0.33x + 33.1 > 34$

$0.33x > 0.9 \Rightarrow x > 2.7$ (approx)

According to the model, the percent of waste recovered first exceeded 34% about 2.7 years after 2007, which was in 2009.

(b) Solve for x for values between 33.9 and 34.5.

$33.9 < 0.33x + 33.1 < 34.5$

$0.8 < 0.33x < 1.4 \Rightarrow 2.4 < x < 4.2$

According to the model, the percent of waste recovered was between 33.9% and 34.5% about 2.4 years after 2007, which was in 2009, until 4.2 years after 2007, which was in 2011.

81. $-16t^2 + 220t \ge 624$

Step 1: Find the values of t that satisfy $-16t^2 + 220t = 624$.

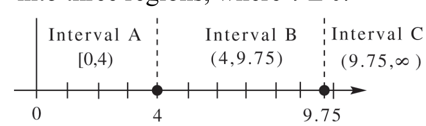
$-16t^2 + 220t = 624 \Rightarrow 0 = 16t^2 - 220t + 624$

$0 = 4t^2 - 55t + 156$

$0 = (t-4)(4t-39)$

$t-4=0 \Rightarrow t=4$ or $4t-39=0 \Rightarrow t=\frac{39}{4}=9.75$

Step 2: The two numbers divide a number line into three regions, where $t \ge 0$.



Step 3: Choose a test value to see if it satisfies the inequality, $-16t^2 + 220t \ge 624$.

Interval	Test Value	Is $-16t^2 + 220t \ge 624$ True or False?
A: $(0,4)$	1	$-16\cdot1^2 + 220\cdot1 \overset{?}{\ge} 624$ $204 \ge 624$ False
B: $(4, 9.75)$	5	$-16\cdot5^2 + 220\cdot5 \overset{?}{\ge} 624$ $700 \ge 624$ True
C: $(9.75, \infty)$	10	$-16\cdot10^2 + 220\cdot10 \overset{?}{\ge} 624$ $600 \ge 624$ False

The projectile will be at least 624 feet above ground between 4 sec and 9.75 sec (inclusive).

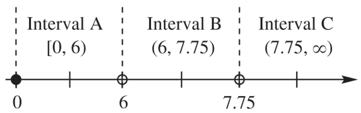
82. $-16t^2 + 220t \ge 744$

Step 1: Find the values of t that satisfy $-16t^2 + 220t = 744$.

$$-16t^2 + 220t = 744 \Rightarrow 0 = 16t^2 - 220t + 744$$
$$0 = 4t^2 - 55t + 186$$
$$0 = (t-6)(4t-31)$$

$t - 6 = 0 \Rightarrow t = 6$ or $4t - 31 = 0 \Rightarrow t = \frac{31}{4} = 7.75$

Step 2: The two numbers divide a number line into three regions, where $t \ge 0$.



Step 3: Choose a test value to see if it satisfies the inequality, $-16t^2 + 220t \ge 744$

Interval	Test Value	Is $-16t^2 + 220t \ge 744$ True or False?
A: $[0, 6)$	1	$-16 \cdot 1^2 + 220 \cdot 1 \overset{?}{\ge} 744$ $204 \ge 744$ False
B: $(6, 7.75)$	7	$-16 \cdot 7^2 + 220 \cdot 7 \overset{?}{\ge} 744$ $756 \ge 744$ True
C: $(7.75, \infty)$	10	$-16 \cdot 10^2 + 220 \cdot 10 \overset{?}{\ge} 744$ $600 \ge 744$ False

The projectile will be at least 744 feet above ground between 6 sec and 7.75 sec (inclusive).

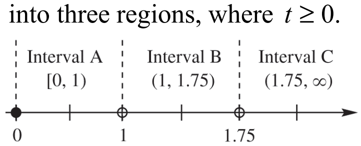
83. $-16t^2 + 44t + 4 \ge 32$

Step 1: Find the values of t that satisfy $-16t^2 + 44t + 4 = 32$.

$$-16t^2 + 44t + 4 = 32$$
$$-16t^2 + 44t - 28 = 0$$
$$4t^2 - 11t + 7 = 0$$
$$(4t-7)(t-1) = 0$$

$4t - 7 = 0 \Rightarrow t = \frac{7}{4} = 1.75$ or $t - 1 = 0 \Rightarrow t = 1$

Step 2: The two numbers divide a number line into three regions, where $t \ge 0$.



Step 3: Choose a test value to see if it satisfies the inequality, $-16t^2 + 44t + 4 \ge 32$

Interval	Test Value	Is $-16t^2 + 44t + 4 \ge 32$ True or False?
A: $(-\infty, 1)$	0	$-16 \cdot 0^2 + 44 \cdot 0 + 4 \overset{?}{\ge} 32$ $4 \ge 32$ False
B: $(1, 1.75)$	1.5	$-16 \cdot 1.5^2 + 44 \cdot 1.5 + 4 \overset{?}{\ge} 32$ $34 \ge 32$ True
C: $(1.75, \infty)$	2	$-16 \cdot 2^2 + 44 \cdot 2 + 4 \overset{?}{\ge} 32$ $28 \ge 32$ False

The baseball will be at least 32 feet above ground between 1 sec and 1.75 sec (inclusive).

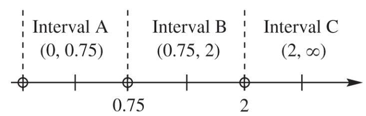
84. $-16t^2 + 44t + 4 > 28$

Step 1: Find the values of t that satisfy $-16t^2 + 44t + 4 = 28$.

$$-16t^2 + 44t + 4 = 28$$
$$-16t^2 + 44t - 24 = 0$$
$$4t^2 - 11t + 6 = 0$$
$$(4x-3)(x-2) = 0$$

$4t - 3 = 0 \Rightarrow t = \frac{3}{4} = 0.75$ or
$t - 2 = 0 \Rightarrow t = 2$

Step 2: The two numbers divide a number line into three regions, where $t \ge 0$.



Step 3: Choose a test value to see if it satisfies the inequality, $-16t^2 + 44t + 4 > 28$

Interval	Test Value	Is $-16t^2 + 44t + 4 > 28$ True or False?
A: $(0, 0.75)$	0.5	$-16 \cdot 0.5^2 + 44 \cdot 0.5 + 4 \overset{?}{>} 28$ $22 > 28$ False
B: $(0.75, 2)$	1.5	$-16 \cdot 1.5^2 + 44 \cdot 1.5 + 4 \overset{?}{>} 28$ $34 > 28$ True
C: $(2, \infty)$	3	$-16 \cdot 3^2 + 44 \cdot 3 + 4 \overset{?}{>} 28$ $-8 \ge 28$ False

The baseball will be greater than 28 feet above ground between 0.75 sec and 2 sec.

85. $2t^2 - 5t - 12 < 0$

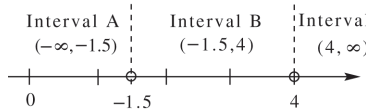
Step 1: Find the values of t that satisfy $2t^2 - 5t - 12 = 0.$

$2t^2 - 5t - 12 = 0 \Rightarrow (2t+3)(t-4) = 0$

$2t + 3 = 0 \Rightarrow t = -\frac{3}{2} = -1.5$ or

$t - 4 = 0 \Rightarrow t = 4$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $2t^2 - 5t - 12 < 0.$

Interval	Test Value	Is $2t^2 - 5t - 12 < 0$ True or False?
A: $(-\infty, -1.5)$	-2	$2(-2)^2 - 5(-2) - 12 \overset{?}{<} 0$ $6 < 0$ False
B: $(-1.5, 4)$	0	$2 \cdot 0^2 - 5 \cdot 0 - 12 \overset{?}{<} 0$ $-12 < 0$ True
C: $(4, \infty)$	5	$2 \cdot 5^2 - 5 \cdot 5 - 12 \overset{?}{<} 0$ $13 < 0$ False

The velocity will be negative between –1.5 sec and 4 sec.

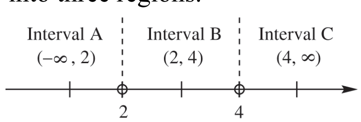
86. $3t^2 - 18t + 24 < 0$

Step 1: Find the values of t that satisfy $3t^2 - 18t + 24 = 0.$

$3t^2 - 18t + 24 = 0 \Rightarrow 3(t-2)(t-4) = 0$

$t - 2 = 0 \Rightarrow t = 2$ or $t - 4 = 0 \Rightarrow t = 4$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $3t^2 - 18t + 24 < 0.$

Interval	Test Value	Is $3t^2 - 18t + 24 < 0$ True or False?
A: $(-\infty, 2)$	0	$3(0)^2 - 18(0) + 24 \overset{?}{<} 0$ $24 < 0$ False
B: $(2, 4)$	3	$3(3)^2 - 18(3) + 24 \overset{?}{<} 0$ $-3 < 0$ True
C: $(4, \infty)$	5	$3(5)^2 - 18(5) + 24 \overset{?}{<} 0$ $9 < 0$ False

The velocity will be negative between 2 sec and 4 sec.

87. $(3x - 4)(x + 2)(x + 6) = 0$

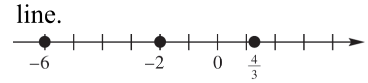
Set each factor to zero and solve.

$3x - 4 = 0 \Rightarrow x = \frac{4}{3}$ or $x + 2 = 0 \Rightarrow x = -2$ or

$x + 6 = 0 \Rightarrow x = -6$

Solution set: $\left\{\frac{4}{3}, -2, -6\right\}$

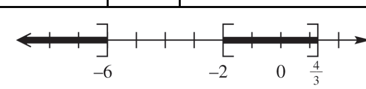
88. Plot the solutions –6, –2, and $\frac{4}{3}$ on a number line.



89.

Interval	Test Value	Is $(3x-4)(x+2)(x+6) \le 0$ True or False?
A: $(-\infty, -6)$	–10	$[3(-10)-4][-10+2]$ $\cdot[-10+6] \overset{?}{\le} 0$ $-1088 \le 0$ True
B: $(-6, -2)$	–4	$[3(-4)-4][-4+2]$ $\cdot[-4+6] \overset{?}{\le} 0$ $64 \le 0$ False
C: $\left(-2, \frac{4}{3}\right)$	0	$[3(0)-4][0+2][0+6] \overset{?}{\le} 0$ $-48 \le 0$ True
D: $\left(\frac{4}{3}, \infty\right)$	4	$[3(4)-4][4+2][4+6] \overset{?}{\le} 0$ $480 \le 0$ False

90. **(a)**

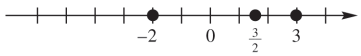


(b) Solution set: $(-\infty, -6] \cup \left[-2, \frac{4}{3}\right]$

91. $(2x-3)(x+2)(x-3) \ge 0$

Step 1: Solve $(2x-3)(x+2)(x-3)=0$.
Set each factor to zero and solve.
$2x-3=0 \Rightarrow x=\frac{3}{2}$ or $x+2=0 \Rightarrow x=-2$ or
$x-3=0 \Rightarrow x=3$
Solution set: $\left\{-2, \frac{3}{2}, 3\right\}$

Step 2: Plot the solutions $-2, \frac{3}{2},$ and 3 on a number line.



Step 3: Choose a test value to see if it satisfies the inequality, $(2x-3)(x+2)(x-3) \ge 0$.

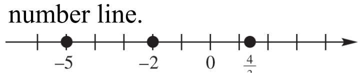
Interval	Test Value	Is $(2x-3)(x+2)(x-3) \ge 0$ True or False?
A: $(-\infty, -2)$	-3	$\left[2(-3)-3\right]\left[-3+2\right] \cdot \left[-3-3\right] \overset{?}{\ge} 0$ $-54 \ge 0$ False
B: $\left(-2, \frac{3}{2}\right)$	0	$\left[2(0)-3\right]\left[0+2\right] \cdot \left[0-3\right] \overset{?}{\ge} 0$ $18 \ge 0$ True
C: $\left(\frac{3}{2}, 3\right)$	2	$\left[2(2)-3\right]\left[2+2\right] \cdot \left[2-3\right] \overset{?}{\ge} 0$ $-4 \ge 0$ False
D: $(3, \infty)$	4	$\left[2(4)-3\right]\left[4+2\right] \cdot \left[4-3\right] \overset{?}{\ge} 0$ $30 \ge 0$ True

Solution set: $\left[-2, \frac{3}{2}\right] \cup [3, \infty)$

92. $(x+5)(3x-4)(x+2) \ge 0$

Step 1: Solve $(x+5)(3x-4)(x+2)=0$.
Set each factor to zero and solve.
$x+5=0 \Rightarrow x=-5$ or $3x-4=0 \Rightarrow x=\frac{4}{3}$ or
$x+2=0 \Rightarrow x=-2$
Solution set: $\left\{-5, -2, \frac{4}{3}\right\}$

Step 2: Plot the solutions $-5, -2,$ and $\frac{4}{3}$ on a number line.



Step 3: Choose a test value to see if it satisfies the inequality, $(x+5)(3x-4)(x+2) \ge 0$.

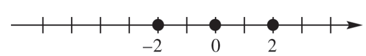
Interval	Test Value	Is $(x+5)(3x-4)(x+2) \ge 0$ True or False?
A: $(-\infty, -5)$	-6	$\left[-6+5\right]\left[3(-6)-4\right] \cdot \left[-6+2\right] \overset{?}{\ge} 0$ $-88 \ge 0$ False
B: $(-5, -2)$	-3	$\left[-3+5\right]\left[3(-3)-4\right] \cdot \left[-3+2\right] \overset{?}{\ge} 0$ $26 \ge 0$ True
C: $\left(-2, \frac{4}{3}\right)$	0	$\left[0+5\right]\left[3(0)-4\right] \cdot \left[0+2\right] \overset{?}{\ge} 0$ $-40 \ge 0$ False
D: $\left(\frac{4}{3}, \infty\right)$	2	$\left[2+5\right]\left[3(2)-4\right] \cdot \left[2+2\right] \ge 0$ $56 \ge 0$ True

Solution set: $[-5, -2] \cup \left[\frac{4}{3}, \infty\right)$

93. $4x - x^3 \ge 0$

Step 1: Solve $4x - x^3 = 0$.
$$4x - x^3 = 0 \Rightarrow x\left(4-x^2\right) = 0 \Rightarrow$$
$x(2+x)(2-x)=0$
Set each factor to zero and solve.
$x=0$ or $2+x=0 \Rightarrow x=-2$ or
$2-x=0 \Rightarrow x=2$
Solution set: $\{-2, 0, 2\}$

Step 2: The values -2, 0, and 2 divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $4x - x^3 \ge 0$.

Interval	Test Value	Is $4x - x^3 \ge 0$ True or False?
A: $(-\infty, -2)$	-3	$4(-3)-(-3)^3 \overset{?}{\ge} 0$ $15 \ge 0$ True
B: $(-2, 0)$	-1	$4(-1)-(-1)^3 \overset{?}{\ge} 0$ $-3 \ge 0$ False

(*continued on next page*)

(*continued*)

Interval	Test Value	Is $4x - x^3 \ge 0$ True or False?
C: $(0,2)$	1	$4(1) - 1^3 \overset{?}{\ge} 0$ $3 \ge 0$ True
D: $(2,\infty)$	3	$4(3) - 3^3 \overset{?}{\ge} 0$ $-15 \ge 0$ False

Solution set: $(-\infty,-2] \cup [0,2]$

94. $16x - x^3 \ge 0$

Step 1: Solve $16x - x^3 = 0.$

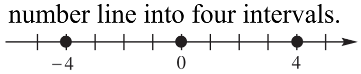
$$16x - x^3 = 0 \Rightarrow x(16 - x^2) = 0 \Rightarrow$$
$$x(4 + x)(4 - x) = 0$$

Set each factor to zero and solve.

$x = 0$ or $4 + x = 0 \Rightarrow x = -4$ or $4 - x = 0 \Rightarrow x = 4$

Solution set: $\{-4, 0, 4\}$

Step 2: The values -4, 0, and 4 divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $16x - x^3 \ge 0.$

Interval	Test Value	Is $16x - x^3 \ge 0$ True or False?
A: $(-\infty,-4)$	-5	$16(-5) - (-5)^3 \overset{?}{\ge} 0$ $45 \ge 0$ True
B: $(-4,0)$	-1	$16(-1) - (-1)^3 \overset{?}{\ge} 0$ $-15 \ge 0$ False
C: $(0,4)$	1	$16(1) - 1^3 \overset{?}{\ge} 0$ $15 \ge 0$ True
D: $(4,\infty)$	5	$16(5) - 5^3 \overset{?}{\ge} 0$ $-45 \ge 0$ False

Solution set: $(-\infty,-4] \cup [0,4]$

95. $(x+1)^2(x-3) < 0$

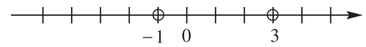
Step 1: Solve $(x+1)^2(x-3) = 0.$

Set each distinct factor to zero and solve.

$x + 1 = 0 \Rightarrow x = -1$ or $x - 3 = 0 \Rightarrow x = 3$

Solution set: $\{-1, 3\}$

Step 2: The values -1 and 3 divide the number line into three intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $(x+1)^2(x-3) < 0.$

Interval	Test Value	Is $(x+1)^2(x-3) < 0$ True or False?
A: $(-\infty,-1)$	-2	$(-2+1)^2(-2-3) \overset{?}{<} 0$ $-5 < 0$ True
B: $(-1,3)$	0	$(0+1)^2(0-3) \overset{?}{<} 0$ $-3 < 0$ True
C: $(3,\infty)$	4	$(4+1)^2(4-3) \overset{?}{<} 0$ $25 < 0$ False

Solution set: $(-\infty,-1) \cup (-1,3)$

96. $(x-5)^2(x+1) < 0$

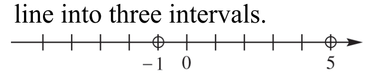
Step 1: Solve $(x-5)^2(x+1) = 0.$

Set each distinct factor to zero and solve.

$x - 5 = 0 \Rightarrow x = 5$ or $x + 1 = 0 \Rightarrow x = -1$

Solution set: $\{-1, 5\}$

Step 2: The values -1 and 5 divide the number line into three intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $(x-5)^2(x+1) < 0.$

Interval	Test Value	Is $(x-5)^2(x+1) < 0$ True or False?
A: $(-\infty,-1)$	-2	$(-2-5)^2(-2+1) \overset{?}{<} 0$ $-49 < 0$ True
B: $(-1,5)$	0	$(0-5)^2(0+1) \overset{?}{<} 0$ $25 < 0$ False
C: $(5,\infty)$	6	$(6-5)^2(6+1) \overset{?}{<} 0$ $7 < 0$ False

Solution set: $(-\infty,-1)$

97. $x^3+4x^2-9x \ge 36$

Step 1: Solve $x^3+4x^2-9x=36$

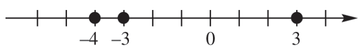
$$x^3+4x^2-9x=36$$
$$x^3+4x^2-9x-36=0$$
$$x^2(x+4)-9(x+4)=0$$
$$(x+4)(x^2-9)=0$$
$$(x+4)(x+3)(x-3)=0$$

Set each factor to zero and solve.

