FUNDAMENTALS OF SUPPLY CHAIN THEORY, 2ND EDITION: INSTRUCTOR'S MANUAL

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Lawrence V. Snyder
Lehigh University

Zuo-Jun Max Shen
University of California, Berkeley

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CHAPTER 2

FORECASTING AND DEMAND MODELING

PROBLEMS

2.1 (Forecasting without Trend) The (a) moving average and (b) exponential smoothing forecasts are given in the table below. The last row lists the forecasts for tomorrow's demand.

Day	Demand	Moving Average (a)	Exp. Smoothing (b)
1	4,804.9	—	4,804.9
2	4,285.0	—	4,804.9
3	3,764.6	—	4,752.9
4	2,486.8	—	4,654.1
5	3,012.2	—	4,437.4
6	2,896.9	—	4,294.8
7	1,985.1	—	4,155.0
8	3,437.0	3,319.4	3,938.0
9	3,345.7	3,123.9	3,887.9
10	1,841.3	2,989.8	3,833.7
11	2,114.6	2,715.0	3,634.5
12	1,803.6	2,661.8	3,482.5
13	2,678.7	2,489.2	3,314.6
14	2,070.5	2,458.0	3,251.0

(*cont'd*)

Day	Demand	Moving Average (a)	Exp. Smoothing (b)
15	2,645.5	2,470.2	3,133.0
16	3,292.6	2,357.1	3,084.2
17	3,844.0	2,349.5	3,105.1
18	4,901.8	2,635.6	3,178.9
19	3,206.5	3,033.8	3,351.2
20	3,362.6	3,234.2	3,336.8
21	2,466.2	3,331.9	3,339.3
22	1,048.5	3,388.5	3,252.0
23	1,431.3	3,160.3	3,031.7
24	2,574.3	2,894.4	2,871.6
25	3,310.7	2,713.0	2,841.9
26	4,415.4	2,485.7	2,888.8
27	2,919.6	2,658.4	3,041.4
28	3,905.5	2,595.1	3,029.3
29	1,332.8	2,800.8	3,116.9
30	1,969.5	2,841.4	2,938.5
		2,918.3	2,841.6

2.2 (Forecasting with Trend)

a) The double exponential smoothing calculations are given in the table below. The last row lists the forecast for next week's demand.

Week	Demand	I_t	S_t	y_t
1	646	646.00	—	—
2	683	683.00	37.00	—
3	708	717.60	36.76	720.00
4	761	755.69	36.89	754.36
5	787	791.46	36.78	792.58
6	809	824.40	36.40	828.25
7	856	859.83	36.30	860.79
8	892	895.31	36.22	896.13
9	944	934.02	36.47	931.53
10	991	974.59	36.88	970.49
11	1034	1,015.97	37.33	1,011.47
12	1091	1,060.84	38.08	1,053.30
13	1123	1,103.74	38.56	1,098.92
14	1144	1,142.64	38.60	1,142.30
15	1164	1,177.79	38.25	1,181.24
16	1186	1,210.04	37.65	1,216.04
17	1231	1,244.35	37.32	1,247.69
18	1255	1,276.33	36.78	1,281.67
19	1298	1,310.10	36.48	1,313.12
20	1337	1,344.66	36.29	1,346.58
21	1389	1,382.56	36.45	1,380.95
22	1436	1,422.41	36.79	1,419.01

(cont'd)

Week	Demand	I_t	S_t	y_t
23	1490	1,465.36	37.41	1,459.20
24	1528	1,507.82	37.91	1,502.77
25	1555	1,547.58	38.10	1,545.73
26	1613	1,591.14	38.64	1,585.68
				1,629.79

b) From (2.23) and (2.24), we have

$$A_{xy} = 1446029$$
$$A_{xx} = 38025.$$

Therefore, from (2.21) and (2.22), we have

$$\beta_1 = 38.03$$
$$\beta_0 = 600.12$$

Finally, y_{27} (the forecast for next week's demand) is

$$y_{27} = \beta_0 + 27\beta_1 = 1626.88.$$

2.3 (Forecasting Cupcake Sales)

a) We initialize the method with $I_1 = D_1 = 47.2$ and $S_1 = D_2 - D_1 = 5.1$. Then

$$y_2 = I_1 + S_1 = 52.3$$
$$I_2 = 0.1D_2 + 0.9(I_1 + S_1) = 52.3$$
$$S_2 = 0.2(I_2 - I_1) + 0.8S_1 = 5.1$$
$$y_3 = I_2 + S_2 = 57.4$$

b)

$$I_3 = 0.1D_3 + 0.9(I_2 + S_2) = 57.6$$
$$S_3 = 0.2(I_3 - I_2) + 0.8S_2 = 5.14$$
$$y_4 = I_3 + S_3 = 62.74$$

2.4 (Forecasting with Seasonality) The triple exponential smoothing calculations are given in the table below. The last row lists the forecast for May.