$x+4=0 \Rightarrow x=-4$ or $x+3=0 \Rightarrow x=-3$ or $x-3=0 \Rightarrow x=3$

Solution set: $\{-4,-3,3\}$

Step 2: The values $-4, -3$, and 3 divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $x^3+4x^2-9x \ge 36$

Interval	Test Value	Is $x^3+4x^2-9x \ge 36$ True or False?
A: $(-\infty,-4)$	-5	$(-5)^3+4(-5)^2-9(-5) \overset{?}{\ge} 36$ $20 \ge 36$ False
B: $(-4,-3)$	-3.5	$(-3.5)^3+4(-3.5)^2$ $-9(-3.5) \overset{?}{\ge} 36$ $37.625 \ge 36$ True
C: $(-3,3)$	0	$0^3+4(0)^2-9(0) \overset{?}{\ge} 36$ $0 \ge 36$ False
D: $(3,\infty)$	4	$4^3+4(4)^2-9(4) \overset{?}{\ge} 36$ $92 \ge 36$ True

Solution set: $[-4,-3] \cup [3,\infty)$

98. $x^3+3x^2-16x \le 48$

Step 1: Solve $x^3+3x^2-16x=48$.

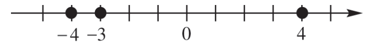
$$x^3+3x^2-16x=48$$
$$x^3+3x^2-16x-48=0$$
$$x^2(x+3)-16(x+3)=0$$
$$(x+3)(x^2-16)=0$$
$$(x+3)(x+4)(x-4)=0$$

Set each factor to zero and solve.

$x+3=0 \Rightarrow x=-3$ or $x+4=0 \Rightarrow x=-4$ or $x-4=0 \Rightarrow x=4$

Solution set: $\{-4,-3,4\}$

Step 2: The values $-4, -3$, and 4 divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $x^3+3x^2-16x \le 48$.

Interval	Test Value	Is $x^3+3x^2-16x \le 48$. True or False?
A: $(-\infty,-4)$	-5	$(-5)^3+3(-5)^2$ $-16(-5) \overset{?}{\le} 48$ $30 \le 48$ True
B: $(-4,-3)$	-3.5	$(-3.5)^3+3(-3.5)^2$ $-16(-3.5) \overset{?}{\le} 48$ $49.875 \le 48$ False
C: $(-3,4)$	0	$0^3+3(0)^2-16(0) \overset{?}{\le} 48$ $0 \le 48$ True
D: $(4,\infty)$	5	$5^3+3(5)^2-16(5) \le 48$ $120 \le 48$ False

Solution set: $(-\infty,-4] \cup [-3,4]$

99. $x^2(x+4)^2 \ge 0$

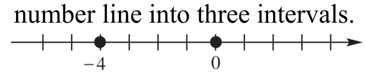
Step 1: Solve $x^2(x+4)^2=0$.

Set each distinct factor to zero and solve.

$x=0$ or $x+4=0 \Rightarrow x=-4$

Solution set: $\{-4,0\}$

Step 2: The values -4 and 0 divide the number line into three intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2(x+4)^2 \ge 0$.

Interval	Test Value	Is $x^2(x+4)^2 \ge 0$ True or False?
A: $(-\infty,-4)$	-5	$(-5)^2(-5+4)^2 \overset{?}{\ge} 0$ $25 \ge 0$ True

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(*continued*)

Interval	Test Value	Is $x^2(x+4)^2 \geq 0$ True or False?
B: $(-4, 0)$	-1	$(-1)^2(-1+4)^2 \overset{?}{\geq} 0$ $9 \geq 0$ True
C: $(0, \infty)$	1	$1^2(1+4)^2 \overset{?}{\geq} 0$ $25 \geq 0$ True

Solution set: $(-\infty, \infty)$

100. $-x^2(2x-3)^2 \leq 0$

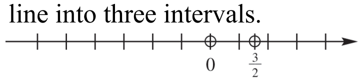
Step 1: Solve $-x^2(2x-3)^2 = 0$.

Set each distinct factor to zero and solve.

$-x^2 = 0 \Rightarrow x = 0$ or $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Solution set: $\left\{0, \frac{3}{2}\right\}$

Step 2: The values 0 and $\frac{3}{2}$ divide the number line into three intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $-x^2(2x-3)^2 \leq 0$.

Interval	Test Value	Is $-x^2(2x-3)^2 \leq 0$ True or False?
A: $(-\infty, 0)$	-1	$-(-1)^2\left[2(-1)-3\right]^2 \overset{?}{\leq} 0$ $-25 \leq 0$ True
B: $\left(0, \frac{3}{2}\right)$	1	$-1^2\left[2(1)-3\right]^2 \overset{?}{\leq} 0$ $-1 \leq 0$ True
C: $\left(\frac{3}{2}, \infty\right)$	2	$-2^2\left[2(2)-3\right]^2 \overset{?}{\leq} 0$ $-4 \leq 0$ True

Solution set: $(-\infty, \infty)$

Section 1.8 Absolute Value Equations and Inequalities

1. F. The solution set includes any value of x whose absolute value is 7; thus $x = 7$ or $x = -7$ are both solutions.
2. B. There is no solution because the absolute value of any real number is never negative.
3. D. The solution set is all real numbers because the absolute value of any real number is always greater than –7. The graph shows the entire number line.
4. E. The solution set includes any value of x whose absolute value is greater than 7; thus $x > 7$ or $x < -7$.
5. G. The solution set includes any value of x whose absolute value is less than 7; thus x must be between –7 and 7, not including –7 or 7.
6. A. The solution set includes any value of x whose absolute value is greater than or equal to 7; thus $x \geq 7$ or $x \leq -7$.
7. C. The solution set includes any value of x whose absolute value is less than or equal to 7; thus x must be between –7 and 7, including –7 and 7.
8. H. The solution set includes any value of x whose absolute value is not equal to 7; thus, x can equal all real numbers except –7 and 7.

9. $|3x-1| = 2$

$3x - 1 = 2 \Rightarrow 3x = 3 \Rightarrow x = 1$ or

$3x - 1 = -2 \Rightarrow 3x = -1 \Rightarrow x = -\frac{1}{3}$

Solution set: $\left\{-\frac{1}{3}, 1\right\}$

10. $|4x+2| = 5$

$4x + 2 = 5 \Rightarrow 4x = 3 \Rightarrow x = \frac{3}{4}$ or

$4x + 2 = -5 \Rightarrow 4x = -7 \Rightarrow x = -\frac{7}{4}$

Solution set: $\left\{-\frac{7}{4}, \frac{3}{4}\right\}$

11. $|5-3x| = 3$

$5 - 3x = 3 \Rightarrow 2 = 3x \Rightarrow \frac{2}{3} = x$ or

$5 - 3x = -3 \Rightarrow 8 = 3x \Rightarrow \frac{8}{3} = x$

Solution set: $\left\{\frac{2}{3}, \frac{8}{3}\right\}$

12. $|7-3x| = 3$

$7 - 3x = 3 \Rightarrow -3x = -4 \Rightarrow x = \frac{4}{3}$ or

$7 - 3x = -3 \Rightarrow -3x = -10 \Rightarrow x = \frac{10}{3}$

Solution set: $\left\{\frac{4}{3}, \frac{10}{3}\right\}$

13. $\left|\frac{x-4}{2}\right| = 5$

$\frac{x-4}{2} = 5 \Rightarrow x - 4 = 10 \Rightarrow x = 14$ or

$\frac{x-4}{2} = -5 \Rightarrow x - 4 = -10 \Rightarrow x = -6$

Solution set: {–6, 14}

14. $\left|\frac{x+2}{2}\right| = 7$

$\frac{x+2}{2} = 7 \Rightarrow x+2 = 14 \Rightarrow x = 12$ or

$\frac{x+2}{2} = -7 \Rightarrow x+2 = -14 \Rightarrow x = -16$

Solution set: {–16, 12}

15. $\left|\frac{5}{x-3}\right| = 10$

$\frac{5}{x-3} = 10 \Rightarrow 5 = 10(x-3) \Rightarrow 5 = 10x - 30 \Rightarrow$

$35 = 10x \Rightarrow x = \frac{35}{10} = \frac{7}{2}$ or

$\frac{5}{x-3} = -10 \Rightarrow 5 = -10(x-3) \Rightarrow$

$5 = -10x + 30 \Rightarrow -25 = -10x \Rightarrow x = \frac{-25}{-10} = \frac{5}{2}$

Solution set: $\left\{\frac{5}{2}, \frac{7}{2}\right\}$

16. $\left|\frac{3}{2x-1}\right| = 4$

$\frac{3}{2x-1} = 4 \Rightarrow 3 = 4(2x-1) \Rightarrow 3 = 8x - 4 \Rightarrow$

$7 = 8x \Rightarrow \frac{7}{8} = x$ or

$\frac{3}{2x-1} = -4 \Rightarrow 3 = -4(2x-1) \Rightarrow$

$3 = -8x + 4 \Rightarrow -1 = -8x \Rightarrow x = \frac{1}{8}$

Solution set: $\left\{\frac{1}{8}, \frac{7}{8}\right\}$

17. $\left|\frac{6x+1}{x-1}\right| = 3$

$\frac{6x+1}{x-1} = 3 \Rightarrow 6x+1 = 3(x-1) \Rightarrow$

$6x+1 = 3x-3 \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$ or

$\frac{6x+1}{x-1} = -3 \Rightarrow 6x+1 = -3(x-1) \Rightarrow$

$6x+1 = -3x+3 \Rightarrow 9x = 2 \Rightarrow x = \frac{2}{9}$

Solution set: $\left\{-\frac{4}{3}, \frac{2}{9}\right\}$

18. $\left|\frac{2x+3}{3x-4}\right| = 1$

$\frac{2x+3}{3x-4} = 1 \Rightarrow 2x+3 = 1(3x-4) \Rightarrow$

$2x+3 = 3x-4 \Rightarrow x = 7$ or

$\frac{2x+3}{3x-4} = -1 \Rightarrow 2x+3 = -1(3x-4) \Rightarrow$

$2x+3 = -3x+4 \Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$

Solution set: $\left\{\frac{1}{5}, 7\right\}$

19. $|2x-3| = |5x+4|$

$2x-3 = 5x+4 \Rightarrow -7 = 3x \Rightarrow -\frac{7}{3} = x$ or

$2x-3 = -(5x+4) \Rightarrow 2x-3 = -5x-4 \Rightarrow$

$7x = -1 \Rightarrow x = \frac{-1}{7} = -\frac{1}{7}$

Solution set: $\left\{-\frac{7}{3}, -\frac{1}{7}\right\}$

20. $|x+1| = |1-3x|$

$x+1 = 1-3x \Rightarrow 4x = 0 \Rightarrow x = 0$ or

$x+1 = -(1-3x) \Rightarrow x+1 = -1+3x \Rightarrow$

$2 = 2x \Rightarrow 1 = x$

Solution set: {0, 1}

21. $|4-3x| = |2-3x|$

$4-3x = 2-3x \Rightarrow 4 = 2$ False or

$4-3x = -(2-3x) \Rightarrow 4-3x = -2+3x \Rightarrow$

$6 = 6x \Rightarrow 1 = x$

Solution set: {1}

22. $|3-2x| = |5-2x|$

$3-2x = 5-2x \Rightarrow 3 = 5$ False or

$3-2x = -(5-2x) \Rightarrow 3-2x = -5+2x \Rightarrow$

$8 = 4x \Rightarrow 2 = x$

Solution set: {2}

23. $|5x-2| = |2-5x|$

$5x-2 = 2-5x \Rightarrow 10x = 4 \Rightarrow x = \frac{4}{10} = \frac{2}{5}$ or

$5x-2 = -(2-5x) \Rightarrow 5x-2 = -2+5x \Rightarrow$

$0 = 0$ True

Solution set: $(-\infty, \infty)$

24. Answers will vary. Sample answer: If *x* is negative, then 3*x* will also be negative. Because the outcome of an absolute value can never be negative, a negative value of *x* is not possible.

25. Answers will vary. Sample answer: If *x* is positive, then –5*x* will be negative. Because the outcome of an absolute value can never be negative, a positive value of x is not possible.

26. **(a)** $-|x| = |x|$

$|x|$ will equal its own opposite only if $x = 0$.

Solution set: {0}

(b) $|-x| = |x|$

Any number and its opposite have the same absolute value.

Solution set: $(-\infty, \infty)$

(c) $\left|x^2\right| = |x|$

Solution set: $\{-1, 0, 1\}$

(d) $-|x| = 9$

$|x| = -9$ is never true.

Solution set: $\varnothing$

27. $|2x+5| < 3$

$-3 < 2x + 5 < 3$
$-8 < 2x < -2$
$-4 < x < -1$

Solution set: $(-4, -1)$

28. $|3x-4| < 2$

$-2 < 3x - 4 < 2$
$2 < 3x < 6$
$\frac{2}{3} < x < 2$

Solution set: $\left(\frac{2}{3}, 2\right)$

29. $|2x+5| \ge 3$

$2x + 5 \le -3 \Rightarrow 2x \le -8 \Rightarrow x \le -4$ or
$2x + 5 \ge 3 \Rightarrow 2x \ge -2 \Rightarrow x \ge -1$

Solution set: $(-\infty, -4] \cup [-1, \infty)$

30. $|3x-4| \ge 2$

$3x - 4 \le -2 \Rightarrow 3x \le 2 \Rightarrow x \le \frac{2}{3}$ or
$3x - 4 \ge 2 \Rightarrow 3x \ge 6 \Rightarrow x \ge 2$

Solution set: $\left(-\infty, \frac{2}{3}\right] \cup [2, \infty)$

31. $\left|\frac{1}{2} - x\right| < 2$

$-2 < \frac{1}{2} - x < 2$
$2(-2) < 2\left(\frac{1}{2} - x\right) < 2(2)$
$-4 < 1 - 2x < 4$
$-5 < -2x < 3$
$\frac{5}{2} > x > -\frac{3}{2}$

Solution set: $\left(-\frac{3}{2}, \frac{5}{2}\right)$

32. $\left|\frac{3}{5} + x\right| < 1$

$-1 < \frac{3}{5} + x < 1$
$5(-1) < 5\left(\frac{3}{5} + x\right) < 5(1)$
$-5 < 3 + 5x < 5$
$-8 < 5x < 2$
$-\frac{8}{5} < x < \frac{2}{5}$

Solution set: $\left(-\frac{8}{5}, \frac{2}{5}\right)$

33. $4|x-3| > 12 \Rightarrow |x-3| > 3$

$x - 3 < -3 \Rightarrow x < 0$ or $x - 3 > 3 \Rightarrow x > 6$

Solution set: $(-\infty, 0) \cup (6, \infty)$

34. $5|x+1| > 10 \Rightarrow |x+1| > 2$

$x + 1 < -2 \Rightarrow x < -3$ or $x + 1 > 2 \Rightarrow x > 1$

Solution set: $(-\infty, -3) \cup (1, \infty)$

35. $|5-3x| > 7$

$5 - 3x < -7 \Rightarrow -3x < -12 \Rightarrow x > 4$ or
$5 - 3x > 7 \Rightarrow -3x > 2 \Rightarrow x < -\frac{2}{3}$

Solution set: $\left(-\infty, -\frac{2}{3}\right) \cup (4, \infty)$

36. $|7-3x| > 4$

$7 - 3x < -4 \Rightarrow -3x < -11 \Rightarrow x > \frac{11}{3}$ or
$7 - 3x > 4 \Rightarrow -3x > -3 \Rightarrow x < 1$

Solution set: $(-\infty, 1) \cup \left(\frac{11}{3}, \infty\right)$

37. $|5-3x| \le 7$

$-7 \le 5 - 3x \le 7$
$-12 \le -3x \le 2$
$4 \ge x \ge -\frac{2}{3}$
$-\frac{2}{3} \le x \le 4$

Solution set: $\left[-\frac{2}{3}, 4\right]$

38. $|7-3x| \le 4$

$-4 \le 7 - 3x \le 4$
$-11 \le -3x \le -3$
$\frac{11}{3} \ge x \ge 1$
$1 \le x \le \frac{11}{3}$

Solution set: $\left[1, \frac{11}{3}\right]$

39. $\left|\frac{2}{3}x + \frac{1}{2}\right| \le \frac{1}{6}$

$-\frac{1}{6} \le \frac{2}{3}x + \frac{1}{2} \le \frac{1}{6}$
$6\left(-\frac{1}{6}\right) \le 6\left(\frac{2}{3}x + \frac{1}{2}\right) \le 6\left(\frac{1}{6}\right)$
$-1 \le 4x + 3 \le 1$
$-4 \le 4x \le -2$
$-1 \le x \le -\frac{1}{2}$

Solution set: $\left[-1, -\frac{1}{2}\right]$

40. $\left|\frac{5}{3}-\frac{1}{2}x\right|>\frac{2}{9}$

$\frac{5}{3}-\frac{1}{2}x<-\frac{2}{9}\Rightarrow 18\left(\frac{5}{3}-\frac{1}{2}x\right)<18\left(-\frac{2}{9}\right)\Rightarrow$
$30-9x<-4\Rightarrow -9x<-34\Rightarrow x>\frac{34}{9}$ or
$\frac{5}{3}-\frac{1}{2}x>\frac{2}{9}\Rightarrow 18\left(\frac{5}{3}-\frac{1}{2}x\right)>18\left(\frac{2}{9}\right)\Rightarrow$
$30-9x>4\Rightarrow -9x>-26\Rightarrow x<\frac{26}{9}$

Solution set: $\left(-\infty,\frac{26}{9}\right)\cup\left(\frac{34}{9},\infty\right)$

41. $|0.01x+1|<0.01$

$-0.01<0.01x+1<0.01$
$-1<x+100<1$
$-101<x<-99$
Solution set: $(-101,-99)$

42. Because $x^2=(-x)^2$, $|x|=\sqrt{x^2}$ for all values of x. The equation is an identity.

43. $|4x+3|-2=-1\Rightarrow|4x+3|=1$

$4x+3=1\Rightarrow 4x=-2\Rightarrow x=\frac{-2}{4}=-\frac{1}{2}$ or
$4x+3=-1\Rightarrow 4x=-4\Rightarrow x=-1$
Solution set: $\left\{-1,-\frac{1}{2}\right\}$

44. $|8-3x|-3=-2\Rightarrow|8-3x|=1$

$8-3x=1\Rightarrow -3x=-7\Rightarrow x=\frac{7}{3}$ or
$8-3x=-1\Rightarrow -3x=-9\Rightarrow x=3$
Solution set: $\left\{\frac{7}{3},3\right\}$

45. $|6-2x|+1=3\Rightarrow|6-2x|=2$

$6-2x=2\Rightarrow -2x=-4\Rightarrow x=2$ or
$6-2x=-2\Rightarrow -2x=-8\Rightarrow x=4$
Solution set: $\{2,4\}$

46. $|4-4x|+2=4\Rightarrow|4-4x|=2$

$4-4x=2\Rightarrow -4x=-2\Rightarrow x=\frac{-2}{-4}=\frac{1}{2}$ or
$4-4x=-2\Rightarrow -4x=-6\Rightarrow x=\frac{-6}{-4}=\frac{3}{2}$
Solution set: $\left\{\frac{1}{2},\frac{3}{2}\right\}$

47. $|3x+1|-1<2\Rightarrow|3x+1|<3$

$-3<3x+1<3$
$-4<3x<2$
$-\frac{4}{3}<x<\frac{2}{3}$
Solution set: $\left(-\frac{4}{3},\frac{2}{3}\right)$

48. $|5x+2|-2<3\Rightarrow|5x+2|<5$

$-5<5x+2<5$
$-7<5x<3$
$-\frac{7}{5}<x<\frac{3}{5}$
Solution set: $\left(-\frac{7}{5},\frac{3}{5}\right)$

49. $\left|5x+\frac{1}{2}\right|-2<5\Rightarrow\left|5x+\frac{1}{2}\right|<7$

$-7<5x+\frac{1}{2}<7$
$2(-7)<2\left(5x+\frac{1}{2}\right)<2(7)$
$-14<10x+1<14$
$-15<10x<13$
$-\frac{15}{10}<x<\frac{13}{10}\Rightarrow -\frac{3}{2}<x<\frac{13}{10}$
Solution set: $\left(-\frac{3}{2},\frac{13}{10}\right)$

50. $\left|2x+\frac{1}{3}\right|+1<4\Rightarrow\left|2x+\frac{1}{3}\right|<3$

$-3<2x+\frac{1}{3}<3$
$3(-3)<3\left(2x+\frac{1}{3}\right)<3(3)$
$-9<6x+1<9$
$-10<6x<8$
$\frac{-10}{6}<x<\frac{8}{6}$
$-\frac{5}{3}<x<\frac{4}{3}$
Solution set: $\left(-\frac{5}{3},\frac{4}{3}\right)$

51. $|10-4x|+1\ge 5\Rightarrow|10-4x|\ge 4$

$10-4x\le -4\Rightarrow -4x\le -14\Rightarrow x\ge\frac{-14}{-4}\Rightarrow x\ge\frac{7}{2}$
or
$10-4x\ge 4\Rightarrow -4x\ge -6\Rightarrow x\le\frac{-6}{-4}\Rightarrow x\le\frac{3}{2}$
Solution set: $\left(-\infty,\frac{3}{2}\right]\cup\left[\frac{7}{2},\infty\right)$

52. $|12-6x|+3\ge 9\Rightarrow|12-6x|\ge 6$

$12-6x\le -6\Rightarrow -6x\le -18\Rightarrow x\ge 3$ or
$12-6x\ge 6\Rightarrow -6x\ge -6\Rightarrow x\le 1$
Solution set: $(-\infty,1]\cup[3,\infty)$

53. $|3x-7|+1<-2\Rightarrow|3x-7|<-3$

An absolute value cannot be negative.
Solution set: $\varnothing$

54. $|-5x+7|-4<-6\Rightarrow|-5x+7|<-2$

An absolute value cannot be negative.
Solution set: $\varnothing$

55. Because the absolute value of a number is always nonnegative, the inequality $|10-4x|\ge -4$ is always true. The solution set is $(-\infty,\infty)$.