Week	Demand	I_t	S_t	c_t	y_t
1	96	96	—	0.0940	—
2	319	319	223	0.3119	—
3	405	405	86	0.3956	—
4	830	830	425	0.8111	—
5	874	874	44	0.8536	—
6	1,719	1,719	845	1.6792	—
7	2,797	2,797	1,077	2.7315	—

(*cont'd*)

Week	Demand	I_t	S_t	c_t	y_t
8	2,235	2,235	-562	2.1826	—
9	1,471	1,471	-764	1.4368	—
10	735	735	-736	0.7175	—
11	383	383	-351	0.3743	—
12	422	422	38	0.4118	—
13	144	674	38	0.1298	43.26
14	364	803	60	0.3544	222.19
15	692	1,040	67	0.4764	341.40
16	656	1,047	84	0.7556	897.74
17	1,223	1,191	76	0.9055	965.46
18	2,199	1,276	83	1.6926	2,128.25
19	3,530	1,345	83	2.6992	3,711.47
20	2,973	1,415	82	2.1580	3,117.92
21	2,099	1,490	81	1.4285	2,150.85
22	1,244	1,603	80	0.7351	1,126.76
23	525	1,627	83	0.3588	630.02
24	209	1,469	77	0.3309	704.30
25	356	1,786	54	0.1507	200.80
26	540	1,777	80	0.3393	651.90
27	770	1,809	71	0.4611	884.46
28	646	1,675	67	0.6446	1,420.37
29	1,355	1,693	47	0.8740	1,577.36
30	2,379	1,673	44	1.6114	2,945.12
31	3,946	1,666	38	2.5999	4,635.39
32	3,503	1,688	33	2.1332	3,677.41
33	2,723	1,758	32	1.4645	2,458.85
34	1,243	1,771	36	0.7253	1,316.27
35	499	1,724	34	0.3381	648.25
36	322	1,601	26	0.2921	581.53
37	238	1,617	11	0.1496	245.02
38	479	1,584	11	0.3282	552.26
39	630	1,550	7	0.4447	735.83
40	921	1,531	3	0.6317	1,003.46
		1,227	1	0.6118	1,340.59

2.5 **(Forecasting Melon Slicers)** Let $\alpha = 0.2$, $\beta = 0.3$, $\gamma = 0.1$.

a)

$$y_{13} = (I_{12} + S_{12})c_9 = 290.4$$

b)

$$I_{13} = 0.2\frac{D_{13}}{c_9} + (1-0.2)(I_{12} + S_{12}) = 751.3$$

$$S_{13} = 0.3(I_{13} - I_{12}) + (1-0.3)S_{12} = 91.59$$

$$c_{13} = 0.1\frac{D_{13}}{I_{13}} + (1-0.1)c_9 = 0.4054$$

2.6 (Forecasting Using Regression)

a) From standard regression analysis, we get

$$\beta_0 = 200.73$$
$$\beta_1 = 18.98$$

b) The forecasts for bottled water for the next three matches are

$$200.73 + 18.98 \times 21.6 = 610.64$$
$$200.73 + 18.98 \times 27.3 = 718.81$$
$$200.73 + 18.98 \times 26.6 = 705.53$$

2.7 (Multiple-Period-Ahead Forecasts)

a) Because moving average assumes the demand is stationary, the forecast for period $t + k$ is identical to that for period t; that is, $y_{t-1,t+k} = y_t$.

b) Double exponential smoothing assumes the demand follows a linear trend. The forecast for period $t + k$ just follows that trend, extended out k periods, using the current estimates for the base signal and slope:

$$y_{t-1,t+k} = I_{t-1} + (k + 1)S_{t-1}.$$

c) Linear regression can be used for multi-period-ahead forecasts without any modification; we simply set

$$y_{t-1,t+k} = \hat{\beta}_0 + (t + k)\hat{\beta}_1.$$

2.8 (Forecasting using Machine Learning Methods) Since there are only 38 data points in the dataset, we choose to fit a decision tree for this problem. As visualized in Figure S.2.1, this model will predict the demand as 598,721 and 707 for 21.6°, 27.3° and 26.6°.

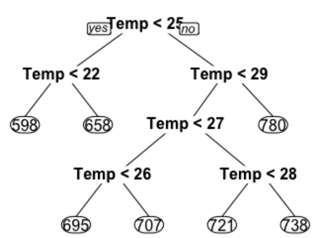


Figure S.2.1 A decision tree model

Corresponding R code stated as follows.

```
library(rpart)
library(rpart.plot)

# input data
```

```
data =  read.csv("bottled-water.csv")
# train a decision tree
mod = rpart(Demand ~ Temp ,
data = data, minbucket=3, cp = 0.001)
# plot the tree
prp(mod)
```