56. Because the absolute value of a number is always nonnegative, the inequality $|12-9x| \geq -12$ is always true. The solution set is $(-\infty, \infty)$.

57. There is no number whose absolute value is less than any negative number. The solution set of $|6-3x| < -11$ is $\varnothing$.

58. There is no number whose absolute value is less than any negative number. The solution set of $|18-3x| < -3$ is $\varnothing$.

59. The absolute value of a number will be 0 if that number is 0. Therefore $|8x+5| = 0$ is equivalent to $8x+5=0$, which has solution set $\left\{-\frac{5}{8}\right\}$.

60. The absolute value of a number will be 0 if that number is 0. Therefore $|7+2x| = 0$ is equivalent to $7+2x=0$, which has solution set $\left\{-\frac{7}{2}\right\}$.

61. Any number less than zero will be negative. There is no number whose absolute value is a negative number. The solution set of $|4.3x+9.8| < 0$ is $\varnothing$.

62. Any number less than zero will be negative. There is no number whose absolute value is a negative number. The solution set of $|1.5x-14| < 0$ is $\varnothing$.

63. Because the absolute value of a number is always nonnegative, $|2x+1| < 0$ is never true, so $|2x+1| \leq 0$ is only true when $|2x+1| = 0$.
$|2x+1| = 0 \Rightarrow 2x+1=0 \Rightarrow 2x=-1 \Rightarrow x=-\frac{1}{2}$
Solution set: $\left\{-\frac{1}{2}\right\}$

64. Because the absolute value of a number is always nonnegative, $|3x+2| < 0$ is never true, so $|3x+2| \leq 0$ is only true when $|3x+2| = 0$.
$|3x+2| = 0 \Rightarrow 3x+2=0 \Rightarrow 3x=-2 \Rightarrow x=-\frac{2}{3}$
Solution set: $\left\{-\frac{2}{3}\right\}$

65. $|3x+2| > 0$ will be false only when $3x+2=0$, which occurs when $x=-\frac{2}{3}$. So the solution set for $|3x+2| > 0$ is $\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, \infty\right)$.

66. $|4x+3| > 0$ will be false only when $4x+3=0$, which occurs when $x=-\frac{3}{4}$. So the solution set for $|4x+3| > 0$ is $\left(-\infty, -\frac{3}{4}\right) \cup \left(-\frac{3}{4}, \infty\right)$.

67. $|p-q| = 2$, which is equivalent to $|q-p| = 2$, indicates that the distance between *p* and *q* is 2 units.

68. $|r-s| = 6$, which is equivalent to $|s-r| = 6$, indicates that the distance between *r* and *s* is 6 units.

69. "*m* is no more than 2 units from 7" means that *m* is 2 units or less from 7. Thus the distance between *m* and 7 is less than or equal to 2, or $|m-7| \leq 2$.

70. "*z* is no less than 5 units from 4" means that *z* is 5 units or more from 4. Thus, the distance between *z* and 4 is greater than or equal to 5, or $|z-4| \geq 5$.

71. "*p* is within 0.0001 unit of 9" means that *p* is less than 0.0001 unit from 9. Thus the distance between *p* and 9 is less than 0.0001, or $|p-9| < 0.0001$.

72. "*k* is within 0.0002 unit of 10" means that *k* is less than 0.0002 unit from 10. Thus the distance between *k* and 10 is less than 0.0002, or $|k-10| < 0.0002$.

73. "*r* is no less than 1 unit from 29" means that *r* is 1 unit or more from 29. Thus the distance between *r* and 29 is greater than or equal to 1, or $|r-29| \geq 1$.

74. "*q* is no more than 8 units from 22" means that *q* is 8 units or less from 22. Thus the distance between *q* and 22 is less than or equal to 8, or $|q-22| \leq 8$.

75. Because we want y to be within 0.002 unit of 6, we have $|y-6|<0.002$ or $|5x+1-6|<0.002.$

$$\begin{aligned}|5x-5|&<0.002\\ -0.002<5x-5&<0.002\\ 4.998<5x&<5.002\\ 0.9996<x&<1.0004\end{aligned}$$

Values of x in the interval $(0.9996, 1.0004)$ would satisfy the condition.

76. Because we want y to be within 0.002 unit of 6, we have $|y-6|<0.002$ or $|10x+2-6|<0.002.$

$$|10x-4|<0.002\Rightarrow -0.002<10x-4<0.002$$
$$3.998<10x<4.002$$
$$0.3998<x<0.4002$$

Values of x in the interval $(0.3998, 0.4002)$ would satisfy the condition.

77. $|x-8.2|\le 1.5$

$$-1.5\le x-8.2\le 1.5$$
$$6.7\le x\le 9.7$$

The range of weights, in pounds, is [6.7, 9.7].

78. $|C+84|\le 56$

$$-56\le C+84\le 56\Rightarrow -140\le C\le -28$$

In degrees Celsius, the range of temperature is the interval [–140, –28].

79. 780 is 50 more than 730 and 680 is 50 less than 730, so all of the temperatures in the acceptable range are within 50° of 730°. That is $|F-730|\le 50$.

80. Let x = the speed of the kite. 148 is 25 more than 123, and 98 is 25 less than 123, so all the speeds are within 25 ft per sec of 123 ft per sec, that is, $|x-123|\le 25.$

Let x = speed of the wind. 26 is 5 more than 21, and 16 is 5 less than 21, so the wind speeds are within 5 ft per sec of 21 ft per sec, that is, $|x-21|\le 5.$

81. $|R_L-26.75|\le 1.42$

$$-1.42\le R_L-26.75\le 1.42$$
$$25.33\le R_L\le 28.17$$

$|R_E-38.75|\le 2.17$

$$-2.17\le R_E-38.75\le 2.17$$
$$36.58\le R_E\le 40.92$$

82. Because there are 225 students and R_L and R_E are individual rates, the total amounts of carbon dioxide emitted would be $T_L=225R_L$ and $T_E=225R_E$. Thus,

$$(225)(25.33)\le T_L\le(225)(28.17)$$
$$5699.25\le T_L\le 6338.25$$
$$(225)(36.58)\le T_E\le(225)(40.92)$$
$$8230.5\le T_E\le 9207$$

83. 6 and the opposite of 6, namely –6

84. $x^2-x=6$

$$x^2-x-6=0\Rightarrow(x+2)(x-3)=0$$
$$x+2=0\Rightarrow x=-2\quad\text{or}\quad x-3=0\Rightarrow x=3$$

Solution set: $\{-2,3\}$

85. $x^2-x=-6\Rightarrow x^2-x+6=0$

The quadratic formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, can be evaluated with a = 1, b = –1, and c = 6.

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ &=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(6)}}{2(1)}=\frac{1\pm\sqrt{1-24}}{2}\\ &=\frac{1\pm\sqrt{-23}}{2}=\frac{1\pm i\sqrt{23}}{2}=\frac{1}{2}\pm\frac{\sqrt{23}}{2}i\end{aligned}$$

Solution set: $\left\{\frac{1}{2}\pm\frac{\sqrt{23}}{2}i\right\}$

86. $\left\{-2,3,\frac{1}{2}\pm\frac{\sqrt{23}}{2}i\right\}$

87. $|3x^2+x|=14\Rightarrow 3x^2+x=14$ or $3x^2+x=-14$

$$3x^2+x=14$$
$$3x^2+x-14=0$$
$$(3x+7)(x-2)=0$$
$$3x+7=0\Rightarrow x=-\tfrac{7}{3}$$
$$x-2=0\Rightarrow x=2$$

$3x^2+x=-14\Rightarrow 3x^2+x+14=0$

We must use the quadratic formula with a = 3, b = 1, and c = 14.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-1\pm\sqrt{1^2-4\cdot 3\cdot 14}}{2\cdot 3}=\frac{-1\pm\sqrt{-167}}{6}$$
$$=\frac{-1\pm i\sqrt{167}}{6}=-\frac{1}{6}\pm\frac{i\sqrt{167}}{6}$$

Solution set: $\left\{-\frac{7}{3},\ 2,\ -\frac{1}{6}\pm\frac{i\sqrt{167}}{6}\right\}$

88. $\left|2x^2-3x\right|=5 \Rightarrow 2x^2-3x=5$ or $2x^2-3x=-5$

$$2x^2-3x=5$$
$$2x^2-3x-5=0$$
$$(2x-5)(x+1)=0$$
$$2x-5=0 \Rightarrow x=\tfrac{5}{2}$$
$$x+1=0 \Rightarrow x=-1$$
$$2x^2-3x=-5$$
$$2x^2-3x+5=0$$

We must use the quadratic formula with $a = 2$, $b = -3$, and $c = 5$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 2\cdot 5}}{2\cdot 2}=\frac{3\pm\sqrt{-31}}{4}$$
$$=\frac{3\pm i\sqrt{31}}{4}=\frac{3}{4}\pm\frac{i\sqrt{31}}{4}$$

Solution set: $\left\{-1, \frac{5}{2}, \frac{3}{4}\pm\frac{i\sqrt{31}}{4}\right\}$

89. $\left|4x^2-23x-6\right|=0$

Because 0 and the opposite of 0 represent the same value, only one equation needs to be solved.

$$4x^2-23x-6=0 \Rightarrow (4x+1)(x-6)=0$$
$$4x+1=0 \quad \text{or} \quad x-6=0$$
$$x=-\tfrac{1}{4} \quad \text{or} \quad x=6$$

Solution set: $\left\{-\frac{1}{4}, 6\right\}$

90. $\left|6x^3+23x^2+7x\right|=0$

Because 0 and the opposite of 0 represent the same value, only one equation needs to be solved.

$$6x^3+23x^2+7x=0$$
$$x\left(6x^2+23x+7\right)=0$$
$$x(2x+7)(3x+1)=0$$
$$x=0 \text{ or } 2x+7=0 \Rightarrow x=-\tfrac{7}{2} \text{ or}$$
$$3x+1=0 \Rightarrow x=-\tfrac{1}{3}$$

Solution set: $\left\{-\frac{7}{2}, -\frac{1}{3}, 0\right\}$

91. $\left|x^2+1\right|-\left|2x\right|=0$

$$\left|x^2+1\right|-\left|2x\right|=0 \Rightarrow \left|x^2+1\right|=\left|2x\right|$$
$$x^2+1=2x \Rightarrow x^2-2x+1=0$$
$$(x-1)^2=0 \Rightarrow x-1=0 \Rightarrow x=1 \text{ or}$$
$$x^2+1=-2x \Rightarrow x^2+2x+1=0 \Rightarrow (x+1)^2=0$$
$$x+1=0 \Rightarrow x=-1$$

Solution set: $\{-1, 1\}$

92. $\left|\frac{x^2+2}{x}\right|-\frac{11}{3}=0$

$$\left|\frac{x^2+2}{x}\right|-\frac{11}{3}=0 \Rightarrow \left|\frac{x^2+2}{x}\right|=\frac{11}{3}$$
$$\frac{x^2+2}{x}=\frac{11}{3} \Rightarrow 3x\left(\frac{x^2+2}{x}\right)=3x\left(\frac{11}{3}\right)$$
$$3\left(x^2+2\right)=11x \Rightarrow 3x^2+6=11x \Rightarrow$$
$$3x^2-11x+6=0 \Rightarrow (3x-2)(x-3)=0$$
$$3x-2=0 \Rightarrow x=\tfrac{2}{3} \quad \text{or} \quad x-3=0 \Rightarrow x=3 \text{ or}$$
$$\frac{x^2+2}{x}=-\frac{11}{3}$$
$$3x\left(\frac{x^2+2}{x}\right)=3x\left(-\frac{11}{3}\right)$$
$$3\left(x^2+2\right)=-11x \Rightarrow 3x^2+6=-11x \Rightarrow$$
$$3x^2+11x+6=0 \Rightarrow (3x+2)(x+3)=0$$
$$3x+2=0 \Rightarrow x=-\tfrac{2}{3} \text{ or } x+3=0 \Rightarrow x=-3$$

Solution set: $\left\{-3, -\frac{2}{3}, \frac{2}{3}, 3\right\}$

93. Any number less than zero will be negative. There is no number whose absolute value is a negative number. The solution set of $\left|x^4+2x^2+1\right|<0$ is $\varnothing$.

94. Any number less than zero will be negative. There is no number whose absolute value is a negative number. The solution set of $\left|x^2+10\right|<0$ is $\varnothing$.

95. $\left|\frac{x-4}{3x+1}\right|\ge 0$

This inequality will be true, except where $\frac{x-4}{3x+1}$ is undefined. This occurs when $3x+1=0$, or $x=-\frac{1}{3}$.

Solution set: $\left(-\infty, -\frac{1}{3}\right)\cup\left(-\frac{1}{3}, \infty\right)$

96. $\left|\dfrac{9-x}{7+8x}\right| \ge 0$

This inequality will be true, except where $\frac{9-x}{7+8x}$ is undefined. This occurs when $7+8x=0$, or $x=-\frac{7}{8}$.

Solution set: $\left(-\infty,-\frac{7}{8}\right)\cup\left(-\frac{7}{8},\infty\right)$

Chapter 1 Review Exercises

1. $2x+8=3x+2 \Rightarrow 8=x+2 \Rightarrow 6=x$
Solution set: $\{6\}$

2. $\frac{1}{6}x-\frac{1}{12}(x-1)=\frac{1}{2}$

Multiply each term by the LCD, 12, to eliminate the fractions. Then solve for *x*.

$$12\left(\tfrac{1}{6}x\right)-12\left[\tfrac{1}{12}(x-1)\right]=12\left(\tfrac{1}{2}\right)$$
$$2x-(x-1)=6$$
$$2x-x+1=6$$
$$x+1=6 \Rightarrow x=5$$

Solution set: $\{5\}$

3. $5x-2(x+4)=3(2x+1)$

$$5x-2x-8=6x+3 \Rightarrow 3x-8=6x+3 \Rightarrow$$
$$-8=3x+3 \Rightarrow -11=3x \Rightarrow -\frac{11}{3}=x$$

Solution set: $\left\{-\frac{11}{3}\right\}$

4. $9x-11(k+p)=x(a-1)$

$$9x-11k-11p=ax-x$$
$$10x-ax=11k+11p$$
$$(10-a)x=11k+11p$$
$$x=\frac{11k+11p}{10-a}=\frac{11(k+p)}{10-a}$$

5. $A=\dfrac{24f}{B(p+1)}$ for *f* (approximate annual interest rate)

$$B(p+1)A=B(p+1)\left(\frac{24f}{B(p+1)}\right)$$
$$AB(p+1)=24f$$
$$\frac{AB(p+1)}{24}=f$$
$$f=\frac{AB(p+1)}{24}$$

6. B and C cannot be equations to solve a geometry problem. The length of a rectangle must be positive.

A. $2x+2(x+2)=20$
$$2x+2x+4=20$$
$$4x+4=20 \Rightarrow 4x=16 \Rightarrow x=4$$

B. $2x+2(5+x)=-2$
$$2x+10+2x=-2$$
$$4x+10=-2 \Rightarrow 4x=-12 \Rightarrow x=-3$$

C. $8(x+2)+4x=16$
$$8x+16+4x=16$$
$$12x+16=16 \Rightarrow 12x=0 \Rightarrow x=0$$

D. $2x+2(x-3)=10$
$$2x+2x-6=10$$
$$4x-6=10 \Rightarrow 4x=16 \Rightarrow x=4$$

7. A and B cannot be equations used to find the number of pennies in a jar. The number of pennies must be a whole number.

A. $5x+3=11 \Rightarrow 5x=8 \Rightarrow x=\frac{8}{5}$

B. $12x+6=-4 \Rightarrow 12x=-10 \Rightarrow$
$$x=-\tfrac{10}{12}=-\tfrac{5}{6}$$

C. $100x=50(x+3)$
$$100x=50x+150 \Rightarrow 50x=150 \Rightarrow x=3$$

D. $6(x+4)=x+24$
$$6x+24=x+24$$
$$5x+24=24 \Rightarrow 5x=0 \Rightarrow x=0$$

8. Let l = the length of the carry-on (in inches).
Let w = the width of the carry-on (in inches).
Let h = the height of the carry-on (in inches).
Linear inches = $l+w+h$.

(a) Linear inches $=l+w+h$
$$=9+12+21=42 \text{ in}$$
No; all airlines on the list will allow the Samsonite carry-on bag.

(b) Linear inches $=l+w+h$
$$=10+14+22=46 \text{ in}$$
The carry-on is allowed on Alaska and Southwest.

9. Let x = the original length of the square (in inches). Because the perimeter of a square is 4 times the length of one side, we have $4(x-4)=\frac{1}{2}(4x)+10$. Solve this equation for x to determine the length of each side of the original square.