2.9 (Ridge Regression) We use the matrix representation in this solution. First let X denotes a $2 \times n$ matrix

$$\begin{bmatrix} x^1, \ldots, x^i, \ldots, x^n \\ 1, \ldots, 1, \ldots, 1 \end{bmatrix}$$

where each column consists of a sample x^i and 1 for intercept. Let $\beta = (beta_1, \beta_0))$ denote the vector of coefficient and $y = (y^1, \cdots, y^n)$ denote the vector of dependent variables. Therefore, under matrix representation, we would like to solve

$$\min_{\beta} \quad (y - \beta^T X)^T (y - \beta^T X) + \lambda \beta^T \beta \tag{S.2.1}$$

And the first order condition of β being optimal is

$$-2(y - \beta^T X)^T X + 2\lambda\beta = 0 \tag{S.2.2}$$
$$\Rightarrow -y^T X + X^T X \beta + \lambda\beta = 0 \tag{S.2.3}$$

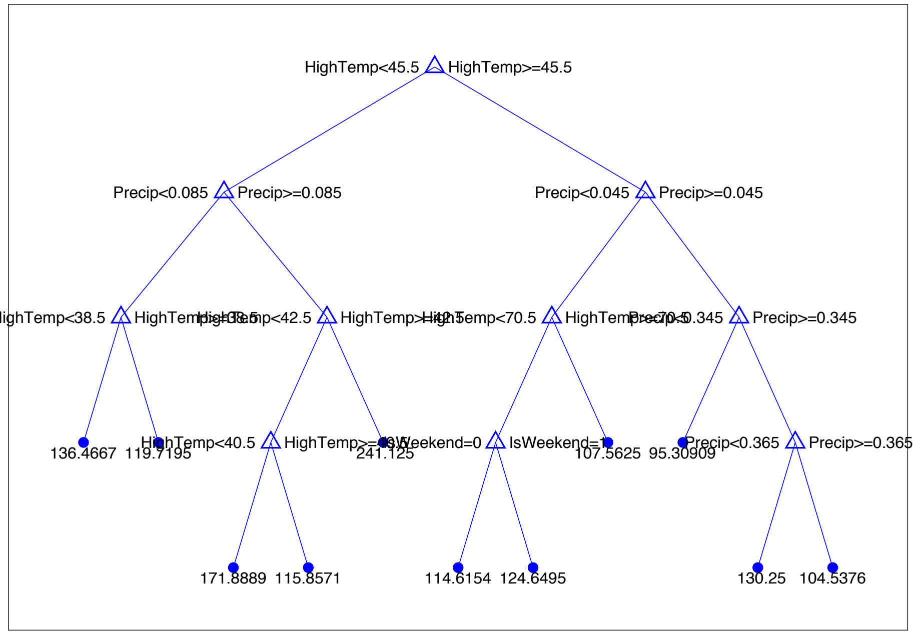
Therefore,

$$\beta = (X^T X + \lambda I)^{-1} y^T X \tag{S.2.4}$$

2.10 (Forecasting Fires)

a) Letting X_1 = high temperature, X_2 = precipitation, and X_3 = 1 if the day is a weekend day, we get $\hat{\beta}_0 = 151.9503$, $\hat{\beta}_1 = -0.6089$, $\hat{\beta}_2 = 2.0906$, $\hat{\beta}_3 = 7.5745$.

b)



c) $\beta_0 = 133.0427$, $\boldsymbol{\beta} = (-0.3426, -2.5369, -4.0248)$

d)

Date	High Temp	Precipitation	IsWeekend	Actual # Fires	Linear Regression Forecast	Linear Regression Error	Regression Tree Forecast	Regression Tree Error	SVR Forecast	SVR Error
1/1/16	42	0	0	142	126.38	-15.62	119.72	-22.28	114.63	-27.37
1/2/16	40	0	1	93	135.17	42.17	119.72	26.72	123.36	30.36
1/3/16	45	0	1	116	132.12	16.12	119.72	3.72	121.65	5.65
1/4/16	36	0	0	110	130.03	20.03	136.47	26.47	116.68	6.68
1/5/16	29	0	0	123	134.29	11.29	136.47	13.47	119.08	-3.92
1/6/16	41	0	0	123	126.98	3.98	119.72	-3.28	114.97	-8.03
1/7/16	46	0	0	114	123.94	9.94	114.62	0.62	113.26	-0.74
1/8/16	46	0	0	98	123.94	25.94	114.62	16.62	113.26	15.26
1/9/16	47	0	1	122	130.91	8.91	124.65	2.65	120.96	-1.04
1/10/16	59	1.8	1	92	127.36	35.36	104.54	12.54	112.29	20.29