$4x-16=2x+10$

$2x-16=10 \Rightarrow 2x=26 \Rightarrow x=13$

The original square is 13 in. on each side.

10. Let x = rate of Becky riding her bike to library. Then $x - 8$ = rate of Becky riding her bike home.

To	r	t	d
Library	x	20 min $=\frac{1}{3}$ hr	$\frac{1}{3}x$
Home	$x-8$	30 min $=\frac{1}{2}$ hr	$\frac{1}{2}(x-8)$

Because the distance going to the library is the same as going home, we solve the following.

$\frac{1}{3}x=\frac{1}{2}(x-8) \Rightarrow 6\left[\frac{1}{3}x\right]=6\left[\frac{1}{2}(x-8)\right]$

$2x=3(x-8) \Rightarrow 2x=3x-24$

$-x=-24 \Rightarrow x=24$

To find the distance, substitute $x = 24$ into $d=\frac{1}{3}x.$ Because $d=\frac{1}{3}(24)=8,$ Becky lives 8 mi from the library.

11. Let x = the amount of 100% alcohol solution (in liters).

Strength	Liters of Solution	Liters of Pure Alcohol
100%	x	$1x=x$
10%	12	$0.10\cdot 12=1.2$
30%	$x+12$	$0.30(x+12)$

The number of liters of pure alcohol in the 100% solution plus the number of liters of pure alcohol in the 10% solution must equal the number of liters of pure alcohol in the 30% solution.

$$\begin{aligned} x+1.2&=0.30(x+12)\\ x+1.2&=0.30x+3.6\\ 0.7x+1.2&=3.6\\ 0.7x&=2.4\\ x&=\frac{2.4}{.7}=\frac{24}{7}=3\tfrac{3}{7}\text{ L}\end{aligned}$$

$3\frac{3}{7}$ L of the 100% solution should be added.

12. Let x = amount borrowed at 5.5%. Then $90{,}000 - x$ = amount borrowed at 6%.

Amount Borrowed	Interest Rate	Interest
x	5.5%	$0.055x$
$90,000-x$	6%	$0.06(90,000-x)$
$90,000$		5125

The amount of interest borrowed at 5.5% plus the amount of interest borrowed at 6% note must equal the total amount of interest.

$$\begin{aligned} 0.055x+0.06(90,000-x)&=5125\\ 0.055x+5400-0.06x&=5125\\ -0.005x+5400&=5125\\ -0.005x&=-275\\ x&=55,000\end{aligned}$$

The amount borrowed at 5.5% is \$55,000 and the amount borrowed at 6% is \$90,000 – \$55,000 = \$35,000.

13. Let x = time (in hours) the mother spent driving to meet plane.
Because Mary Lynn has been in the plane for 15 minutes, and 15 minutes is $\frac{1}{4}$ hr, she has been traveling by plane for $x+\frac{1}{4}$ hr.

	d	r	t
Mary Lynn by plane	420		$x+\frac{1}{4}$
Mother by car	20	40	x

The time driven by Mary Lynn's mother can be found by $20=40x \Rightarrow x=\frac{1}{2}$ hr. Mary Lynn, therefore, flew for $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$ hr. The rate of Mary Lynn's plane can be found by

$$r=\frac{d}{t}=\frac{420}{\frac{3}{4}}=420\cdot\frac{4}{3}=560 \text{ km per hour.}$$

14. Let x = number of hours for slower plant (Plant II) to release that amount. Then $\frac{1}{2}x$ = number of hours for faster plant (Plant I) to release that amount. (If the plant is twice as fast, it takes half the time.)

	Rate	Time	Part of the Job Accomplished
Plant II	$\frac{1}{x}$	3	$\frac{1}{x}(3)=\frac{3}{x}$
Plant I	$\frac{1}{\frac{1}{2}x}=\frac{2}{x}$	3	$\frac{2}{x}(3)=\frac{6}{x}$

Because Plant I and Plant II accomplish 1 job (releasing toxic waste) we must solve the following equation.

(*continued on next page*)

(*continued*)

$\frac{3}{x}+\frac{6}{x}=1 \Rightarrow \frac{9}{x}=1 \Rightarrow x\left(\frac{9}{x}\right)=x\cdot 1 \Rightarrow 9=x$

It takes the slower plant (Plant I) 9 hours to release that same amount.

15. **(a)** In one year, the maximum amount of lead ingested would be

$$0.05\ \frac{\text{mg}}{\text{liter}}\cdot 2\ \frac{\text{liters}}{\text{day}}\cdot 365.25\ \frac{\text{days}}{\text{year}} = 36.525\ \frac{\text{mg}}{\text{year}}.$$

The maximum amount A of lead (in milligrams) ingested in x years would be $A = 36.525x$.

(b) If $x = 72$, then $A = 36.525(72) = 2629.8$ mg. The EPA maximum lead intake from water over a lifetime is 2629.8 mg.

16. In 2018, $x = 8$.

$y = 40.892x + 150.53$

$y = 40.892(8) + 150.53 = 477.67$

Based on the model, retail e-commerce sales will be approximately \$477.67 billion in 2018.

17. **(a)** Using 1956 for $x = 0$, then for 1990, $x = 34$.

$y = 0.1132x + 0.4609$

$y = 0.1132\cdot 34 + 0.4609 \approx 4.31$

According to the model, the minimum wage in 1990 was \$4.31. This is \$0.51 more than the actual value of \$3.80.

(b) Let $y = \$5.85$ and then solve for x.

$$\begin{aligned} y &= 0.1132x + 0.4609 \\ 5.85 &= 0.1132x + 0.4609 \\ 5.3891 &= 0.1132x \\ 47.6 &\approx x \end{aligned}$$

The model predicts the minimum wage to be \$5.85 about 47.6 years after 1956, which is mid-2003. This is close to the minimum wage changing to \$5.85 in 2007.

18. **(a)** 1980: 226.5 million (0.078) ≈ 17.7 million
1990: 248.7 million (0.072) ≈ 17.9 million
2000: 281.4 million (0.068) ≈ 19.1 million
2010: 308.7 million (0.063) ≈ 19.4 million
2014: 318.9 million (0.062) ≈ 19.8 million

(b) The number of Americans living in New York State is increasing.

19. $$\begin{aligned}(6-i)+(7-2i) &= (6+7)+\left[-1+(-2)\right]i \\ &= 13+(-3)i = 13-3i\end{aligned}$$

20. $$\begin{aligned}(-11+2i)-(8-7i) &= (-11-8)+\left[2-(-7)\right]i \\ &= -19+9i\end{aligned}$$

21. $$\begin{aligned}15i-(3+2i)-11 &= (-3-11)+(15-2)i \\ &= -14+13i\end{aligned}$$

22. $$\begin{aligned}-6+4i-(8i-2) &= \left[-6-(-2)\right]+(4-8)i \\ &= -4+(-4)i = -4-4i\end{aligned}$$

23. $$\begin{aligned}(5-i)(3+4i) &= 5(3)+5(4i)-i(3)-i(4i) \\ &= 15+20i-3i-4i^2 \\ &= 15+17i-4(-1) \\ &= 15+17i+4 = 19+17i\end{aligned}$$

24. $$\begin{aligned}&(-8+2i)(-1+i) \\ &= -8(-1)-8(i)+2i(-1)+2i(i) \\ &= 8-8i-2i+2i^2 = 8-10i+2(-1) \\ &= 8-10i-2 = 6-10i\end{aligned}$$

25. $(5-11i)(5+11i) = 5^2-(11i)^2$ Product of the sum and difference of two terms

$$\begin{aligned}&= 25-121i^2 \\ &= 25-121(-1) \\ &= 25+121 = 146\end{aligned}$$

26. $(4-3i)^2 = 4^2-2(4)(3i)+(3i)^2$ Square of a binomial

$$\begin{aligned}&= 16-24i+9i^2 \\ &= 16-24i+9(-1) \\ &= 16-24i-9 = 7-24i\end{aligned}$$

27. $$\begin{aligned}-5i(3-i)^2 &= -5i\left[3^2-2(3)(i)+i^2\right] \\ &= -5i\left[9-6i+(-1)\right] \\ &= -5i(8-6i) = -40i+30i^2 \\ &= -40i+30(-1) = -40i+(-30) \\ &= -30-40i\end{aligned}$$

28. $$\begin{aligned}4i(2+5i)(2-i) &= \left(8i+20i^2\right)(2-i) \\ &= (8i-20)(2-i) \\ &= 16i-8i^2-40+20i \\ &= 16i+8-40+20i \\ &= -32+36i\end{aligned}$$

29. $\dfrac{-12-i}{-2-5i}=\dfrac{(-12-i)(-2+5i)}{(-2-5i)(-2+5i)}$
$=\dfrac{24-60i+2i-5i^2}{(-2)^2-(5i)^2}=\dfrac{24-58i-5(-1)}{4-25i^2}$
$=\dfrac{24-58i+5}{4-25(-1)}=\dfrac{29-58i}{4+25}=\dfrac{29-58i}{29}$
$=\dfrac{29}{29}-\dfrac{58}{29}i=1-2i$

30. $\dfrac{-7+i}{-1-i}=\dfrac{(-7+i)(-1+i)}{(-1-i)(-1+i)}=\dfrac{7-7i-i+i^2}{(-1)^2-i^2}$
$=\dfrac{7-8i+(-1)}{1-(-1)}=\dfrac{6-8i}{2}=\dfrac{6}{2}-\dfrac{8}{2}i$
$=3-4i$

31. $i^{11}=i^8\cdot i^3=1\cdot(-i)=-i$

32. $i^{60}=\left(i^4\right)^{15}=1^{15}=1$

33. $i^{1001}=i^{1000}\cdot i=\left(i^4\right)^{250}\cdot i=1^{250}\cdot i=i$

34. $i^{110}=i^{108}\cdot i^2=\left(i^4\right)^{27}\cdot(-1)=1^{27}\cdot(-1)=-1$

35. $i^{-27}=i^{-28}\cdot i=\left(i^4\right)^{-7}\cdot i=1^{-7}\cdot i=i$

36. $\frac{1}{i^{17}}=i^{-17}=i^{-20}\cdot i^3=\left(i^4\right)^{-5}\cdot i^3=1^{-5}\cdot(-i)=-i$

37. $(x+7)^2=5\Rightarrow x+7=\pm\sqrt{5}\Rightarrow x=-7\pm\sqrt{5}$
Solution set: $\{-7\pm\sqrt{5}\}$

38. $(2-3x)^2=8$
$2-3x=\pm\sqrt{8}\Rightarrow 2-3x=\pm2\sqrt{2}\Rightarrow$
$2\pm2\sqrt{2}=3x\Rightarrow \dfrac{2\pm2\sqrt{2}}{3}=x$
Solution set: $\left\{\frac{2\pm2\sqrt{2}}{3}\right\}$

39. $2x^2+x-15=0$
$(x+3)(2x-5)=0$
$x+3=0\Rightarrow x=-3$ or $2x-5=0\Rightarrow x=\frac{5}{2}$
Solution set: $\left\{-3,\frac{5}{2}\right\}$

40. $12x^2=8x-1\Rightarrow 12x^2-8x+1=0$
$(6x-1)(2x-1)=0$
$6x-1=0\Rightarrow x=\frac{1}{6}$ or $2x-1=0\Rightarrow x=\frac{1}{2}$
Solution set: $\left\{\frac{1}{6},\frac{1}{2}\right\}$

41. $-2x^2+11x=-21\Rightarrow -2x^2+11x+21=0$
$2x^2-11x-21=0\Rightarrow(2x+3)(x-7)=0$
$2x+3=0\Rightarrow x=-\frac{3}{2}$ or $x-7=0\Rightarrow x=7$
Solution set: $\left\{-\frac{3}{2},7\right\}$

42. $-x(3x+2)=5\Rightarrow -3x^2-2x=5$
$-3x^2-2x-5=0\Rightarrow 3x^2+2x+5=0$
Solve by completing the square.
$3x^2+2x+5=0$
$x^2+\frac{2}{3}x+\frac{5}{3}=0$
$x^2+\frac{2}{3}x+\frac{1}{9}=-\frac{5}{3}+\frac{1}{9}$
Note: $\left[\frac{1}{2}\cdot\frac{2}{3}\right]^2=\left(\frac{1}{3}\right)^2=\frac{1}{9}$
$\left(x+\frac{1}{3}\right)^2=-\frac{14}{9}$
$x+\frac{1}{3}=\pm\sqrt{-\frac{14}{9}}$
$x+\frac{1}{3}=\pm\frac{\sqrt{14}}{3}i$
$x=-\frac{1}{3}\pm\frac{\sqrt{14}}{3}i$
Solve by the quadratic formula.
Let $a=3$, $b=2$, and $c=5$.
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
$=\dfrac{-2\pm\sqrt{2^2-4(3)(5)}}{2(3)}$
$=\dfrac{-2\pm\sqrt{4-60}}{6}=\dfrac{-2\pm\sqrt{-56}}{6}$
$=\dfrac{-2\pm2i\sqrt{14}}{6}=-\frac{2}{6}\pm\frac{2\sqrt{14}}{6}i=-\frac{1}{3}\pm\frac{\sqrt{14}}{3}i$
Solution set: $\left\{-\frac{1}{3}\pm\frac{\sqrt{14}}{3}i\right\}$

43. $(2x+1)(x-4)=x\Rightarrow 2x^2-8x+x-4=x\Rightarrow$
$2x^2-7x-4=x\Rightarrow 2x^2-8x-4=0\Rightarrow$
$x^2-4x-2=0$
Solve by completing the square.
$x^2-4x-2=0$
$x^2-4x+4=2+4$
Note: $\left[\frac{1}{2}\cdot(-4)\right]^2=(-2)^2=4$
$(x-2)^2=6\Rightarrow x-2=\pm\sqrt{6}\Rightarrow x=2\pm\sqrt{6}$

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Solve by the quadratic formula.
Let $a=1,\ b=-4,$ and $c=-2.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-2)}}{2(1)}=\frac{4\pm\sqrt{16+8}}{2}$$
$$=\frac{4\pm\sqrt{24}}{2}=\frac{4\pm2\sqrt{6}}{2}=2\pm\sqrt{6}$$

Solution set: $\{2\pm\sqrt{6}\}$

44. $\sqrt{2}x^2-4x+\sqrt{2}=0$
Using the quadratic formula would be the most direct approach.
$a=\sqrt{2},\ b=-4,$ and $c=\sqrt{2}$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot\sqrt{2}\cdot\sqrt{2}}}{2\cdot\sqrt{2}}$$
$$=\frac{4\pm\sqrt{16-8}}{2\sqrt{2}}=\frac{4\pm\sqrt{8}}{2\sqrt{2}}=\frac{4\pm2\sqrt{2}}{2\sqrt{2}}=\frac{2\pm\sqrt{2}}{\sqrt{2}}$$
$$=\frac{(2\pm\sqrt{2})(\sqrt{2})}{\sqrt{2}(\sqrt{2})}=\frac{2\sqrt{2}\pm2}{2}=\sqrt{2}\pm1$$

Solution set: $\{\sqrt{2}\pm1\}$

45. $x^2-\sqrt{5}x-1=0$
Using the quadratic formula would be the most direct approach.
$a=1,\ b=-\sqrt{5},$ and $c=-1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-\sqrt{5})\pm\sqrt{(-\sqrt{5})^2-4\cdot1\cdot(-1)}}{2\cdot1}$$
$$=\frac{\sqrt{5}\pm\sqrt{5+4}}{2}=\frac{\sqrt{5}\pm\sqrt{9}}{2}=\frac{\sqrt{5}\pm3}{2}$$

Solution set: $\left\{\frac{\sqrt{5}\pm3}{2}\right\}$

46. $(x+4)(x+2)=2x\Rightarrow x^2+2x+4x+8=2x$
$x^2+6x+8=2x\Rightarrow x^2+4x+8=0$
Solve by completing the square.

$x^2+4x+8=0$
$x^2+4x+4=-8+4$
Note: $\left[\frac{1}{2}\cdot4\right]^2=(2)^2=4$
$(x+2)^2=-4\Rightarrow x+2=\pm\sqrt{-4}\Rightarrow$
$x+2=\pm2i\Rightarrow x=-2\pm2i$

Solve using the quadratic formula.
Let $a=1,\ b=4,$ and $c=8.$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{4^2-4(1)(8)}}{2(1)}$$
$$=\frac{-4\pm\sqrt{16-32}}{2}=\frac{-4\pm\sqrt{-16}}{2}$$
$$=\frac{-4\pm4i}{2}=-2\pm2i$$

Solution set: $\{-2\pm2i\}$

47. D. The equation $(7x+4)^2=11$ has two real, distinct solutions because the positive number 11 has a positive square root and a negative square root.

48. **(a)** B and C are the equations that have exactly one real solution because the positive and negative square root of 0 represent the same number.

(b) A. The equation $(3x-4)^2=-9$ has two imaginary solutions because the negative number –9 has two imaginary square roots.

49. $-6x^2+2x=-3\Rightarrow-6x^2+2x+3=0$
$a=-6,\ b=2,$ and $c=3$
$b^2-4ac=2^2-4(-6)(3)=4+72=76$
The equation has two distinct irrational solutions because the discriminant is positive but not a perfect square.

50. $8x^2=-2x-6\Rightarrow8x^2+2x+6=0$
$a=8,\ b=2,$ and $c=6$
$b^2-4ac=2^2-4(8)(6)=4-192=-188$
The equation has two distinct nonreal complex solutions because the discriminant is negative.

51. $-8x^2+10x=7\Rightarrow0=8x^2-10x+7$
$a=8,\ b=-10,$ and $c=7$
$b^2-4ac=(-10)^2-4(8)(7)$
$=100-224=-124$
The equation has two distinct nonreal complex solutions because the discriminant is negative.

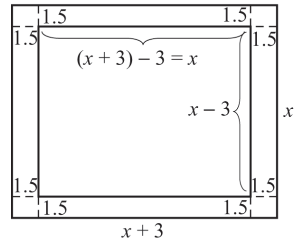
52. $16x^2 + 3 = -26x \Rightarrow 16x^2 + 26x + 3 = 0$
$a = 16$, $b = 26$, and $c = 3$
$$b^2 - 4ac = 26^2 - 4(16)(3) = 676 - 192 = 484 = 22^2$$
The equation has two distinct rational solutions because the discriminant is a positive perfect square.

53. $x(9x+6) = -1 \Rightarrow 9x^2 + 6x = -1 \Rightarrow$
$9x^2 + 6x + 1 = 0$
$a = 9$, $b = 6$, and $c = 1$
$$b^2 - 4ac = 6^2 - 4(9)(1) = 36 - 36 = 0$$
The equation has one rational solution (a double solution) because the discriminant is equal to zero.

54. $25x^2 + 110x + 121 = 0$
$a = 25$, $b = 110$, and $c = 121$
$$b^2 - 4ac = 110^2 - 4(25)(121) = 12{,}100 - 12{,}100 = 0$$
The equation has one rational solution (a double solution) because the discriminant is equal to zero.