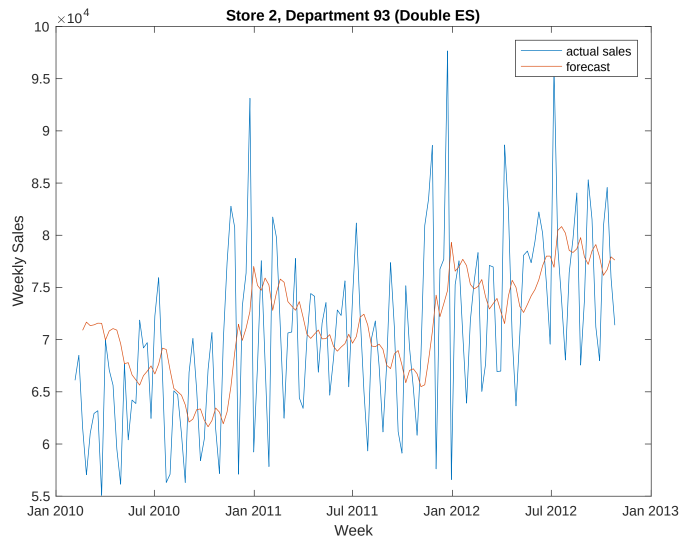
The MSE values are: 1.09×10^3 for linear regression, 1.36×10^3 for regression tree, and 1.09×10^3 for SVR.

2.11 (Exponential Smoothing for Retail Sales)

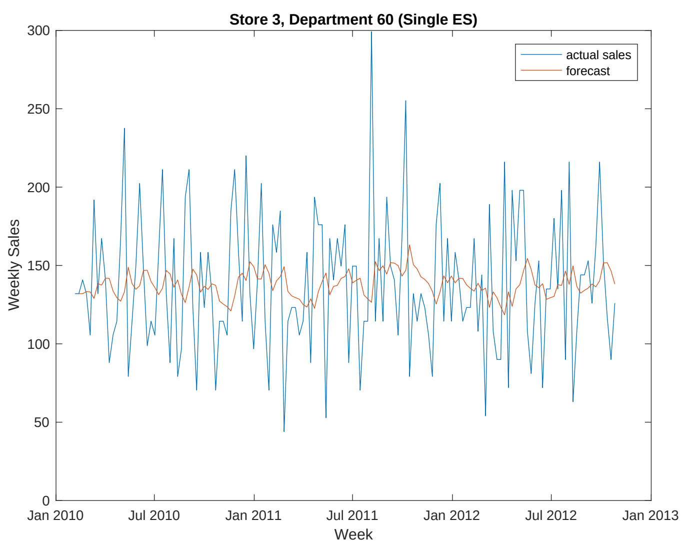
Store 2 Dept 93		Store 3 Dept 60		Store 1 Dept 16	
Week	Forecast	Week	Forecast	Week	Forecast
2/19/10	70,896.65	2/12/10	132.00	2/3/12	8,821.94
2/26/10	71,679.40	2/19/10	132.00	2/10/12	10,740.17
3/5/10	71,341.40	2/26/10	133.32	2/17/12	14,951.53
3/12/10	71,418.10	3/5/10	133.12	2/24/12	18,604.71
3/19/10	71,577.46	3/12/10	128.99	3/2/12	23,894.78
3/26/10	71,563.45	3/19/10	138.41	3/9/12	27,954.71
4/2/10	69,962.75	3/26/10	137.45	3/16/12	47,788.14
4/9/10	70,833.01	4/2/10	141.91	3/23/12	42,955.89
4/16/10	71,058.82	4/9/10	141.75	3/30/12	51,485.23
4/23/10	70,910.45	4/16/10	133.69	4/6/12	60,960.74
4/30/10	69,610.49	4/23/10	129.47	4/13/12	64,941.58
5/7/10	67,697.51	4/30/10	127.21	4/20/12	59,775.00
5/14/10	67,793.90	5/7/10	133.21	4/27/12	50,860.55
5/21/10	66,623.68	5/14/10	148.87	5/4/12	50,422.75
5/28/10	66,146.74	5/21/10	138.42	5/11/12	46,349.49
6/4/10	65,639.98	5/28/10	134.82	5/18/12	39,351.53
6/11/10	66,548.59	6/4/10	137.03	5/25/12	44,076.58
6/18/10	66,980.57	6/11/10	146.84	6/1/12	47,542.75
6/25/10	67,485.30	6/18/10	146.95	6/8/12	35,530.11
7/2/10	66,712.28	6/25/10	139.73	6/15/12	36,524.94

Error Measure	Store 2 Dept 93	Store 3 Dept 60	Store 1 Dept 16
MAD	6815.50	37.58	3367.55
MSE	7.2070×10^7	2234.72	2.2666×10^7
MAPE	32.07	9.86	11.38