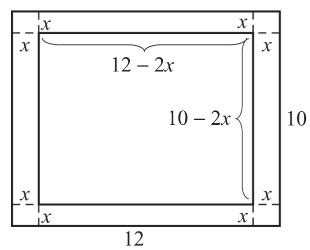
55. The projectile will be 750 ft above the ground whenever $-16t^2 + 220t = 750$.
Solve this equation for t.
$$-16t^2 + 220t = 750$$
$$-16t^2 + 220t - 750 = 0$$
$$16t^2 - 220t + 750 = 0$$
$$8t^2 - 110t + 375 = 0$$
$$(4t - 25)(2t - 15) = 0$$
$$4t - 25 = 0 \quad \text{or} \quad 2t - 15 = 0$$
$$t = \frac{25}{4} = 6.25 \quad \text{or} \quad t = \frac{15}{2} = 7.5$$
The projectile will be 750 ft high at 6.25 sec and at 7.5 sec.

56. Let x = width of the frame.
Then $x + 3$ = length of the frame.
Set up an equation that represents the area of the unframed picture.
$$x(x-3) = 70$$
$$x^2 - 3x = 70$$
$$x^2 - 3x - 70 = 0$$
$$(x+7)(x-10) = 0$$
$$x + 7 = 0 \Rightarrow x = -7 \quad \text{or} \quad x - 10 = 0 \Rightarrow x = 10$$
We disregard the negative solution.



Because x represents the width of the frame, the frame is 10 in wide and 10 + 3 = 13 in. long.

57. Let x = width of border.
Apply the formula $A = LW$ to both the outside and inside rectangles.
Inside area= Outside area – Border area
$$(12 - 2x)(10 - 2x) = 12 \cdot 10 - 21$$
$$120 - 24x - 20x + 4x^2 = 120 - 21$$
$$120 - 44x + 4x^2 = 99$$
$$4x^2 - 44x + 120 = 99$$
$$4x^2 - 44x + 21 = 0$$
$$(2x - 21)(2x - 1) = 0$$
$$2x = 21 \Rightarrow x = \frac{21}{2} = 10\tfrac{1}{2} \quad \text{or}$$
$$2x - 1 = 0 \Rightarrow x = \tfrac{1}{2}$$



The border width cannot be $10\frac{1}{2}$ because this exceeds the width of the outside rectangle, so reject this solution. The width of the border is $\frac{1}{2}$ ft.

58.
$$D = 0.1s^2 - 3s + 22$$
$$800 = 0.1s^2 - 3s + 22$$
$$0 = 0.1s^2 - 3s - 778$$
$$10 \cdot 0 = 10\left(0.1s^2 - 3s - 778\right)$$
$$0 = s^2 - 30s - 7780$$
Solve using the quadratic formula.
Let $a = 1$, $b = -30$, and $c = -7780$.
$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(-7780)}}{2(1)} = \frac{30 \pm \sqrt{900 + 31{,}120}}{2} = \frac{30 \pm \sqrt{32{,}020}}{2}$$

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$$s = \frac{30 - \sqrt{32,020}}{2} \approx -74.5 \text{ or}$$
$$s = \frac{30 + \sqrt{32,020}}{2} \approx 104.5$$

We disregard the negative solution. The appropriate landing speed would be approximately 104.5 ft per sec.

59. In 2009, $x = 19$.

$$y = 1.016x^2 + 12.49x + 197.8$$
$$y = 1.016(19)^2 + 12.49(19) + 197.8 \approx 801.9$$

According to the model, in 2009 the U.S. government spent approximately \$801.9 billion on medical care.

60. Let x = the length of the middle side. Then $x - 7$ = the length of the shorter side and $x + 1$ = the length of the hypotenuse. Use the Pythagorean theorem.

$$x^2 + (x-7)^2 = (x+1)^2$$
$$x^2 + x^2 - 14x + 49 = x^2 + 2x + 1$$
$$x^2 - 16x + 48 = 0$$
$$(x-12)(x-4) = 0 \Rightarrow x = 12 \text{ or } x = 4$$

If $x = 12$, then $x - 7 = 5$ and $x + 1 = 13$.
If $x = 4$, then $x - 7 = -3$, which is not possible.
The sides are 5 inches, 12 inches, and 13 inches long.

61. $4x^4 + 3x^2 - 1 = 0$

Let $u = x^2$; then $u^2 = x^4$.
With this substitution, the equation becomes $4u^2 + 3u - 1 = 0$.
Solve this equation by factoring.

$$(u+1)(4u-1) = 0$$
$$u + 1 \Rightarrow u = -1 \text{ or } 4u - 1 = 0 \Rightarrow u = \tfrac{1}{4}$$

To find x, replace u with x^2.

$$x^2 = -1 \Rightarrow x = \pm\sqrt{-1} \Rightarrow x = \pm i \text{ or}$$
$$x^2 = \tfrac{1}{4} \Rightarrow x = \pm\sqrt{\tfrac{1}{4}} \Rightarrow x = \pm\tfrac{1}{2}$$

Solution set: $\left\{\pm i, \pm\frac{1}{2}\right\}$

62. $x^2 - 2x^4 = 0 \Rightarrow x^2\left(1 - 2x^2\right) = 0$

$$x^2 = 0 \Rightarrow x = \pm\sqrt{0} \Rightarrow x = 0 \text{ or}$$
$$1 - 2x^2 = 0 \Rightarrow 1 = 2x^2$$
$$\tfrac{1}{2} = x^2 \Rightarrow \pm\sqrt{\tfrac{1}{2}} = x$$
$$x = \pm\tfrac{1}{\sqrt{2}} \cdot \tfrac{\sqrt{2}}{\sqrt{2}} \Rightarrow x = \pm\tfrac{\sqrt{2}}{2}$$

Solution set: $\left\{0, \pm\frac{\sqrt{2}}{2}\right\}$

63. $\frac{2}{x} - \frac{4}{3x} = 8 + \frac{3}{x}$

$$3x\left(\frac{2}{x} - \frac{4}{3x}\right) = 3x\left(8 + \frac{3}{x}\right)$$
$$6 - 4 = 24x + 9$$
$$2 = 24x + 9$$
$$-7 = 24x \Rightarrow -\tfrac{7}{24} = x$$

Solution set: $\left\{-\frac{7}{24}\right\}$

64. $2 - \frac{5}{x} = \frac{3}{x^2}$

$$x^2\left(2 - \frac{5}{x}\right) = x^2\left(\frac{3}{x^2}\right)$$
$$2x^2 - 5x = 3$$
$$2x^2 - 5x - 3 = 0$$
$$(2x+1)(x-3) = 0$$
$$2x + 1 = 0 \Rightarrow x = -\tfrac{1}{2} \text{ or } x - 3 = 0 \Rightarrow x = 3$$

Solution set: $\left\{-\frac{1}{2}, 3\right\}$

65. $\frac{10}{4x-4} = \frac{1}{1-x} \Rightarrow \frac{10}{4(x-1)} = \frac{1}{1-x} \Rightarrow$

$$\frac{10}{4(x-1)} = \frac{(-1)\cdot 1}{(-1)(1-x)} \Rightarrow \frac{10}{4(x-1)} = \frac{-1}{x-1}$$

Multiply each term in the equation by the least common denominator, $4(x-1)$, assuming $x \neq 1$.

$$4(x-1)\left[\frac{10}{4(x-1)}\right] = 4(x-1)\left(\frac{-1}{x-1}\right)$$
$$10 = -4 \Rightarrow 14 = 0$$

This is a false statement, the solution set is $\varnothing$.
Alternate solution:

$$\frac{10}{4x-4} = \frac{1}{1-x} \text{ or } \frac{10}{4(x-1)} = \frac{1}{1-x}$$

Multiply each term in the equation by the least common denominator, $4(x-1)(1-x)$, assuming $x \neq 1$.

$$4(x-1)(1-x)\left[\frac{10}{4(x-1)}\right]$$
$$= 4(x-1)(1-x)\left(\frac{1}{1-x}\right)$$
$$10(1-x) = 4(x-1)$$
$$10 - 10x = 4x - 4$$
$$10 = 14x - 4$$
$$14 = 14x$$
$$1 = x$$

Because of the restriction $x \neq 1$, the solution set is $\varnothing$.

66. $\dfrac{13}{x^2+10}=\dfrac{2}{x}$

Multiply both sides by the least common denominator, $x\left(x^2+10\right)$, assuming $x \neq 0$.

$$x(x^2+10)\left(\frac{13}{x^2+10}\right)=x\left(x^2+10\right)\left(\frac{2}{x}\right)$$
$$13x=2\left(x^2+10\right)$$
$$13x=2x^2+20$$
$$0=2x^2-13x+20$$
$$0=\left(2x-5\right)\left(x-4\right)$$

$2x-5=0 \Rightarrow x=\frac{5}{2}$ or $x-4=0 \Rightarrow x=4$

The restriction $x \neq 0$ does not affect the result. Therefore, the solution set is $\left\{\frac{5}{2}, 4\right\}$.

67. $\left(x-4\right)^{2/5}=9$

$$\left[\left(x-4\right)^{2/5}\right]^{5/2}=\pm 9^{5/2}=\pm\left(9^{1/2}\right)^5$$
$$x-4=\left(\pm 3\right)^5$$

$x-4=\left(-3\right)^5$ or $x-4=3^5$

$x-4=-243$ $x-4=243$

$x=-239$ $x=247$

Solution set: {–239, 247}

68. $\left(x^2-6x\right)^{1/4}=2$

$$\left[\left(x^2-6x\right)^{1/4}\right]^4=2^4=16$$
$$x^2-6x=16$$
$$x^2-6x-16=0$$
$$\left(x+2\right)\left(x-8\right)=0 \Rightarrow x=-2,\ x=8$$

Solution set: {–2, 8}

69. $(x-2)^{2/3}=x^{1/3}$

$$\left[(x-2)^{2/3}\right]^3=\left(x^{1/3}\right)^3$$
$$(x-2)^2=x$$
$$x^2-4x+4=x$$
$$x^2-5x+4=0$$
$$(x-4)(x-1)=0$$

$x-4=0 \Rightarrow x=4$ or $x-1=0 \Rightarrow x=1$

Check $x=1$.

$$(x-2)^{2/3}=x^{1/3} \Rightarrow$$
$$(1-2)^{2/3}=1^{1/3} \Rightarrow (-1)^{2/3}=1$$
$$\left[\left(-1\right)^{1/3}\right]^2=1 \Rightarrow (-1)^2=1 \Rightarrow 1=1$$

This is a true statement. 1 is a solution.

Check $x=4$.

$$(x-2)^{2/3}=x^{1/3} \Rightarrow$$
$$(4-2)^{2/3}=4^{1/3} \Rightarrow (2)^{2/3}=4^{1/3}$$
$$\left[2^2\right]^{1/3}=4^{1/3} \Rightarrow 4^{1/3}=4^{1/3}$$

This is a true statement. 4 is a solution.

Solution set: {1, 4}

70. $\sqrt{2x+3}=x+2$

$$\left(\sqrt{2x+3}\right)^2=\left(x+2\right)^2$$
$$2x+3=x^2+4x+4$$
$$0=x^2+2x+1$$
$$0=\left(x+1\right)^2$$

$x+1=0 \Rightarrow x=-1$

Check $x=-1$.

$$\sqrt{2x+3}=x+2$$
$$\sqrt{2(-1)+3}=-1+2$$
$$\sqrt{-2+3}=1$$
$$\sqrt{1}=1$$
$$1=1$$

This is a true statement.

Solution set: $\{-1\}$

71. $\sqrt{x+2}-x=2 \Rightarrow \sqrt{x+2}=2+x$

$$\left(\sqrt{x+2}\right)^2=\left(2+x\right)^2 \Rightarrow x+2=4+4x+x^2$$
$$0=x^2+3x+2 \Rightarrow 0=\left(x+2\right)\left(x+1\right)$$

$x+2=0 \Rightarrow x=-2$ or $x+1=0 \Rightarrow x=-1$

Check $x=-2$.

$$\sqrt{x+2}=2+x$$
$$\sqrt{-2+2}=2+\left(-2\right)$$
$$\sqrt{0}=0 \Rightarrow 0=0$$

This is a true statement. –2 is a solution.

Check $x=-1$.

$$\sqrt{x+2}=2+x$$
$$\sqrt{-1+2}=2+\left(-1\right)\,?$$
$$\sqrt{1}=1 \Rightarrow 1=1$$

This is a true statement. –1 is a solution.

Solution set: {–2, –1}

72. $\sqrt{x}-\sqrt{x+3}=-1 \Rightarrow \sqrt{x}=\sqrt{x+3}-1$

$$\left(\sqrt{x}\right)^2=\left(\sqrt{x+3}-1\right)^2$$
$$x=(x+3)-2\sqrt{x+3}+1$$
$$x=x+4-2\sqrt{x+3}$$
$$0=4-2\sqrt{x+3}$$
$$2\sqrt{x+3}=4 \Rightarrow \sqrt{x+3}=2$$
$$\left(\sqrt{x+3}\right)=2^2 \Rightarrow x+3=4 \Rightarrow x=1$$

Check $x=1$.

$$\sqrt{x}-\sqrt{x+3}=-1$$
$$\sqrt{1}-\sqrt{1+3}=-1\ ?$$
$$1-\sqrt{4}=-1$$
$$1-2=-1$$
$$-1=-1$$

This is a true statement.
Solution set: $\{1\}$

73. $\sqrt{4x-2}=\sqrt{3x+1}$

$$\left(\sqrt{4x-2}\right)^2=\left(\sqrt{3x+1}\right)^2$$
$$4x-2=3x+1$$
$$x-2=1$$
$$x=3$$

Check $x=3$.

$$\sqrt{4x-2}=\sqrt{3x+1}$$
$$\sqrt{4(3)-2}=\sqrt{3(3)+1}$$
$$\sqrt{12-2}=\sqrt{9+1}$$
$$\sqrt{10}=\sqrt{10}$$

This is a true statement.
Solution set: $\{3\}$

74. $\sqrt{5x-15}-\sqrt{x+1}=2$

$$\sqrt{5x-15}=\sqrt{x+1}+2$$
$$\left(\sqrt{5x-15}\right)^2=\left(\sqrt{x+1}+2\right)^2$$
$$5x-15=(x+1)+4\sqrt{x+1}+4$$
$$5x-15=x+5+4\sqrt{x+1}$$
$$4x-20=4\sqrt{x+1}$$
$$x-5=\sqrt{x+1}$$
$$(x-5)^2=\left(\sqrt{x+1}\right)^2$$
$$x^2-10x+25=x+1$$
$$x^2-11x+24=0 \Rightarrow (x-3)(x-8)=0$$
$$x-3=0 \Rightarrow x=3 \quad \text{or} \quad x-8=0 \Rightarrow x=8$$

Check $x=3$.

$$\sqrt{5x-15}-\sqrt{x+1}=2$$
$$\sqrt{5(3)-15}-\sqrt{3+1}=2$$
$$\sqrt{15-15}-\sqrt{4}=2$$
$$\sqrt{0}-2=2$$
$$0-2=2$$
$$-2=2$$

This is a false statement. 3 is not a solution.
Check $x=8$.

$$\sqrt{5x-15}-\sqrt{x+1}=2$$
$$\sqrt{5(8)-15}-\sqrt{8+1}=2$$
$$\sqrt{40-15}-\sqrt{9}=2$$
$$\sqrt{25}-3=2$$
$$5-3=2$$
$$2=2$$

This is a true statement. 8 is a solution.
Solution set: $\{8\}$

75. $\sqrt{x+3}-\sqrt{3x+10}=1$

$$\sqrt{x+3}=1+\sqrt{3x+10}$$
$$\left(\sqrt{x+3}\right)^2=\left(1+\sqrt{3x+10}\right)^2$$
$$x+3=1+2\sqrt{3x+10}+(3x+10)$$
$$x+3=3x+11+2\sqrt{3x+10}$$
$$-2x-8=2\sqrt{3x+10}$$
$$x+4=-\sqrt{3x+10}$$
$$(x+4)^2=\left(-\sqrt{3x+10}\right)^2$$
$$x^2+8x+16=3x+10$$
$$x^2+5x+6=0 \Rightarrow (x+2)(x+3)=0$$
$$x+3=0 \Rightarrow x=-3 \quad \text{or} \quad x+2=0 \Rightarrow x=-2$$

Check $x=-3$.

$$\sqrt{x+3}-\sqrt{3x+10}=1$$
$$\sqrt{-3+3}-\sqrt{3(-3)+10}=1$$
$$\sqrt{0}-\sqrt{-9+10}=1$$
$$0-\sqrt{1}=1$$
$$0-1=1 \Rightarrow -1=1$$

This is a false statement. –3 is not a solution.
Check $x=-2$.

$$\sqrt{x+3}-\sqrt{3x+10}=1$$
$$\sqrt{-2+3}-\sqrt{3(-2)+10}=1$$
$$\sqrt{1}-\sqrt{-6+10}=1$$
$$1-\sqrt{4}=1$$
$$1-2=1 \Rightarrow -1=1$$

This is a false statement. –2 is not a solution.
Because neither of the proposed solutions satisfies the original equation, the equation has no solution. Solution set: $\varnothing$

76. $\sqrt[5]{2x} = \sqrt[5]{3x+2}$

$\left(\sqrt[5]{2x}\right)^5 = \left(\sqrt[5]{3x+2}\right)^5$

$2x = 3x+2 \Rightarrow -x = 2 \Rightarrow x = -2$

Check $x = -2$.

$$\begin{aligned}\sqrt[5]{2x} &= \sqrt[5]{3x+2}\\ \sqrt[5]{2(-2)} &= \sqrt[5]{3(-2)+2}\\ \sqrt[5]{-4} &= \sqrt[5]{-6+2}\\ -\sqrt[5]{4} &= \sqrt[5]{-4}\\ -\sqrt[5]{4} &= -\sqrt[5]{4}\end{aligned}$$

This is a true statement.
Solution set: $\{-2\}$

77. $\sqrt[3]{6x+2} - \sqrt[3]{4x} = 0$

$\sqrt[3]{6x+2} = \sqrt[3]{4x}$

$\left(\sqrt[3]{6x+2}\right)^3 = \left(\sqrt[3]{4x}\right)^3$

$6x+2 = 4x \Rightarrow 2 = -2x \Rightarrow -1 = x$

Check $x = -1$.

$$\begin{aligned}\sqrt[3]{6x+2} - \sqrt[3]{4x} &= 0\\ \sqrt[3]{6(-1)+2} - \sqrt[3]{4(-1)} &= 0\\ \sqrt[3]{-6+2} - \sqrt[3]{-4} &= 0\\ \sqrt[3]{-4} - \left(-\sqrt[3]{4}\right) &= 0\\ -\sqrt[3]{4} + \sqrt[3]{4} = 0 &\Rightarrow 0 = 0\end{aligned}$$

This is a true statement.
Solution set: $\{-1\}$

78. $\sqrt{x^2+3x} - 2 = 0$

$\sqrt{x^2+3x} = 2 \Rightarrow \left(\sqrt{x^2+3x}\right)^2 = 2^2$

$x^2+3x = 4 \Rightarrow x^2+3x-4 = 0$

$(x-1)(x+4) = 0$

$x-1 = 0 \Rightarrow x = 1$ or $x+4 = 0 \Rightarrow x = -4$

Check $x = -4$.

$$\begin{aligned}\sqrt{x^2+3x} - 2 &= 0\\ \sqrt{(-4)^2+3(-4)} - 2 &= 0\\ \sqrt{16+(-12)} - 2 &= 0\\ \sqrt{4} - 2 &= 0\\ 2-2 &= 0\\ 0 &= 0\end{aligned}$$

This is a true statement. −4 is a solution.