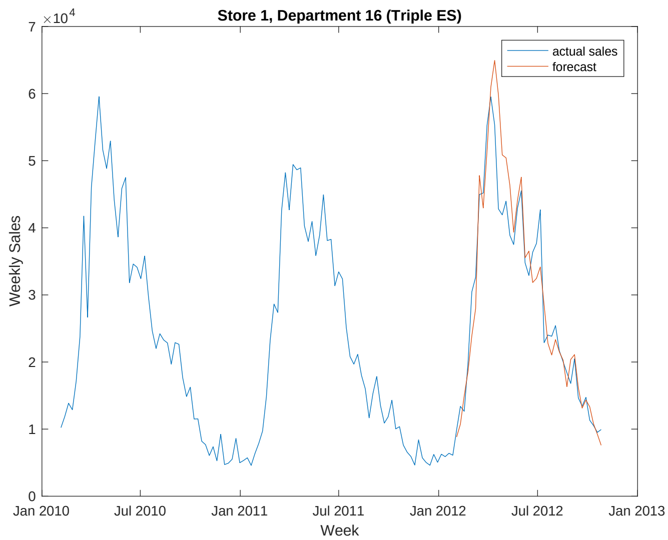
a) These data exhibit a trend, so double exponential smoothing is most appropriate. The first 20 forecast values are given in the first table above. The MAD, MSE, and MAPE are given in the second table. The actual and forecast sales are plotted below.



b) These data do not exhibit a trend or seasonality, so single exponential smoothing is most appropriate. The first 20 forecast values are given in the first table above. The MAD, MSE, and MAPE are given in the second table. The actual and forecast sales are plotted below.



c) These data exhibit seasonality, so triple exponential smoothing is most appropriate. The first 20 forecast values are given in the first table above. The MAD, MSE, and MAPE are given in the second table. The actual and forecast sales are plotted below.



2.12 (Mean and Variance of Exponential Smoothing Forecast Error)

$$
\begin{aligned}
\mu_e = \mathbb{E}[y_t - D_t] &= \mathbb{E}[y_t] - \mathbb{E}[D_t] \\
&= \mathbb{E}\left[\sum_{i=0}^{\infty} \alpha(1-\alpha)^i D_{t-i-1}\right] - \mu \\
&= 1 \cdot \mu - \mu \text{ (by (C.50)))} \\
&= 0 \\
\sigma_e = \sqrt{\text{Var}[y_t - D_t]} &= \sqrt{\text{Var}[y_t] + \text{Var}[D_t]} \\
&= \sqrt{\text{Var}\left[\sum_{i=0}^{\infty} \alpha(1-\alpha)^i D_{t-i-1}\right] + \sigma^2} \\
&= \sqrt{\sum_{i=0}^{\infty} \alpha^2(1-\alpha)^{2i}\sigma^2 + \sigma^2} \\
&= \sqrt{\frac{\alpha^2\sigma^2}{1-(1-\alpha)^2} + \sigma^2} \\
&= \sqrt{\frac{\alpha^2\sigma^2 + \sigma^2\left(1-(1-\alpha)^2\right)}{1-(1-\alpha)^2}} \\
&= \sigma\sqrt{\frac{2}{2-\alpha}}.
\end{aligned}
$$

2.13 (Forecasting Simulation)

a) Our simulation resulted in MSE = 41.10 and MAD = 5.12. The standard deviation of the forecast error is 6.42. The approximation in (2.28) is $1.25\text{MAD} = 6.40$, which is quite close to the actual value.

b) MSE = 37.30, MAD = 4.86, standard deviation of forecast error = 6.11, 1.25MAD = 6.08, which is again quite close.

c) Since the MSE and MAD are smaller for exponential smoothing, exponential smoothing appears to work slightly better for this data set.

2.14 **(Bass Diffusion for LPhone)** The peak demand occurs at time

$$t = \frac{1}{p+q} \ln\left(\frac{q}{p}\right) = 9.238.$$

The cumulative sales by then is given by

$$\frac{m(q-p)}{2q} = 2.845 \text{ million}.$$

2.15 **(Bass Diffusion for iPeel)**

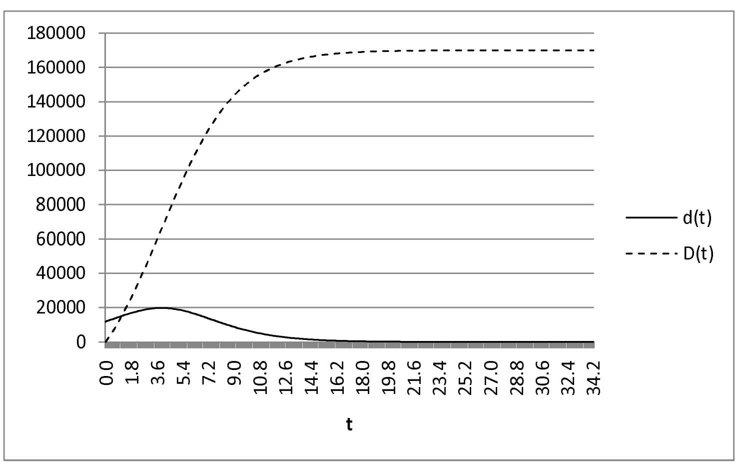
a) The parameters are $m = 170,000$, $p = 0.07$, and $q = 0.31$. From Corollary 2.2, we have

$$t^* = 3.9160$$
$$d(t^*) = 19796.77$$
$$D(t^*) = 65806.45$$

Therefore, sales will peak at year 3.9160, i.e., on November 30 of the third year after sales begin. At that time, the demand rate will be 19796.77 and the total sales will be 65806.45.

b) The reader can verify that $D(t) = 170,000 \times 0.9 = 153,000$ when $t = 10.2873$, i.e., on April 12 of the tenth year after sales begin.

c) The plot is given below.



2.16 **(Bass Diffusion for Books)** Standard regression analysis gives estimates of a, b, and c in

$$d_t = a + bD_{t-1} + c(D_{t-1})^2$$

as

$$
\begin{aligned}
a &= 227.5654 \\
b &= 0.1361 \\
c &= -0.000009715.
\end{aligned}
$$

Then from (2.51)–(2.53), we have

$$
\begin{aligned}
m &= 15513.1268 \\
p &= 0.0147 \\
q &= 0.1507.
\end{aligned}
$$

2.17 (Proof of Corollary 2.2) The derivative of $d(t)$ from (2.44) is given by

$$d'(t) = \frac{mp(p+q)^3(p+qe^{-(p+q)t})e^{-(p+q)t}[-(p+qe^{-(p+q)t})+2qe^{-(p+q)t}]}{(p+qe^{-(p+q)t})^4}.$$

If $t = \frac{1}{p+q}\ln\left(\frac{q}{p}\right)$, then $e^{-(p+q)t} = \frac{p}{q}$. Then $d'(t) = 0$ since

$$-(p+qe^{-(p+q)t}) + 2qe^{-(p+q)t} = -\left(p + q\frac{p}{q}\right) + 2q\frac{p}{q} = 0.$$

2.18 (Influentials and Imitators) [*This problem is adapted from Ho et al. (2011).*]

We use the subscripts 1 and 2 to denote each type (1 = influentials, 2 = imitators) and subscript m to denote the total population.

a) Each type's instantaneous adoption behavior is captured by the following functions:

$$d_1(t) = (p_1 + q_1 D_1(t))(\theta - D_1(t)) \tag{S.2.5}$$
$$d_2(t) = (p_2 + q_c D_1(t) + q_2 D_2(t))(\bar{\theta} - D_2(t)) \tag{S.2.6}$$
$$d_m(t) = d_1(t) + d_2(t) \tag{S.2.7}$$

Parameters p_i and q_i ($i = 1, 2$) are Type i's within-segment innovation and imitation parameters. As Type 2's adoption behavior can also be influenced by Type 1, we use q_c to denote the cross-segment imitation parameter. Equation (S.2.5) means that an influential's likelihood of adopting at time t, conditioned on no adoption in the past, is determined by her intrinsic motivation and the within-segment social influence at that time. Equation (S.2.6) tells us that an imitator's likelihood of buying at t given that she didn't adopt in the past depends on her intrinsic motivation as well as the social contagion of both the influentials segment and the imitators segment at that time.

b) When $\theta = 0$ or $\theta = 1$, all customers fall into a single segment and the model is reduced to the traditional Bass diffusion model. When $0 < \theta < 1$ and $q_c = 0$, customers of different types are disconnected, and the diffusion process of each type independently experiences its own Bass-type contagion process.

c) If there are no pre-release purchases (i.e., $D_1(0) = D_2(0) = 0$), the cumulative adoption at t can be written as:

$$
\begin{aligned}
D_1(t) =& \frac{1 - e^{-(p_1+q_1\theta)t}}{\frac{1}{\theta} + \frac{q_1}{p_1} e^{-(p_1+q_1\theta)t}} && \text{(S.2.8)} \\
A =& \exp\left(-q_2\bar{\theta}t - p_2 t - \frac{q_c}{q_1} \ln\left(\frac{1}{\theta} + \frac{q_1}{p_1} e^{-(p_1+q_1\theta)t} \right) \right. \\
& \left. + \frac{\theta q_c}{p_1 + q_1\theta} \ln\left(\frac{q_1}{p_1} e^{-(p_1+q_1\theta)t} \right) \right) && \text{(S.2.9)} \\
B =& q_2 \left(\frac{q_1}{p_1} \right)^{\frac{\theta q_c}{p_1+\theta q_1}} \int_0^t e^{-(q_2\bar{\theta}+p_2+\theta q_c)s} \left(\frac{1}{\theta} + \frac{q_1}{p_1} e^{-(p_1+q_1\theta)s} \right)^{-\frac{q_c}{q_1}} ds && \text{(S.2.10)} \\
C =& -\left(\frac{1}{\theta} + \frac{q_1}{p_1} \right)^{-\frac{q_c}{q_1}} \left(\frac{q_1}{p_1} \right)^{\frac{\theta q_c}{p_1+q_1\theta}} \bar{\theta} && \text{(S.2.11)} \\
D_2(t) =& \bar{\theta} + \frac{A}{B+C} && \text{(S.2.12)} \\
D_m(t) =& D_1(t) + D_2(t) && \text{(S.2.13)}
\end{aligned}
$$

2.19 (Demand Diffusion Across Multiple Markets) [*This problem is adapted from Wu et al. (2009).*]

This lifecycle demand can be expressed as a bell-shaped time-series curve or as a cumulative curve, in which each point on the curve represents the percentage of lifecycle demand satisfied up to that time. We use $D(T+\tau)$ to denote the cumulative percentage of total market demand that has been observed by time $T+\tau$, given that actual demand observations up to time T, $\Theta(T) = D(1), D(2), \ldots, D(T)$, are available.