Check $x = 1$.

$$\begin{aligned}\sqrt{x^2+3x} - 2 &= 0\\ \sqrt{1^2+3(1)} - 2 &= 0\\ \sqrt{1+3} - 2 &= 0\\ \sqrt{4} - 2 &= 0\\ 2-2 &= 0\\ 0 &= 0\end{aligned}$$

This is a true statement. 1 is a solution.
Solution set: $\{-4, 1\}$

79. $\dfrac{x}{x+2} + \dfrac{1}{x} + 3 = \dfrac{2}{x^2+2x} \Rightarrow$

$\dfrac{x}{x+2} + \dfrac{1}{x} + 3 = \dfrac{2}{x(x+2)}$

Multiply each term in the equation by the least common denominator, $x(x+2)$, assuming $x \neq 0, -2$.

$$x(x+2)\left[\frac{x}{x+2} + \frac{1}{x} + 3\right] = x(x+2)\left(\frac{2}{x(x+2)}\right)$$

$$\begin{aligned}x^2 + (x+2) + 3x(x+2) &= 2\\ x^2 + x + 2 + 3x^2 + 6x &= 2\\ 4x^2 + 7x + 2 &= 2\\ 4x^2 + 7x = 0 &\Rightarrow x(4x+7) = 0\end{aligned}$$

$x = 0$ or $4x+7 = 0 \Rightarrow x = -\frac{7}{4}$

Because of the restriction $x \neq 0$, the only valid solution is $-\frac{7}{4}$. The solution set is $\left\{-\frac{7}{4}\right\}$.

80. $\dfrac{2}{x+2} + \dfrac{1}{x+4} = \dfrac{4}{x^2+6x+8} \Rightarrow$

$\dfrac{2}{x+2} + \dfrac{1}{x+4} = \dfrac{4}{(x+4)(x+2)}$

The least common denominator is $(x+4)(x+2)$, which is equal to 0 if $x = -4$ or $x = -2$. Therefore, −4 and −2 cannot possibly be solutions of this equation.

$$(x+4)(x+2)\left[\frac{2}{x+2} + \frac{1}{x+4}\right] = (x+4)(x+2)\left(\frac{4}{(x+4)(x+2)}\right)$$

$2(x+4) + (x+2) = 4$

$2x+8+x+2 = 4 \Rightarrow 3x = -6 \Rightarrow x = -2$

The only possible solution is –2. However, the variable is restricted to real numbers except –4 and –2. Therefore, the solution set is: $\varnothing$.

81. $(2x+3)^{2/3}+(2x+3)^{1/3}-6=0$

Let $u=(2x+3)^{1/3}$. Then

$u^2=[(2x+3)^{1/3}]^2=(2x+3)^{2/3}$.

With this substitution, the equation becomes

$u^2+u-6=0.$ Solve by factoring.

$(u+3)(u-2)=0$

$u+3=0\Rightarrow u=-3$ or $u-2=0\Rightarrow u=2$

To find *x*, replace *u* with $(2x+3)^{1/3}$.

$(2x+3)^{1/3}=-3\Rightarrow\left[(2x+3)^{1/3}\right]^3=(-3)^3\Rightarrow$

$2x+3=-27\Rightarrow 2x=-30\Rightarrow x=-15$ or

$(2x+3)^{1/3}=2\Rightarrow\left[(2x+3)^{1/3}\right]^3=2^3\Rightarrow$

$2x+3=8\Rightarrow 2x=5\Rightarrow x=\frac{5}{2}$

Check $x=-15$.

$$\begin{aligned}(2x+3)^{2/3}+(2x+3)^{1/3}&=6\\ \left[2(-15)+3\right]^{2/3}+\left[2(-15)+3\right]^{1/3}&=6\\ (-30+3)^{2/3}+(-30+3)^{1/3}&=6\\ (-27)^{2/3}+(-27)^{1/3}&=6\\ \left[(-27)^{1/3}\right]^2+(-3)&=6\\ (-3)^2-3&=6\\ 9-3=6&\Rightarrow 6=6\end{aligned}$$

This is a true statement. –15 is a solution.

Check $x=\frac{5}{2}$.

$$\begin{aligned}(2x+3)^{2/3}+(2x+3)^{1/3}&=6\\ \left[2\left(\tfrac{5}{2}\right)+3\right]^{2/3}+\left[2\left(\tfrac{5}{2}\right)+3\right]^{1/3}&=6\\ (5+3)^{2/3}+(5+3)^{1/3}&=6\\ 8^{2/3}+8^{1/3}&=6\\ \left[8^{1/3}\right]^2+2&=6\\ (2)^2+2&=6\\ 4+2=6&\Rightarrow 6=6\end{aligned}$$

This is a true statement. $\frac{5}{2}$ is a solution.

Solution set: $\left\{-15,\frac{5}{2}\right\}$

82. $(x+3)^{-2/3}-2(x+3)^{-1/3}=3\Rightarrow$

$(x+3)^{-2/3}-2(x+3)^{-1/3}-3=0$

Let $u=(x+3)^{-1/3}$; then

$u^2=\left[(x+3)^{-1/3}\right]^2=(x+3)^{-2/3}$.

$u^2-2u-3=0\Rightarrow(u+1)(u-3)=0$

$u=-1$ or $u=3$

To find *x*, replace *u* with $(x+3)^{-1/3}$.

$(x+3)^{-1/3}=-1\Rightarrow\left[(x+3)^{-1/3}\right]^{-3}=(-1)^{-3}\Rightarrow$

$x+3=\frac{1}{(-1)^3}\Rightarrow x+3=\frac{1}{-1}=-1\Rightarrow x=-4$

or

$(x+3)^{-1/3}=3\Rightarrow\left[(x+3)^{-1/3}\right]^{-3}=3^{-3}\Rightarrow$

$x+3=\frac{1}{3^3}\Rightarrow x+3=\frac{1}{27}\Rightarrow$

$x=-3+\frac{1}{27}=-\frac{81}{27}+\frac{1}{27}=-\frac{80}{27}$

Check $x=-4$.

$$\begin{aligned}(x+3)^{-2/3}-2(x+3)^{-1/3}-3&=0\\ (-4+3)^{-2/3}-2(-4+3)^{-1/3}-3&=0\\ (-1)^{-2/3}-2(-1)^{-1/3}-3&=0\\ (-1)^{2/3}-2(-1)^{1/3}-3&=0\\ \left[(-1)^{1/3}\right]^2-2(-1)^{1/3}-3&=0\\ (-1)^2-2(-1)-3&=0\\ 1+2-3&=0\\ 0&=0\end{aligned}$$

This is a true statement. –4 is a solution.

Check $x=-\frac{80}{27}$.

$$\begin{aligned}(x+3)^{-2/3}-2(x+3)^{-1/3}-3&=0\\ \left(-\tfrac{80}{27}+3\right)^{-2/3}-2\left(-\tfrac{80}{27}+3\right)^{-1/3}-3&=0\ ?\\ \left(-\tfrac{80}{27}+\tfrac{81}{27}\right)^{-2/3}-2\left(-\tfrac{80}{27}+\tfrac{81}{27}\right)^{-1/3}-3&=0\\ \left(\tfrac{1}{27}\right)^{-2/3}-2\left(\tfrac{1}{27}\right)^{-1/3}-3&=0\\ (27)^{2/3}-2(27)^{1/3}-3&=0\\ \left[27^{1/3}\right]^2-2(27)^{1/3}-3&=0\\ 3^2-2(3)-3&=0\\ 9-6-3&=0\\ 0&=0\end{aligned}$$

This is a true statement. $-\frac{80}{27}$ is a solution.

Solution set: $\left\{-4,-\frac{80}{27}\right\}$

83. $-9x+3<4x+10$
$-13x<7$
$x>-\frac{7}{13}$
Solution set: $\left(-\frac{7}{13},\infty\right)$

84. $11x\ge 2(x-4)$
$11x\ge 2x-8$
$9x\ge -8$
$x\ge -\frac{8}{9}$
Solution set: $\left[-\frac{8}{9},\infty\right)$

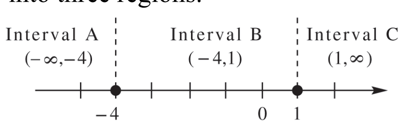
85. $-5x-4\ge 3(2x-5)$
$-5x-4\ge 6x-15$
$-11x-4\ge -15$
$-11x\ge -11$
$x\le 1$
Solution set: $(-\infty,1]$

86. $7x-2(x-3)\le 5(2-x)$
$7x-2x+6\le 10-5x$
$5x+6\le 10-5x$
$10x+6\le 10$
$10x\le 4$
$x\le \frac{4}{10}\Rightarrow x\le \frac{2}{5}$
Solution set: $\left(-\infty,\frac{2}{5}\right]$

87. $5\le 2x-3\le 7$
$8\le 2x\le 10$
$4\le x\le 5$
Solution set: [4, 5]

88. $-8>3x-5>-12$
$-3>3x>-7$
$-1>x>-\frac{7}{3}$
$-\frac{7}{3}<x<-1$
Solution set: $\left(-\frac{7}{3},-1\right)$

89. $x^2+3x-4\le 0$
Step 1: Find the values of x that satisfy $x^2+3x-4=0$.
$x^2+3x-4=0$
$(x+4)(x-1)=0$
$x+4=0\Rightarrow x=-4$ or $x-1=0\Rightarrow x=1$
Step 2: The two numbers divide a number line into three regions.

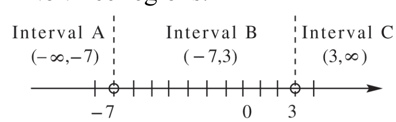


Step 3: Choose a test value to see if it satisfies the inequality, $x^2+3x-4\le 0$.

Interval	Test Value	Is $x^2+3x-4\le 0$ True or False?
A: $(-\infty,-4)$	-5	$(-5)^2+3(-5)-4\stackrel{?}{\le}0$; $6\le 0$; False
B: $(-4,1)$	0	$0^2+3(0)-4\stackrel{?}{\le}0$; $-4\le 0$; True
C: $(1,\infty)$	2	$2^2+3(2)-4\stackrel{?}{\le}0$; $6\le 0$; False

Solution set: $[-4,1]$

90. $x^2+4x-21>0$
Step 1: Find the values of x that satisfy $x^2+4x-21=0$.
$x^2+4x-21=0$
$(x+7)(x-3)=0$
$x+7=0\Rightarrow x=-7$ or $x-3=0\Rightarrow x=3$
Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2+4x-21>0$

Interval	Test Value	Is $x^2+4x-21>0$ True or False?
A: $(-\infty,-7)$	-8	$(-8)^2+4(-8)-21\stackrel{?}{>}0$; $11>0$; True
B: $(-7,3)$	0	$0^2+4(0)-21\stackrel{?}{>}0$; $-21>0$; False
C: $(3,\infty)$	4	$4^2+4(4)-21\stackrel{?}{>}0$; $11>0$; True

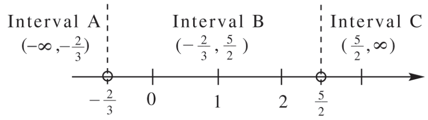
Solution set: $(-\infty,-7)\cup(3,\infty)$

91. $6x^2 - 11x < 10$

Step 1: Find the values of x that satisfy $6x^2 - 11x = 10$.

$$6x^2 - 11x = 10$$
$$6x^2 - 11x - 10 = 0$$
$$(3x+2)(2x-5) = 0$$
$$3x + 2 = 0 \Rightarrow x = -\tfrac{2}{3} \quad \text{or} \quad 2x - 5 = 0 \Rightarrow x = \tfrac{5}{2}$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $6x^2 - 11x < 10$

Interval	Test Value	Is $6x^2 - 11x < 10$ True or False?
A: $\left(-\infty, -\frac{2}{3}\right)$	-1	$6(-1)^2 - 11(-1) \overset{?}{<} 10$ $17 < 10$ False
B: $\left(-\frac{2}{3}, \frac{5}{2}\right)$	0	$6 \cdot 0^2 - 11 \cdot 0 - 10 \overset{?}{<} 10$ $-10 < 10$ True
C: $\left(\frac{5}{2}, \infty\right)$	3	$6 \cdot 3^2 - 11 \cdot 3 - 10 \overset{?}{<} 10$ $11 < 10$ False

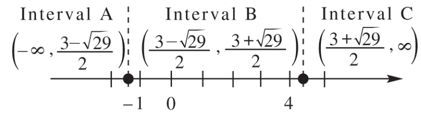
Solution set: $\left(-\frac{2}{3}, \frac{5}{2}\right)$

92. $x^2 - 3x \ge 5$

Step 1: Find the values of x that satisfy $x^2 - 3x = 5 \Rightarrow x^2 - 3x - 5 = 0$ Use the quadratic formula Let $a = 1$, $b = -3$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)} = \frac{3 \pm \sqrt{9+20}}{2} = \frac{3 \pm \sqrt{29}}{2}$$
$$x = \frac{3 - \sqrt{29}}{2} \approx -1.2 \text{ or } x = \frac{3 + \sqrt{29}}{2} \approx 4.2$$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $x^2 - 3x \ge 5$

Interval	Test Value	Is $x^2 - 3x \ge 5$ True or False?
A: $\left(-\infty, \frac{3-\sqrt{29}}{2}\right)$	-2	$(-2)^2 - 3(-2) \overset{?}{\ge} 5$ $10 \ge 5$ True
B: $\left(\frac{3-\sqrt{29}}{2}, \frac{3+\sqrt{29}}{2}\right)$	0	$0^2 - 3 \cdot 0 \overset{?}{\ge} 5$ $0 \ge 5$ False
C: $\left(\frac{3+\sqrt{29}}{2}, \infty\right)$	5	$5^2 - 3 \cdot 5 \overset{?}{\ge} 5$ $10 \ge 5$ True

Solution set: $\left(-\infty, \frac{3-\sqrt{29}}{2}\right] \cup \left[\frac{3+\sqrt{29}}{2}, \infty\right)$

93. $x^3 - 16x \le 0$

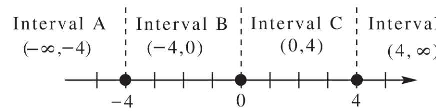
Step 1: Solve $x^3 - 16x = 0$.

$$x^3 - 16x = 0 \Rightarrow x\left(x^2 - 16\right) = 0 \Rightarrow$$
$$x(x+4)(x-4) = 0$$

Set each factor to zero and solve.

$x = 0$ or $x + 4 = 0 \Rightarrow x = -4$ or $x - 4 = 0 \Rightarrow x = 4$

Step 2: The values -4, 0, and, 4 divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $x^3 - 16x \le 0$.

Interval	Test Value	Is $x^3 - 16x \le 0$ True or False?
A: $(-\infty, -4)$	-5	$(-5)^3 - 16(-5) \overset{?}{\le} 0$ $-45 \le 0$ True
B: $(-4, 0)$	-1	$(-1)^3 - 16(-1) \overset{?}{\le} 0$ $15 \le 0$ False

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Interval	Test Value	Is $x^3 - 16x \le 0$ True or False?
C: $(0,4)$	1	$1^3 - 16 \cdot 1 \overset{?}{\le} 0$ $-15 \le 0$ True
D: $(4,\infty)$	5	$5^3 - 16 \cdot 5 \overset{?}{\le} 0$ $45 \le 0$ False

Solution set: $(-\infty,-4] \cup [0,4]$

94. $2x^3 - 3x^2 - 5x < 0$

Step 1: Solve $2x^3 - 3x^2 - 5x = 0$.

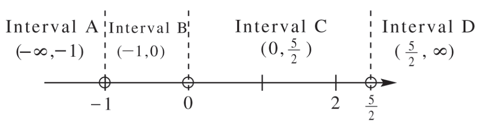
$2x^3 - 3x^2 - 5x = 0 \Rightarrow x(2x^2 - 3x - 5) = 0 \Rightarrow$
$x(x+1)(2x-5) = 0$

Set each factor to zero and solve.

$x = 0$ or $x + 1 = 0$ or $2x - 5 = 0$

$x = 0$ or $x = -1$ or $x = \frac{5}{2}$

Step 2: The values -1, 0, and, $\frac{5}{2}$ divide the number line into four intervals.



Step 3: Choose a test value to see if it satisfies the inequality, $2x^3 - 3x^2 - 5x < 0$.

Interval	Test Value	Is $2x^3 - 3x^2 - 5x < 0$ True or False?
A: $(-\infty,-1)$	-2	$2(-2)^3 - 3(-2)^2 - 5(-2) \overset{?}{<} 0$ $-18 < 0$ True
B: $(-1,0)$	-0.5	$2(-0.5)^3 - 3(-0.5)^2 - 5(-0.5) \overset{?}{<} 0$ $1.5 < 0$ False
C: $\left(0,\frac{5}{2}\right)$	1	$2 \cdot 1^3 - 3 \cdot 1^2 - 5 \cdot 1 \overset{?}{<} 0$ $-6 < 0$ True
D: $\left(\frac{5}{2},\infty\right)$	3	$2 \cdot 3^3 - 3 \cdot 3^2 - 5 \cdot 3 \overset{?}{<} 0$ $12 < 0$ False

Solution set: $(-\infty,-1) \cup \left(0,\frac{5}{2}\right)$

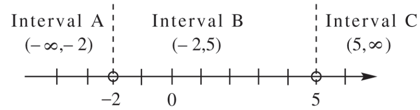
95. $\dfrac{3x+6}{x-5} > 0$

Because one side of the inequality is already 0, we start with Step 2.

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$3x + 6 = 0 \Rightarrow x = -2$ or $x - 5 = 0 \Rightarrow x = 5$

The values –2 and 5 to divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{3x+6}{x-5} > 0$.

Interval	Test Value	Is $\frac{3x+6}{x-5} > 0$ True or False?
A: $(-\infty,-2)$	-3	$\frac{3(-3)+6}{-3-5} \overset{?}{>} 0$ $\frac{3}{8} > 0$ True
B: $(-2,5)$	0	$\frac{3(0)+6}{0-5} \overset{?}{>} 0$ $-\frac{6}{5} > 0$ False
C: $(5,\infty)$	6	$\frac{3(6)+6}{6-5} \overset{?}{>} 0$ $24 > 0$ True

Solution set: $(-\infty,-2) \cup (5,\infty)$

96. $\dfrac{x+7}{2x+1} \le 1$

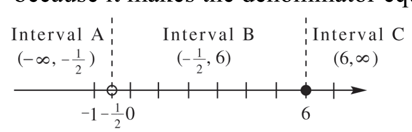
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{x+7}{2x+1} \le 1 \Rightarrow \frac{x+7}{2x+1} - 1 \le 0 \Rightarrow$$
$$\frac{x+7}{2x+1} - \frac{2x+1}{2x+1} \le 0 \Rightarrow \frac{x+7-(2x+1)}{2x+1} \le 0 \Rightarrow$$
$$\frac{x+7-2x-1}{2x+1} \le 0 \Rightarrow \frac{6-x}{2x+1} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$6 - x = 0 \Rightarrow x = 6$ or $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

The values $-\frac{1}{2}$ and 6 divide the number line into three regions. Use an open circle on $-\frac{1}{2}$ because it makes the denominator equal 0.