Let K be the set of different diffusion models that are used in forecasting ($k \in K$). Let $\hat{D}_k(T+\tau|\Theta(T))$ denote an estimate of cumulative demand percentage observed by time $T+\tau$, projected by diffusion model k. Then

$$D(T+\tau) = \hat{D}_k(T+\tau|\Theta(T)) + \varepsilon(T+\tau|\Theta(T)).$$

We are interested to know if combining diffusion models derived from different vertical markets would help in improving the overall market forecast. Given a particular diffusion model, the actual cumulative demand at $T+\tau$, $D_k(T+\tau)$, can be represented as a normally distributed random variable (assuming normally distributed fitting errors). With the combination of multiple diffusion models to forecast demands τ-period ahead, $\hat{D}(T+\tau)$, the combined forecast is a linear combination of independent normal random variables $\hat{D}_k(T+\tau|\Theta(T))$, it is also normally distributed with mean $\sum_{k\in K} w_k \hat{D}_k(T+\tau|\Theta(T))$ and variance $\sum_{k\in K} w_k^2 \sigma_k^2$, where w_k is the weight assigned to model k's forecast by the combination method. Note that combined forecast's variance is:

$$\sigma_c^2 = \sum_{k\in K} \left(\frac{1/\sigma_k^2}{\sum_{i\in K} 1/\sigma_i^2} \right)^2 \sigma_k^2 = \sum_{k\in K} \frac{1}{\sigma_k^2 (\sum_{i\in K} 1/\sigma_i^2)^2} = \frac{1}{\sum_{i\in K} 1/\sigma_i^2} < \sigma_k^2,$$

for all $k \in K$.

2.20 (Leading Indicators)

a) The ρ_{ik} values are given in the table below. The (i,k) pairs for which $\rho_{ik} \geq \rho_{\min} = 0.85$ are $(5,5)$, $(7,4)$, $(7,5)$, and $(7,6)$.

Product	$k = 3$	4	5	6	7	8	9
1	0.7306	0.6625	0.4829	0.1950	−0.1499	−0.4654	−0.6842
2	−0.3573	−0.4878	−0.5728	−0.6191	−0.6283	−0.5509	−0.3432
3	0.5031	0.2981	0.0420	−0.2302	−0.5298	−0.7759	−0.8717
4	0.5315	0.7005	0.8032	0.7579	0.5760	0.3223	0.0548
5	0.6332	0.7833	0.8580	0.8486	0.7687	0.5857	0.3604
6	−0.5133	−0.5585	−0.5678	−0.5429	−0.4740	−0.3057	0.0000
7	0.7918	0.8700	0.9119	0.8573	0.6326	0.3045	0.0757
8	−0.6116	−0.5698	−0.4908	−0.3899	−0.2762	−0.1409	0.0000
9	−0.4198	−0.5885	−0.6871	−0.7131	−0.6909	−0.5831	−0.3718
10	0.6804	0.5132	0.2625	−0.0548	−0.4028	−0.6986	−0.8594
11	0.2557	0.0519	−0.1817	−0.3911	−0.5841	−0.7489	−0.7987
12	0.6836	0.7289	0.6964	0.5485	0.2975	−0.0019	−0.2556
13	−0.4310	−0.3536	−0.2119	−0.0213	0.1673	0.2957	0.3566
14	0.1039	−0.1601	−0.4077	−0.6302	−0.8229	−0.9008	−0.8281
15	0.1927	−0.0886	−0.3620	−0.5990	−0.8108	−0.9210	−0.8783
16	−0.3664	−0.5326	−0.6537	−0.7199	−0.7407	−0.6850	−0.5272
17	−0.0161	0.2274	0.3666	0.3851	0.3850	0.4555	0.5434
18	0.5056	0.6522	0.7573	0.7921	0.6930	0.4625	0.1693
19	0.6174	0.6906	0.6889	0.5715	0.3498	0.0708	−0.1751
20	0.5941	0.4279	0.1844	−0.1241	−0.4533	−0.7112	−0.8356
21	−0.6465	−0.5978	−0.5000	−0.3713	−0.2374	−0.1078	0.0000
22	0.7444	0.6189	0.4086	0.1445	−0.1511	−0.4366	−0.6487
23	−0.0360	0.2008	0.4008	0.4606	0.4447	0.4773	0.5297
24	0.0695	−0.1474	−0.3529	−0.5636	−0.7773	−0.8901	−0.8353
25	−0.6564	−0.5713	−0.4386	−0.2758	−0.1219	0.0000	0.0000

b) We'll use $k = 5$, $i = 5$. The regression model in (2.54) yields $\beta_0 = 63985.00$, $\beta_1 = 11.65$. Then we get the following forecasts:

$$\tilde{D}_{27}^{-5} = \beta_0 + \beta_1 D_{5,22} = 64019.96$$
$$\tilde{D}_{28}^{-5} = \beta_0 + \beta_1 D_{5,23} = 64008.30$$