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Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{x+7}{2x+1} \le 1$

Interval	Test Value	Is $\frac{x+7}{2x+1} \le 1$ True or False?
A: $\left(-\infty, -\frac{1}{2}\right)$	-1	$\frac{-1+7}{2(-1)+1} \overset{?}{\le} 1$ $-6 \le 1$ True
B: $\left(-\frac{1}{2}, 6\right)$	0	$\frac{0+7}{2\cdot 0+1} \overset{?}{\le} 1$ $7 \le 1$ False
C: $(6, \infty)$	7	$\frac{7+7}{2\cdot 7+1} \overset{?}{\le} 1$ $\frac{14}{15} \le 1$ True

Intervals A and C satisfy the inequality. The endpoint $-\frac{1}{2}$ is not included because it makes the denominator 0.

Solution set: $\left(-\infty, -\frac{1}{2}\right) \cup [6, \infty)$

97. $\dfrac{3x-2}{x} - 4 > 0$

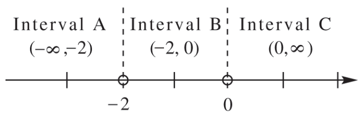
Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3x-2}{x} - 4 > 0 \Rightarrow \frac{3x-2}{x} - \frac{4x}{x} > 0 \Rightarrow$$
$$\frac{3x-2-4x}{x} > 0 \Rightarrow \frac{-x-2}{x} > 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-x-2 = 0 \Rightarrow x = -2$ or $x = 0$

The values –2 and 0 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{3x-2}{x} - 4 > 0$.

Interval	Test Value	Is $\frac{3x-2}{x} - 4 > 0$ True or False?	
A: $(-\infty, -2)$	-3	$\frac{3(-3)-2}{-3} - 4 \overset{?}{>} 0$ $-\frac{1}{3} > 0$ False	
B: $(-2, 0)$	-1	$\frac{3(-1)-2}{-1} - 4 \overset{?}{>} 0$ $1 > 0$ True	
C: $(0, \infty)$	1	$\frac{3\cdot 1-2}{1} - 4 \overset{?}{>} 0$ $-3 > 0$ False	

Solution set: (–2, 0)

98. $\dfrac{5x+2}{x} + 1 < 0$

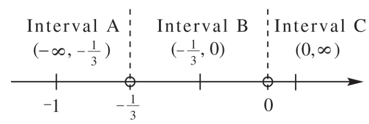
Step 1: Rewrite the inequality to compare a single fraction with 0.

$$\frac{5x+2}{x} + 1 < 0 \Rightarrow \frac{5x+2}{x} + \frac{x}{x} < 0 \Rightarrow$$
$$\frac{5x+2+x}{x} < 0 \Rightarrow \frac{6x+2}{x} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$6x + 2 = 0 \Rightarrow x = -\frac{1}{3}$ or $x = 0$

The values $-\frac{1}{3}$ and 0 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{5x+2}{x} + 1 < 0$.

Interval	Test Value	Is $\frac{5x+2}{x} + 1 < 0$ True or False?
A: $\left(-\infty, -\frac{1}{3}\right)$	-1	$\frac{5(-1)+2}{-1} + 1 \overset{?}{<} 0$ $4 < 0$ False
B: $\left(-\frac{1}{3}, 0\right)$	-0.1	$\frac{5(-0.1)+2}{-0.1} + 1 \overset{?}{<} 0$ $-14 < 0$ True
C: $(0, \infty)$	1	$\frac{5\cdot 1+2}{1} + 1 \overset{?}{<} 0$ $8 < 0$ False

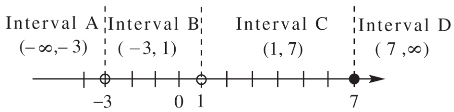
Solution set: $\left(-\frac{1}{3}, 0\right)$

99. $\dfrac{3}{x-1} \le \dfrac{5}{x+3}$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{3}{x-1} - \frac{5}{x+3} \le 0$$
$$\frac{3(x+3)}{(x-1)(x+3)} - \frac{5(x-1)}{(x+3)(x-1)} \le 0$$
$$\frac{3(x+3)-5(x-1)}{(x-1)(x+3)} \le 0$$
$$\frac{3x+9-5x+5}{(x-1)(x+3)} \le 0$$
$$\frac{-2x+14}{(x-1)(x+3)} \le 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.
$-2x+14=0 \Rightarrow x=7$ or $x-1=0 \Rightarrow x=1$ or
$x+3=0 \Rightarrow x=-3$
The values –3, 1 and 7 divide the number line into four regions. Use an open circle on -3 and 1 because they make the denominator equal 0.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{3}{x-1} \le \dfrac{5}{x+3}$.

Interval	Test Value	Is $\frac{3}{x-1} \le \frac{5}{x+3}$ True or False?
A: $(-\infty,-3)$	-4	$\frac{3}{-4-1} \overset{?}{\le} \frac{5}{-4+3}$ $-\frac{3}{5} \le -5$ False
B: $(-3,1)$	0	$\frac{3}{0-1} \overset{?}{\le} \frac{5}{0+3}$ $-3 \le \frac{5}{3}$ True
C: $(1,7)$	2	$\frac{3}{2-1} \overset{?}{\le} \frac{5}{2+3}$ $3 \le 1$ False
D: $(7,\infty)$	8	$\frac{3}{8-1} \overset{?}{\le} \frac{5}{8+3}$ $\frac{3}{7} \overset{?}{\le} \frac{5}{11}$ $\frac{33}{77} \le \frac{35}{77}$ True

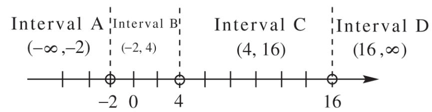
Intervals B and D satisfy the inequality. The endpoints -3 and 1 are not included because they make the denominator 0.
Solution set: $(-3,1) \cup [7,\infty)$

100. $\dfrac{3}{x+2} > \dfrac{2}{x-4}$

Step 1: Rewrite the inequality to compare a single fraction with 0.

$$\frac{3}{x+2} - \frac{2}{x-4} > 0$$
$$\frac{3(x-4)}{(x+2)(x-4)} - \frac{2(x+2)}{(x-4)(x+2)} > 0$$
$$\frac{3(x-4)-2(x+2)}{(x+2)(x-4)} > 0$$
$$\frac{3x-12-2x-4}{(x+2)(x-4)} > 0$$
$$\frac{x-16}{(x+2)(x-4)} > 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.
$x-16=0 \Rightarrow x=16$ or $x+2=0 \Rightarrow x=-2$ or
$x-4=0 \Rightarrow x=4$
The values –2, 4 and 16 divide the number line into four regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\dfrac{3}{x+2} > \dfrac{2}{x-4}$.

Interval	Test Value	Is $\frac{3}{x+2} > \frac{2}{x-4}$ True or False?
A: $(-\infty,-2)$	-3	$\frac{3}{-3+2} \overset{?}{>} \frac{2}{-3-4}$ $-3 > -\frac{2}{7}$ False
B: $(-2,4)$	0	$\frac{3}{0+2} \overset{?}{>} \frac{2}{0-4}$ $\frac{3}{2} > -\frac{1}{2}$ True
C: $(4,16)$	5	$\frac{3}{5+2} \overset{?}{>} \frac{2}{5-4}$ $\frac{3}{7} > 2$ False
D: $(16,\infty)$	17	$\frac{3}{17+2} \overset{?}{>} \frac{2}{17-4}$ $\frac{3}{19} \overset{?}{>} \frac{2}{13}$ $\frac{39}{247} > \frac{38}{247}$ True

Solution set: $(-2,4) \cup (16,\infty)$

101. **(a)** Let x = the ozone concentration after the Purafil air filter is used.
$x = 140 - 0.43(140) = 79.8$
The ozone concentration after the Purafil air filter is used is 79.8 ppb.

(b) Let x = the maximum initial concentration of ozone.

$x - 0.43x \le 50 \Rightarrow 0.57x \le 50$

$x \le 87.7$ (approximately)

The filter will reduce ozone concentrations that don't exceed 87.7 ppb.

102. $C = 3x + 1500$, $R = 8x$

The company will at least break even when $R \ge C$.

$8x \ge 3x + 1500 \Rightarrow 5x > 1500 \Rightarrow x \ge 300$

The break-even point is at $x = 300$. The company will at least break even if the number of units produced is in the interval $[300, \infty)$.

103. $s = -16t^2 + 320$

(a) When $s = 0$, the projectile will be at ground level.

$0 = -16t^2 + 320t \Rightarrow 16t^2 - 320t = 0 \Rightarrow$

$t^2 - 20t = 0 \Rightarrow t(t - 20) = 0 \Rightarrow$

$t = 0$ or $t = 20$

The projectile will return to the ground after 20 sec.

(b) Solve $s > 576$ for t.

$320t - 16t^2 > 576$

$0 > 16t^2 - 320t + 576$

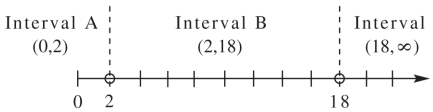
$0 > t^2 - 20t + 36$

Step 1: Find the values of t that satisfy $t^2 - 20t + 36 = 0$.

$t^2 - 20t + 36 = 0 \Rightarrow (t-2)(t-18) = 0 \Rightarrow$

$t - 2 = 0 \Rightarrow t = 2$ or $t - 18 = 0 \Rightarrow t = 18$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $320t - 16t^2 > 576$.

Interval	Test Value	Is $320t - 16t^2 > 576$ True or False?
A: $(0,2)$	1	$320(1) - 16(1)^2 \overset{?}{>} 576$ $304 > 576$ False
B: $(2,18)$	3	$320(3) - 16(3)^2 \overset{?}{>} 576$ $816 > 576$ True
C: $(18,\infty)$	20	$320(20) - 16(20)^2 \overset{?}{>} 576$ $0 > 576$ False

The projectile will be more than 576 ft above the ground between 2 and 18 sec.

104. $y = 35.7x + 486$

$35.7x + 486 > 800$

$35.7x > 314$

$x > 8.8$ (approximately)

Based on the model, the amount paid by the government first exceeds \$800 billion about 8.8 years after 2004, which is in 2012. This is very close to the graph.

105. Answers will vary. 3 cannot be in the solution set because when 3 is substituted into $\frac{14x+9}{x-3}$, division by zero occurs.

106. Answers will vary. −4 must be in the solution set because when −4 is substituted into $\frac{x+4}{2x+1}$, the result is zero, which makes the nonstrict inequality true.

107. $|x + 4| = 7$

$x + 4 = 7 \Rightarrow x = 3$ or $x + 4 = -7 \Rightarrow x = -11$

Solution set: $\{-11, 3\}$

108. $|2 - x| = 3 \Rightarrow 2 - x = \pm 3$

$2 - x = 3 \Rightarrow x = -1$ or $2 - x = -3 \Rightarrow x = 5$

Solution set: $\{-1, 5\}$

109. $\left|\frac{7}{2-3x}\right| - 9 = 0 \Rightarrow \left|\frac{7}{2-3x}\right| = 9$

$\frac{7}{2-3x} = 9 \Rightarrow 7 = 9(2-3x) \Rightarrow 7 = 18 - 27x \Rightarrow$

$-11 = -27x \Rightarrow \frac{-11}{-27} = x \Rightarrow x = \frac{11}{27}$ or

$\frac{7}{2-3x} = -9 \Rightarrow 7 = -9(2-3x) \Rightarrow$

$7 = -18 + 27x \Rightarrow 25 = 27x \Rightarrow \frac{25}{27} = x \Rightarrow x = \frac{25}{27}$

Solution set: $\left\{\frac{11}{27}, \frac{25}{27}\right\}$

110. $\left|\frac{8x-1}{3x+2}\right|-7=0$

$\left|\frac{8x-1}{3x+2}\right|-7=0 \Rightarrow \frac{8x-1}{3x+2}=7 \Rightarrow$
$8x-1=7(3x+2) \Rightarrow 8x-1=21x+14 \Rightarrow$
$-1=13x+14 \Rightarrow -15=13x \Rightarrow x=-\frac{15}{13}$ or

$\frac{8x-1}{3x+2}=-7 \Rightarrow 8x-1=-7(3x+2) \Rightarrow$
$8x-1=-21x-14 \Rightarrow 29x-1=-14 \Rightarrow$
$29x=-13 \Rightarrow x=-\frac{13}{29}$

Solution set: $\left\{-\frac{15}{13}, -\frac{13}{29}\right\}$

111. $|5x-1|=|2x+3|$

$5x-1=2x+3 \Rightarrow 3x-1=3 \Rightarrow 3x=4 \Rightarrow x=\frac{4}{3}$
or
$5x-1=-(2x+3) \Rightarrow 5x-1=-2x-3 \Rightarrow$
$7x-1=-3 \Rightarrow 7x=-2 \Rightarrow x=-\frac{2}{7}$

Solution set: $\left\{-\frac{2}{7}, \frac{4}{3}\right\}$

112. $|x+10|=|x-11|$

$x+10=x-11 \Rightarrow 10=-11$ False
or
$x+10=-(x-11) \Rightarrow x+10=-x+11 \Rightarrow$
$2x+10=11 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$

Solution set: $\left\{\frac{1}{2}\right\}$

113. $|2x+9| \le 3$

$-3 \le 2x+9 \le 3$
$-12 \le 2x \le -6$
$-6 \le x \le -3$
Solution set: $[-6,-3]$

114. $|8-5x| \ge 2$

$8-5x \ge 2 \Rightarrow -5x \ge -6 \Rightarrow x \le \frac{6}{5}$ or
$8-5x \le -2 \Rightarrow -5x \le -10 \Rightarrow x \ge 2$
Solution set: $\left(-\infty, \frac{6}{5}\right] \cup [2,\infty)$

115. $|7x-3| > 4$

$7x-3 < -4 \Rightarrow 7x < -1 \Rightarrow x < -\frac{1}{7}$ or
$7x-3 > 4 \Rightarrow 7x > 7 \Rightarrow x > 1$
Solution set: $\left(-\infty, -\frac{1}{7}\right) \cup (1,\infty)$

116. $\left|\frac{1}{2}x+\frac{2}{3}\right| < 3$

$-3 < \frac{1}{2}x+\frac{2}{3} < 3$
$6(-3) < 6\left(\frac{1}{2}x+\frac{2}{3}\right) < 6(3)$
$-18 < 3x+4 < 18$
$-22 < 3x < 14 \Rightarrow -\frac{22}{3} < x < \frac{14}{3}$

Solution set: $\left(-\frac{22}{3}, \frac{14}{3}\right)$

117. $|3x+7|-5 < 5 \Rightarrow |3x+7| < 10$

$-10 < 3x+7 < 10$
$-17 < 3x < 3$
$-\frac{17}{3} < x < 1$

Solution set: $\left(-\frac{17}{3}, 1\right)$

118. $|7x+8|-6 > -3 \Rightarrow |7x+8| > 3$

$7x+8 < -3 \Rightarrow 7x < -11 \Rightarrow x < -\frac{11}{7}$ or
$7x+8 > 3 \Rightarrow 7x > -5 \Rightarrow x > -\frac{5}{7}$

Solution set: $\left(-\infty, -\frac{11}{7}\right) \cup \left(-\frac{5}{7}, \infty\right)$

119. Because the absolute value of a number is always nonnegative, the inequality $|4x-12| \ge -3$ is always true. The solution set is $(-\infty,\infty)$.

120. There is no number whose absolute value is less than or equal to any negative number. The solution set of $|7-2x| \le -9$ is $\varnothing$.

121. Because the absolute value of a number is always nonnegative, $\left|x^2+4x\right| < 0$ is never true, so $\left|x^2+4x\right| \le 0$ is only true when $\left|x^2+4x\right| = 0$.

$\left|x^2+4x\right|=0 \Rightarrow x^2+4x=0 \Rightarrow x(x+4)=0$
$x=0$ or $x+4=0$
$x=0$ or $x=-4$

Solution set: $\{-4,0\}$

122. $\left|x^2+4x\right| > 0$ will be false only when $x^2+4x=0,$ which occurs when $x=-4$ or $x=0$ (see last exercise). So the solution set for $\left|x^2+4x\right| > 0$ is $(-\infty,-4) \cup (-4,0) \cup (0,\infty)$.

123. "k is 12 units from 6" means that the distance between k and 6 is 12 units, or $|k-6|=12$ or $|6-k|=12$.

124. "p is at least 3 units from 1" means that p is 3 units or more from 1. Thus, the distance between p and 1 is greater than or equal to 3, or $|p-1|\ge 3$ or $|1-p|\ge 3$.

125. "t is no less than 0.01 unit from 5" means that t is 0.01 unit or more from 5. Thus, the distance between t and 5 is greater than or equal to 0.01, or $|t-5|\ge 0.01$ or $|5-t|\ge 0.01$.

Chapter 1 Test

1.
$$\begin{aligned}3(x-4)-5(x+2)&=2-(x+24)\\3x-12-5x-10&=2-x-24\\-2x-22&=-x-22\\-22&=x-22\\0&=x\end{aligned}$$
Solution set: $\{0\}$

2.
$$\begin{aligned}\frac{2}{3}x+\frac{1}{2}(x-4)&=x-4\\6\left[\frac{2}{3}x+\frac{1}{2}(x-4)\right]&=6(x-4)\\4x+3(x-4)&=6x-24\\4x+3x-12&=6x-24\\7x-12&=6x-24\\x-12&=-24\\x&=-12\end{aligned}$$
Solution set: $\{-12\}$

3.
$$\begin{aligned}6x^2-11x-7&=0\\(2x+1)(3x-7)&=0\end{aligned}$$
$2x+1=0\Rightarrow x=-\frac{1}{2}$ or $3x-7=0\Rightarrow x=\frac{7}{3}$

Solution set: $\left\{-\frac{1}{2},\frac{7}{3}\right\}$

4.
$$\begin{aligned}(3x+1)^2&=8\\3x+1&=\pm\sqrt{8}=\pm 2\sqrt{2}\\3x&=-1\pm 2\sqrt{2}\Rightarrow x=\frac{-1\pm 2\sqrt{2}}{3}\end{aligned}$$
Solution set: $\left\{\frac{-1\pm 2\sqrt{2}}{3}\right\}$

5. $3x^2+2x=-2$

Solve by completing the square.
$$\begin{aligned}3x^2+2x&=-2\\3x^2+2x+2&=0\end{aligned}$$
$$x^2+\tfrac{2}{3}x+\tfrac{2}{3}=0\Rightarrow x^2+\tfrac{2}{3}x+\tfrac{1}{9}=-\tfrac{2}{3}+\tfrac{1}{9}$$

Note: $\left[\frac{1}{2}\cdot\frac{2}{3}\right]^2=\left(\frac{1}{3}\right)^2=\frac{1}{9}$

$$\left(x+\tfrac{1}{3}\right)^2=-\tfrac{5}{9}\Rightarrow x+\tfrac{1}{3}=\pm\sqrt{-\tfrac{5}{9}}\Rightarrow$$
$$x+\tfrac{1}{3}=\pm\tfrac{\sqrt{5}}{3}i\Rightarrow x=-\tfrac{1}{3}\pm\tfrac{\sqrt{5}}{3}i$$

Solve by the quadratic formula.
Let $a=3$, $b=2$, and $c=2$.
$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-2\pm\sqrt{2^2-4(3)(2)}}{2(3)}=\frac{-2\pm\sqrt{4-24}}{6}\\&=\frac{-2\pm\sqrt{-20}}{6}=\frac{-2\pm 2i\sqrt{5}}{6}\\&=-\tfrac{2}{6}\pm\tfrac{2\sqrt{5}}{6}i=-\tfrac{1}{3}\pm\tfrac{\sqrt{5}}{3}i\end{aligned}$$
Solution set: $\left\{-\frac{1}{3}\pm\frac{\sqrt{5}}{3}i\right\}$

6.
$$\begin{aligned}\frac{12}{x^2-9}&=\frac{2}{x-3}-\frac{3}{x+3}\\\frac{12}{(x+3)(x-3)}+\frac{3}{x+3}&=\frac{2}{x-3}\end{aligned}$$
Multiply each term in the equation by the least common denominator, $(x+3)(x-3)$ assuming $x\ne -3,3$.
$$\begin{aligned}(x+3)(x-3)\left[\frac{12}{(x+3)(x-3)}+\frac{3}{x+3}\right]&=(x+3)(x-3)\left(\frac{2}{x-3}\right)\\12+3(x-3)&=2(x+3)\\12+3x-9&=2x+6\\3x+3&=2x+6\\x+3&=6\Rightarrow x=3\end{aligned}$$
The only possible solution is 3. However, the variable is restricted to real numbers except –3 and 3. Therefore, the solution set is $\varnothing$.

7. $\dfrac{4x}{x-2}+\dfrac{3}{x}=\dfrac{-6}{x^2-2x}$ or $\dfrac{4x}{x-2}+\dfrac{3}{x}=\dfrac{-6}{x(x-2)}$

Multiply each term in the equation by the least common denominator, $x(x-2)$ assuming $x\ne 0,2$.
$$x(x-2)\left[\frac{4x}{x-2}+\frac{3}{x}\right]=x(x-2)\left(\frac{-6}{x(x-2)}\right)$$
$$4x^2+3(x-2)=-6\Rightarrow 4x^2+3x-6=-6$$
$$4x^2+3x=0\Rightarrow x(4x+3)=0$$
$x=0$ or $4x+3=0\Rightarrow x=-\frac{3}{4}$

Because of the restriction $x\ne 0$, the only valid solution is $-\frac{3}{4}$. The solution set is $\left\{-\frac{3}{4}\right\}$.

8. $\sqrt{3x+4}+5=2x+1 \Rightarrow \sqrt{3x+4}=2x-4$

$$\left(\sqrt{3x+4}\right)^2=(2x-4)^2$$
$$3x+4=4x^2-16x+16$$
$$0=4x^2-19x+12$$
$$0=(4x-3)(x-4)$$

$4x-3=0 \Rightarrow x=\frac{3}{4}$ or $x-4=0 \Rightarrow x=4$

Check $x=\frac{3}{4}$.

$$\sqrt{3x+4}+5=2x+1$$
$$\sqrt{3\left(\frac{3}{4}\right)+4}+5=2\left(\frac{3}{4}\right)+1$$
$$\sqrt{\frac{9}{4}+4}+5=\frac{5}{2} \Rightarrow \sqrt{\frac{25}{4}}+5=\frac{5}{2}$$
$$\frac{5}{2}+5=\frac{5}{2} \Rightarrow \frac{15}{2}=\frac{5}{2}$$

This is a false statement. $\frac{3}{4}$ is a not solution.

Check $x=4$.

$$\sqrt{3x+4}+5=2x+1$$
$$\sqrt{3(4)+4}+5=2(4)+1$$
$$\sqrt{12+4}+5=8+1$$
$$\sqrt{16}+5=8+1$$
$$4+5=9 \Rightarrow 9=9$$

This is a true statement. 4 is a solution.
Solution set: {4}

9. $\sqrt{-2x+3}+\sqrt{x+3}=3$

$$\sqrt{-2x+3}=3-\sqrt{x+3}$$
$$\left(\sqrt{-2x+3}\right)^2=\left(3-\sqrt{x+3}\right)^2$$
$$-2x+3=9-6\sqrt{x+3}+(x+3)$$
$$-2x+3=12+x-6\sqrt{x+3}$$
$$-3x-9=-6\sqrt{x+3}$$
$$x+3=2\sqrt{x+3}$$
$$(x+3)^2=\left(2\sqrt{x+3}\right)^2$$
$$x^2+6x+9=4(x+3)$$
$$x^2+6x+9=4x+12$$
$$x^2+2x-3=0 \Rightarrow (x+3)(x-1)=0$$

$x+3=0 \Rightarrow x=-3$ or $x-1=0 \Rightarrow x=1$

Check $x=-3$.

$$\sqrt{-2x+3}+\sqrt{x+3}=3$$
$$\sqrt{-2(-3)+3}+\sqrt{-3+3}=3$$
$$\sqrt{6+3}+\sqrt{0}=3$$
$$\sqrt{9}+0=3$$
$$3+0=3 \Rightarrow 3=3$$

This is a true statement. –3 is a solution.

Check $x=1$.

$$\sqrt{-2x+3}+\sqrt{x+3}=3$$
$$\sqrt{-2(1)+3}+\sqrt{1+3}=3$$
$$\sqrt{-2+3}+\sqrt{4}=3$$
$$\sqrt{1}+2=3$$
$$1+2=3 \Rightarrow 3=3$$

This is a true statement. 1 is a solution.
Solution set: {–3, 1}

10. $\sqrt[3]{3x-8}=\sqrt[3]{9x+4}$

$$\left(\sqrt[3]{3x-8}\right)^3=\left(\sqrt[3]{9x+4}\right)^3$$
$$3x-8=9x+4 \Rightarrow -8=6x+4 \Rightarrow$$
$$-12=6x \Rightarrow -2=x$$

Check $x=-2$.

$$\sqrt[3]{3x-8}=\sqrt[3]{9x+4}$$
$$\sqrt[3]{3(-2)-8}=\sqrt[3]{9(-2)+4}$$
$$\sqrt[3]{-6-8}=\sqrt[3]{-18+4}$$
$$\sqrt[3]{-14}=\sqrt[3]{-14} \Rightarrow -\sqrt[3]{14}=-\sqrt[3]{14}$$

This is a true statement.
Solution set: {–2}

11. $x^4-17x^2+16=0$

Let $u=x^2$; then $u^2=x^4$.
With this substitution, the equation becomes $u^2-17u+16=0$.
Solve this equation by factoring.

$(u-1)(u-16)=0$

$u-1=0 \Rightarrow u=1$ or $u-16=0 \Rightarrow u=16$

To find *x*, replace *u* with x^2.

$x^2=1 \Rightarrow x=\pm\sqrt{1} \Rightarrow x=\pm 1$ or

$x^2=16 \Rightarrow x=\pm\sqrt{16} \Rightarrow x=\pm 4$

Solution set: $\{\pm 1, \pm 4\}$

12. $(x+3)^{2/3}+(x+3)^{1/3}-6=0$

Let $u=(x+3)^{1/3}$. Then

$u^2=\left[(x+3)^{1/3}\right]^2=(x+3)^{2/3}$.

$u^2+u-6=0 \Rightarrow (u+3)(u-2)=0$

$u+3=0 \Rightarrow u=-3$ or $u-2=0 \Rightarrow u=2$

To find *x*, replace *u* with $(x+3)^{1/3}$.

$(x+3)^{1/3}=-3 \Rightarrow \left[(x+3)^{1/3}\right]^3=(-3)^3 \Rightarrow$

$x+3=-27 \Rightarrow x=-30$ or

$(x+3)^{1/3}=2 \Rightarrow \left[(x+3)^{1/3}\right]^3=2^3 \Rightarrow$

$x+3=8 \Rightarrow x=5$

(*continued on next page*)

(*continued*)

Check $x = -30$.

$$(x+3)^{2/3} + (x+3)^{1/3} - 6 = 0$$
$$(-30+3)^{2/3} + (-30+3)^{1/3} - 6 = 0$$
$$(-27)^{2/3} + (-27)^{1/3} - 6 = 0$$
$$\left[(-27)^{1/3}\right]^2 + (-3) - 6 = 0$$
$$(-3)^2 - 3 - 6 = 0$$
$$9 - 3 - 6 = 0 \Rightarrow 0 = 0$$

This is a true statement. –30 is a solution.

Check $x = 5$.

$$(x+3)^{2/3} + (x+3)^{1/3} - 6 = 0$$
$$(5+3)^{2/3} + (5+3)^{1/3} - 6 = 0$$
$$8^{2/3} + 8^{1/3} - 6 = 0$$
$$\left[8^{1/3}\right]^2 + 2 - 6 = 0$$
$$2^2 + 2 - 6 = 0$$
$$4 + 2 - 6 = 0 \Rightarrow 0 = 0$$

This is a true statement. 5 is a solution.

Solution set: $\{-30, 5\}$

13. $|4x+3| = 7$

$4x+3 = 7 \Rightarrow 4x = 4 \Rightarrow x = 1$ or

$4x+3 = -7 \Rightarrow 4x = -10 \Rightarrow x = -\frac{10}{4} = -\frac{5}{2}$

Solution set: $\left\{-\frac{5}{2}, 1\right\}$

14. $|2x+1| = |5-x|$

$2x+1 = 5-x \Rightarrow 3x+1 = 5 \Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3}$

or

$2x+1 = -(5-x) \Rightarrow 2x+1 = -5+x \Rightarrow x = -6$

Solution set: $\left\{-6, \frac{4}{3}\right\}$

15.

$$S = 2HW + 2LW + 2LH$$
$$S - 2LH = 2HW + 2LW$$
$$S - 2LH = W(2H + 2L)$$
$$\frac{S - 2LH}{2H + 2L} = W$$
$$W = \frac{S - 2LH}{2H + 2L}$$

16. **(a)** $(9-3i) - (4+5i) = (9-4) + (-3-5)i$
$= 5 - 8i$

(b) $(4+3i)(-5+3i) = -20 + 12i - 15i + 9i^2$
$= -20 - 3i + 9(-1)$
$= -20 - 3i - 9 = -29 - 3i$

(c) $(8+3i)^2 = 8^2 + 2(8)(3i) + (3i)^2$
$= 64 + 48i + 9i^2$
$= 64 + 48i + 9(-1)$
$= 64 + 48i - 9 = 55 + 48i$

(d) $\frac{3+19i}{1+3i} = \frac{(3+19i)(1-3i)}{(1+3i)(1-3i)}$
$= \frac{3 - 9i + 19i - 57i^2}{1 - (3i)^2}$
$= \frac{3 + 10i - 57(-1)}{1 - 9i^2} = \frac{3 + 10i + 57}{1 - 9(-1)}$
$= \frac{60 + 10i}{1 + 9} = \frac{60 + 10i}{10} = 6 + i$

17. **(a)** $i^{42} = i^{40} \cdot i^2 = \left(i^4\right)^{10} \cdot (-1) = 1^{10} \cdot (-1) = -1$

(b) $i^{-31} = i^{-32} \cdot i = \left(i^4\right)^{-8} \cdot i = 1^{-8} \cdot i = i$

(c) $\frac{1}{i^{19}} = i^{-19} = i^{-20} \cdot i = \left(i^4\right)^{-5} \cdot i = 1^{-5} \cdot i = i$

18. **(a)** Minimum:

$1120 \frac{\text{gal}}{\text{min}} \cdot 60 \frac{\text{min}}{\text{hr}} \cdot 12 \frac{\text{hr}}{\text{day}} = 806{,}400 \frac{\text{gal}}{\text{day}}$

The equation that will calculate the minimum amount of water pumped after x days would be $A = 806{,}400x$.

(b) $A = 806{,}400x$ when $x = 30$ would be $A = 806{,}400(30) = 24{,}192{,}000$ gal.

(c) Because there would be $806{,}400 \frac{\text{gal}}{\text{day}}$ minimum and each pool requires 20,000 gal, there would be a minimum of $\frac{806{,}400}{20{,}000} = 40.32$ pools that could be filled each day. The equation that will calculate the minimum number of pools that could be filled after x days would be $P = 40.32x$. Approximately 40 pools could be filled each day.

(d) Solve $P = 40.32x$ where $P = 1000$.

$1000 = 40.32x \Rightarrow x = \frac{1000}{40.32} \approx 24.8$ days.

A minimum of 1000 pools could be filled in 25 days.

19. Let $w =$ width of rectangle. Then

$2w - 20 =$ length of rectangle.

Use the formula for the perimeter of a rectangle.

$P = 2l + 2w$

$620 = 2(2w - 20) + 2w$

$620 = 4w - 40 + 2w$

$620 = 6w - 40 \Rightarrow 660 = 6w \Rightarrow 110 = w$

The width is 110 m and the length is $2(110) - 20 = 220 - 20 = 200$ m.

20. Let x = amount of cashews (in pounds). Then $35 - x$ = amount of walnuts (in pounds).

	Cost per Pound	Amount of Nuts	Total Cost
Cashews	7.00	x	$7.00x$
Walnuts	5.50	$35 - x$	$5.50(35 - x)$
Mixture	6.50	35	$35 \cdot 6.50$

Solve the following equation.

$$7.00x + 5.50(35 - x) = 35 \cdot 6.50$$
$$7x + 192.5 - 5.5x = 227.5$$
$$1.5x + 192.5 = 227.5$$
$$1.5x = 35$$
$$x = \tfrac{35}{1.5} = \tfrac{350}{15} = \tfrac{70}{3} = 23\tfrac{1}{3}$$

The fruit and nut stand owner should mix $23\frac{1}{3}$ lbs of cashews with $35 - 23\frac{1}{3} = 11\frac{2}{3}$ lbs of walnuts.

21. Let x = average speed upriver.
Then $x + 5$ = average speed on return trip.

	r	t	d
Upriver	x	1.2	$1.2x$
Downriver	$x + 5$	0.9	$0.9(x + 5)$

Because the distance upriver and downriver are the same, we solve the following.

$1.2x = 0.9(x + 5)$

$1.2x = 0.9x + 4.5 \Rightarrow 0.3x = 4.5 \Rightarrow x = 15$

The average speed of the boat upriver is 15 mph.

22. $y = -0.461x + 6.32$

(a) The year 2014 is represented by $x = 10$.

$y = -0.461(10) + 6.32 = 1.71$

According to the model, in 2014 about 1.7% of college freshmen smoked.

(b) $4.9 = -0.461x + 6.32$

$-1.42 = -0.461x \Rightarrow 3.1 \approx x$

According to the model, 4.9% of freshman smoked about 3.1 years after 2004 or in 2007.

23. $s = -16t^2 + 96t$

(a) Let $s = 80$ and solve for t.

$80 = -16t^2 + 96t \Rightarrow 16t^2 - 96t + 80 = 0$

$t^2 - 6t + 5 = 0$

$(t - 1)(t - 5) = 0$

$t - 1 = 0 \Rightarrow t = 1$ or $t - 5 = 0 \Rightarrow t = 5$

The projectile will reach a height of 80 ft at 1 sec and 5 sec.

(b) Let $s = 0$ and solve for t.

$0 = -16t^2 + 96t$

$0 = -16t(t - 6)$

$t = 0$ or $t - 6 = 0 \Rightarrow t = 6$

The projectile will return to the ground at 6 sec.

24. $-2(x - 1) - 12 < 2(x + 1)$

$$-2x + 2 - 12 < 2x + 2$$
$$-2x - 10 < 2x + 2$$
$$-4x - 10 < 2$$
$$-4x < 12$$
$$x > -3$$

Solution set: $(-3, \infty)$

25. $-3 \le \frac{1}{2}x + 2 \le 3$

$$2(-3) \le 2\left(\frac{1}{2}x + 2\right) \le 2(3)$$
$$-6 \le x + 4 \le 6$$
$$-10 \le x \le 2$$

Solution set: $[-10, 2]$

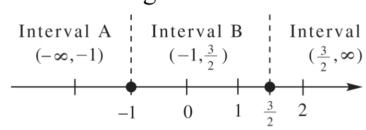
26. $2x^2 - x \ge 3$

Step 1: Find the values of x that satisfy $2x^2 - x = 3$.

$$2x^2 - x = 3$$
$$2x^2 - x - 3 = 0$$
$$(x + 1)(2x - 3) = 0$$

$x + 1 = 0 \Rightarrow x = -1$ or $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Step 2: The two numbers divide a number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $2x^2 - x \ge 3$

(*continued on next page*)

(*continued*)

Interval	Test Value	Is $2x^2 - x \ge 3$ True or False?
A: $(-\infty, -1)$	-2	$2(-2)^2 - (-2) \stackrel{?}{\ge} 3$ $10 \ge 3$ True
B: $\left(-1, \frac{3}{2}\right)$	0	$2 \cdot 0^2 - 0 \stackrel{?}{\ge} 3$ $0 \ge 3$ False
C: $\left(\frac{3}{2}, \infty\right)$	2	$2 \cdot 2^2 - 2 \stackrel{?}{\ge} 3$ $6 \ge 3$ True

Solution set: $(-\infty, -1] \cup \left[\frac{3}{2}, \infty\right)$

27. $\dfrac{x+1}{x-3} < 5$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{x+1}{x-3} < 5 \Rightarrow \frac{x+1}{x-3} - 5 < 0$$

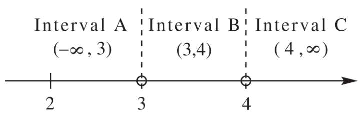
$$\frac{x+1}{x-3} - \frac{5(x-3)}{x-3} < 0 \Rightarrow \frac{x+1-5(x-3)}{x-3} < 0$$

$$\frac{x+1-5x+15}{x-3} < 0 \Rightarrow \frac{-4x+16}{x-3} < 0$$

Step 2: Determine the values that will cause either the numerator or denominator to equal 0.

$-4x + 16 = 0 \Rightarrow x = 4$ or $x - 3 = 0 \Rightarrow x = 3$

The values 3 and 4 divide the number line into three regions.



Step 3: Choose a test value to see if it satisfies the inequality, $\frac{x+1}{x-3} < 5$.

Interval	Test Value	Is $\frac{x+1}{x-3} < 5$ True or False?
A: $(-\infty, 3)$	0	$\frac{0+1}{0-3} \stackrel{?}{<} 5$ $-\frac{1}{3} < 5$ True
B: $(3, 4)$	3.5	$\frac{3.5+1}{3.5-3} \stackrel{?}{<} 5$ $9 < 5$ False
C: $(4, \infty)$	5	$\frac{5+1}{5-3} \stackrel{?}{<} 5$ $3 < 5$ True

Solution set: $(-\infty, 3) \cup (4, \infty)$

28. $|2x - 5| < 9$
$-9 < 2x - 5 < 9$
$-4 < 2x < 14$
$-2 < x < 7$
Solution set: $(-2, 7)$

29. $|2x + 1| - 11 \ge 0 \Rightarrow |2x + 1| \ge 11$

$2x + 1 \le -11$ or $2x + 1 \ge 11$
$2x \le -12$ $\quad$ $2x \ge 10$
$x \le -6$ or $x \ge 5$

Solution set: $(-\infty, -6] \cup [5, \infty)$

30. $|3x + 7| \le 0 \Rightarrow 3x + 7 \le 0 \Rightarrow x \le -\frac{7}{3}$

However, if $x < -\frac{7}{3}$, then $3x + 7 < 0$, and $|3x + 7|$ is not defined. Thus, the solution set of $|3x + 7| \le 0$ is $\left\{-\frac{7}{3}\right\}$.