2.21 (Discrete Choice with Uniform Errors) ϵ_{ni} has pdf and cdf

$$f(x) = \frac{1}{2}$$
$$F(x) = \frac{x+1}{2}$$

Then

$$\begin{aligned} P_{ni} &= P\left(\epsilon_{nj} < V_{ni} + \epsilon_{ni} - V_{nj}\right) \\ &= \frac{V_{ni} + \epsilon_{ni} - V_{nj} + 1}{2} \\ \implies P_{ni}|\epsilon_{ni} &= \prod_{j \neq i} \frac{V_{ni} + \epsilon_{ni} - V_{nj} + 1}{2} \end{aligned}$$

$$\implies P_{ni} = \int_{-1}^{1} \prod_{j \neq i} \frac{V_{ni} + \epsilon_{ni} - V_{nj} + 1}{2} \cdot \frac{1}{2} \cdot d\epsilon_{ni}$$

2.22 (Discrete Choices for Day Care)

$$\frac{e^{-0.45PP_A - 0.23D_A}}{e^{-0.45PP_A - 0.23D_A} + e^{-0.45PP_B - 0.23D_B}}$$

2.23 (Using Discrete Choice to Forecast Movie Sales)

a) The table in the problem gives V_{ni} values, where i represents the movie (or no movie at all for $i = 0$) and n represents the age range. Using (2.62), we can calculate the following table of P_{ni} values:

	Age Range		
Movie	16–25	26–35	36+
Prognosis Negative	0.219	0.279	0.286
Rochelle, Rochelle	0.287	0.287	0.256
Sack Lunch	0.299	0.221	0.225
No movie	0.194	0.213	0.232

Therefore the expected demand for *Prognosis Negative* is

$$0.219 \cdot 700 + 0.279 \cdot 1900 + 0.286 \cdot 1150 = 1012.213.$$

The expected demand for *Rochelle, Rochelle* is

$$0.287 \cdot 700 + 0.287 \cdot 1900 + 0.256 \cdot 1150 = 1041.623.$$

The expected demand for *Sack Lunch* is

$$0.299 \cdot 700 + 0.221 \cdot 1900 + 0.225 \cdot 1150 = 888.958.$$

b) We have the following table of P_{ni} values:

	Age Range		
Movie	16–25	26–35	36+
Prognosis Negative	0.164	0.320	0.323
Rochelle, Rochelle	0.366	0.337	0.266
Sack Lunch	0.396	0.183	0.198
No movie	0.075	0.160	0.214

Therefore the expected demand for *Prognosis Negative* is

$$0.164 \cdot 700 + 0.320 \cdot 1900 + 0.323 \cdot 1150 = 1093.380.$$

The expected demand for *Rochelle, Rochelle* is

$$0.366 \cdot 700 + 0.337 \cdot 1900 + 0.266 \cdot 1150 = 1202.267.$$

The expected demand for *Sack Lunch* is

$$0.396 \cdot 700 + 0.183 \cdot 1900 + 0.198 \cdot 1150 = 852.991.$$

2.24 (Proof of (2.62)) From (2.61),

$$P_{ni} = \int \left(\prod_{j \neq i} e^{-e^{-(\epsilon_{ni} + V_{ni} - V_{nj})}} \right) e^{-\epsilon_{ni}} e^{-e^{-\epsilon_{ni}}} d\epsilon_{ni}.$$

Define $u_i \equiv e^{-\epsilon_{ni}}$ and $v_i \equiv e^{-u_i}$, then

$$P_{ni} = \int \left(\prod_{j \neq i} v_i e^{-(V_{ni} - V_{nj})} \right) dv_i \tag{S.2.14}$$

$$= \int \left(v_i^{\sum_{j \neq i} e^{-(V_{ni} - V_{nj})}} \right) dv_i \tag{S.2.15}$$

$$= \frac{1}{\sum_{j \neq i} e^{-(V_{ni} - V_{nj})+1}} v_i^{\sum_{j \neq i} e^{-(V_{ni} - V_{nj})+1}} \Bigg|_0^1 \tag{S.2.16}$$

Notice that v_i goes from 0 to 1 when ϵ_{ni} goes from $-\infty$ to $+\infty$.
Thus,

$$\begin{aligned} P_{ni} &= \frac{1}{\sum_{j \neq i} e^{-(V_{ni} - V_{nj})+1}} \\ &= \frac{1}{\sum_j e^{-(V_{ni} - V_{nj})}} \\ &= \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}. \end{aligned}$$